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# Thursday 14 June 2012 – Morning

## A2 GCE MATHEMATICS (MEI)

**4754B** Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the Question Paper.

**OCR supplied materials:**
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
- Scientific or graphical calculator
- Rough paper

**Duration:** Up to 1 hour

| Candidate forename | | Candidate surname | |
|---|---|---|---|

| Centre number | | | | | | Candidate number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

### INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and to do some calculations as you read the passage.
- You are **not** required to hand in these notes with your Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

**1** Use Fig. 4 to estimate the number of 50–54 year olds in the UK in 2001. (These were born in the post World War 2 baby boom.) **[1]**

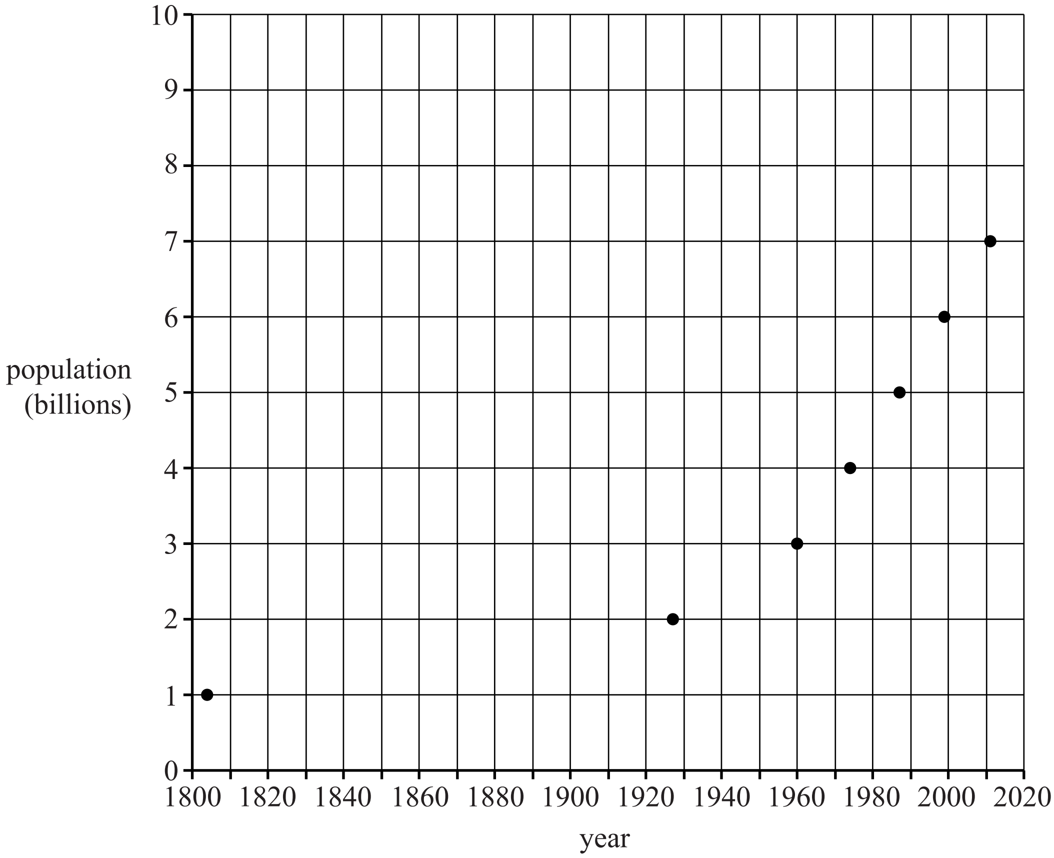
| | |
|---|---|
| **1** | |
| | |
| | |

**2** A copy of Fig. 2 is given below.

**(i)** Join the points with a curve and hence estimate the rate of population growth in the year 1927 in people per year. **[3]**

**(ii)** Estimate this rate as a percentage of the population at that time. **[2]**

**2 (i)**




| | |
|---|---|
| **2 (ii)** | |
| | |
| | |
| | |

**3** **(i)** In line 21, the solution of the differential equation $\frac{\mathrm{d}p}{\mathrm{d}t} = kp$ is stated to be $p = p_0 \mathrm{e}^{kt}$.

Use integration to derive this result. **[3]**

**(ii)** The article then goes on to say

"If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

1804 $t = 0,\ p = p_0 = 10^9,$

1927 $t = 123,\ p = 2\times10^9,$

and these correspond to $k = 0.005\,63\ldots$ ."

Show how this value of $k$ is obtained. **[2]**

| | |
|---|---|
| **3 (i)** | |
| **3 (ii)** | |

**4** In Table 6, the population profile of an imaginary country was predicted. Complete the table subject to the same general assumptions except that, after 2010:

- the average number of children per female is 2.2;
- 60% of those in the 40–59 age group survive into the 60–79 age group;
- 20% of those in the 60–79 age group survive into the 80+ age group. **[3]**

**4**

| **Age group** | **2010** | **2030** | **2050** | **2070** |
|---|---|---|---|---|
| **80+** | 1 | | | |
| **60–79** | 10 | | | |
| **40–59** | 20 | | | |
| **20–39** | 20 | | | |
| **0–19** | 20 | | | |
| **Total** | **71** | | | |

As in Table 6, the figures are in millions.

**5** In constructing Table 6, some assumptions were made about the proportion of people surviving from one age group to the next. Use Table 6 to find

**(i)** the proportion of people in the 40–59 age group surviving into the 60–79 age group, **[1]**

**(ii)** the proportion of those in the 60–79 age group surviving into the 80+ age group. **[1]**

**5 (i)**

**5 (ii)**

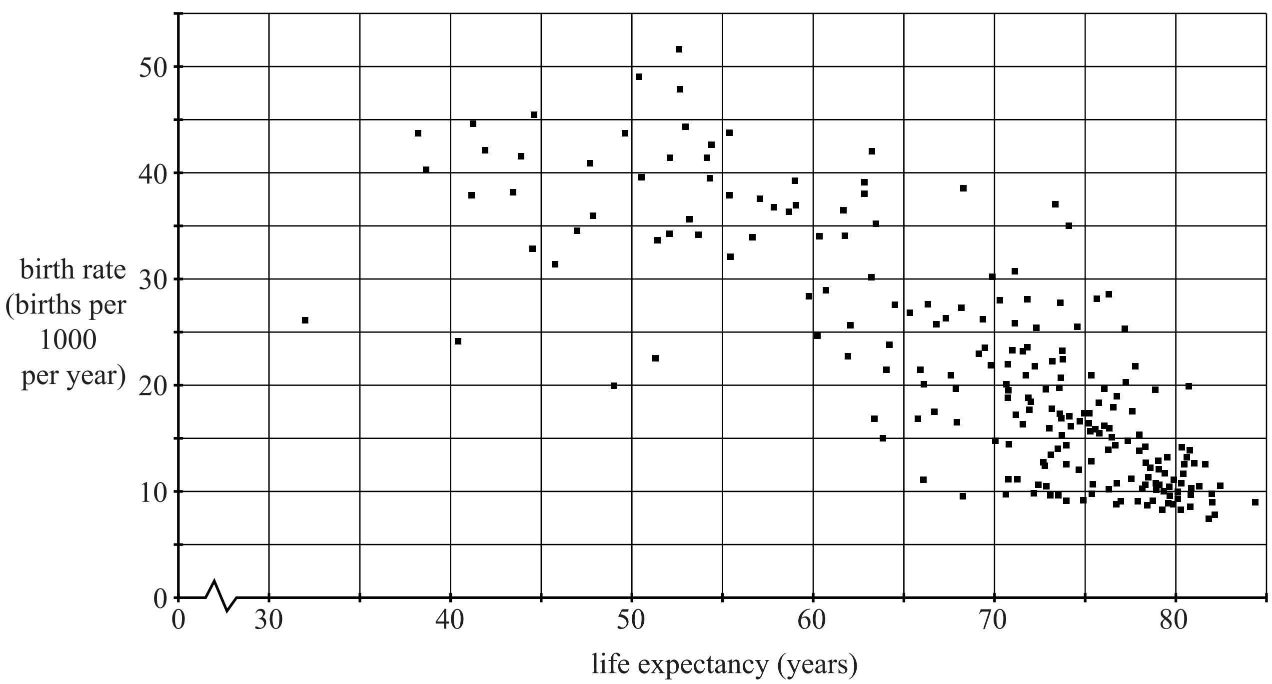
6 This table refers to the UK. It gives life expectancy and birth rate every 20 years from 1901 to 2001.

| Year | Life expectancy | Birth rate (births / 1000) |
|---|---|---|
| **1901** | 47 | 28.5 |
| **1921** | 58 | 22.7 |
| **1941** | 64 | 14.5 |
| **1961** | 71 | 17.8 |
| **1981** | 74 | 12.9 |
| **2001** | 78 | 12.0 |

Explain how these data relate to the conclusions of the article. **[2]**

[A copy of Fig. 7 is given below. You do not need to use it but may find it helpful.]

| **6** | |
|---|---|
| | |
| | |
| | |
| | |



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**INSERT**

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**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

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# The world population

## Population pressure on our planet

During the last 200 years, the human population has increased by a factor of about 7. Table 1 gives the years when it reached 1, 2, 3 and so on billions of people, where 1 billion is $10^9$.

| Year | 1804 | 1927 | 1960 | 1974 | 1987 | 1999 | 2011 |
|---|---|---|---|---|---|---|---|
| Population (billions) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

**Table 1 World population**

The increase in population is placing rising demands on the resources of our planet and on the whole eco-system that supports us. This raises very important questions.

- Is the world's population going to continue to increase indefinitely or will there be a limit?
- Will the world's population reach a level that the planet is unable to support?
- Should we be taking measures to restrict the world's population, and if so what?

The first two of these questions require mathematical modelling of the situation. The third involves political and ethical decisions which should be informed by that modelling.

The modelling involved is complicated; this article introduces some of the issues involved.

## The exponential model

A simple mathematical model is that the world's population is increasing at a rate which is directly proportional to its existing size,

$$\frac{dp}{dt} = kp ,$$

where $p$ is the number of people,
$t$ is time, measured in years,
$k$ is a constant.

The solution of this differential equation is

$$p = p_0 e^{kt}$$

where $p_0$ is the population at the time from which $t$ is measured.

If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

$$1804 \qquad t = 0,\ p = p_0 = 10^9,$$
$$1927 \qquad t = 123,\ p = 2\times10^9,$$

and these correspond to $k = 0.005\,63\ldots$ .

With this value of $k$, this model would predict that the population in 2011 would be 3.2 billion but in fact it was 7 billion. This model, based on the first two data points, is clearly not suitable.

In fact no exponential model fits the data in Table 1 well. You can see this just by looking at the graph of the data in Fig. 2. The graph of an exponential function is a curve which gets steeper and steeper but for the last 50 years this graph is virtually a straight line, indicating a constant rate of growth.

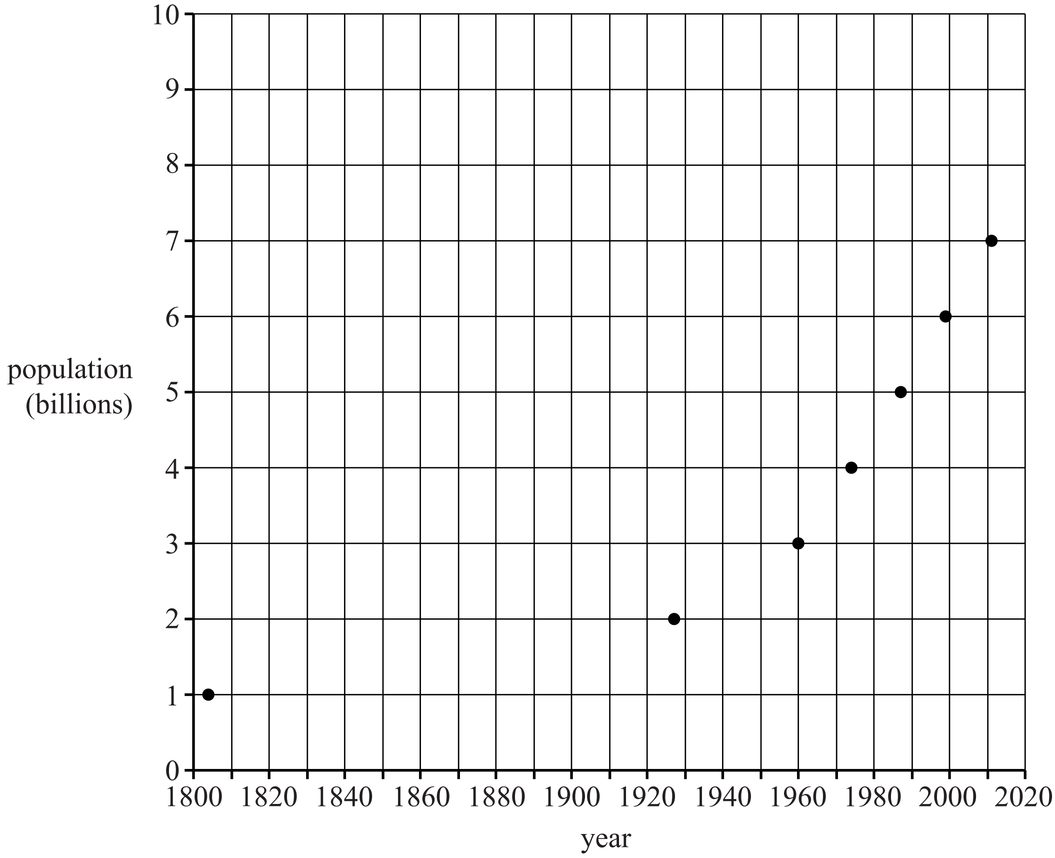



**Fig. 2 World population from 1800 to the present**

**The logistic model**

A standard mathematical model for a population which increases towards a limiting value of $m$ is given by the differential equation

$$\frac{\mathrm{d}p}{\mathrm{d}t} = kp(m-p) .$$

This is known as the *logistic equation*. A typical solution curve is shown in Fig. 3.

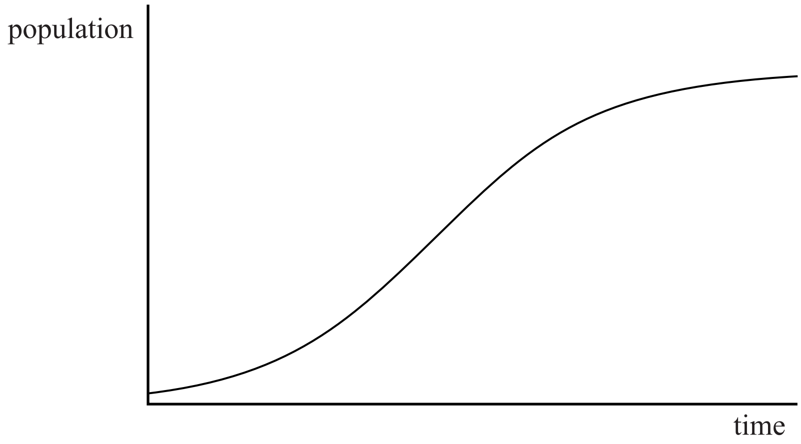



**Fig. 3 The logistic model**

While this looks as though it starts with the same sort of shape as a curve through the data points in Fig. 2, the resemblance is only superficial; it is not actually possible to find values of $k$ and $m$ that produce anything like a good fit. So this model is also unsatisfactory.

Like the previous model, this is an attempt to find a simple, neat solution to a very complicated problem. A different approach is needed and a starting point is provided by population profiles.

**Population profiles**

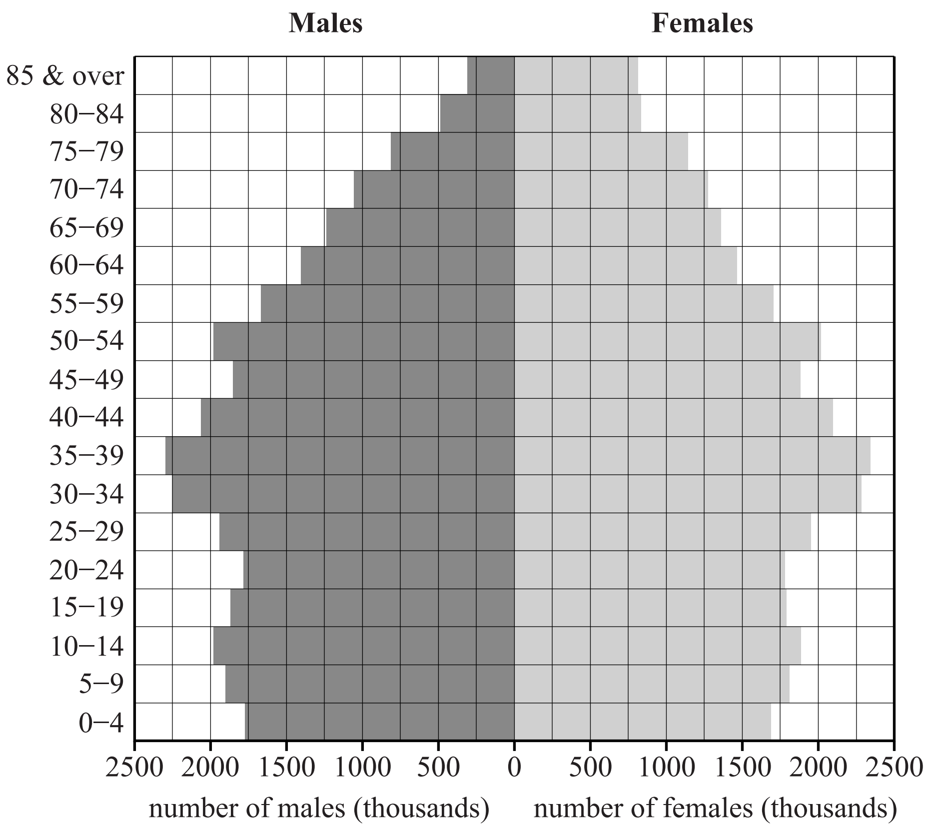



**Fig. 4 Population profile of the UK in 2001**

Population profiles are often illustrated by population pyramids, like that in Fig. 4. The lengths of the horizontal bars indicate the numbers of males and females in the UK population in 2001, in 5-year age intervals. In this case, the numbers on the horizontal scale are in thousands. Those on the vertical scale refer to age in completed years so that, for example, 10–14 means from 10 years 0 days to 14 years 364 days.

The UK population profile shows that in 2001 the number of children in the 0–4 age range was among the lowest for 50 years. Because there are fewer people in that age group, they in turn can be expected to have fewer children.

The shapes of the population profiles vary considerably between countries. In some countries the profiles have very wide bases, indicating large numbers of children.

It is worth noting that population figures for the UK are often affected by emigration and immigration.

**Modelling using population profiles**

If the individual population profiles of the large number of countries in the world are combined, a profile for the whole world emerges. It is possible to predict the changes in any country's profile in the years ahead, and hence the changes in the world's population. Each country is different and so needs to be looked at separately before combining the profiles.

The following model, for an imaginary country, is designed to highlight the key factors. Table 5 illustrates its profile in 2010 and part of that for 2030.

| Age group | 2010 | 2030 |
|---|---|---|
| **80+** | 1 | ? |
| **60–79** | 10 | ? |
| **40–59** | 20 | 20 |
| **20–39** | 20 | 20 |
| **0–19** | 20 | ? |
| **Total** | **71** | **?** |

**Table 5 Population profile of an imaginary country (in millions)**

In 2010, this country has a stable population with the same numbers in the youngest three age groups, up to the age of 60; however, life expectancy is quite low with very few people reaching the age of 80.

Two of the figures for 2030 have also been filled in. The 20 million people in the 0–19 age group in 2010 will move into the 20–39 group. Similarly those in the 20–39 group will move into the 40–59 group. (It is assumed, for simplicity, that no one in these age groups dies.) What will the other figures for 2030 be?

Two different factors are involved: the birth rate and the life expectancy.

The 2010 profile in Table 5 was constructed using a number of assumptions:

- that those in the 0–19 age group are all children of females in the 20–39 group;
- that 50% of those in the 20–39 age group are female;
- that on average each female has 2 children;
- that there is no immigration or emigration.

While these assumptions are obviously somewhat artificial, particularly with regard to the age at which women have children, they are good enough to demonstrate the key features of a country's population.

Throughout the world, life expectancy is rising. The proportion of the population in Table 5 who reach the age of 80 could be expected to increase.

In Table 6, the population profile of the country in Table 5 is predicted for the next 100 years, on the basis of the following new assumptions about the birth rate and life expectancy.

- Every 20 years, each group of people moves up a level.
- The average figure of 2 children per female is assumed to fall to 1.8, from 2010 onwards.
- The proportion of those in the 40–59 age group surviving into the 60–79 group increases from the 2010 figure of 50%; similarly there is an increase in survival from the 60–79 group into the 80+ group.

The figures used in these assumptions have been chosen to illustrate the modelling process. Their use does not mean that they will actually apply to the population of any real country.

| Age group | 2010 | 2030 | 2050 | 2070 | 2090 | 2110 |
|---|---|---|---|---|---|---|
| **80+** | 1 | 4 | 6.4 | 6.4 | 6.4 | 5.76 |
| **60–79** | 10 | 16 | 16 | 16 | 14.4 | 12.96 |
| **40–59** | 20 | 20 | 20 | 18 | 16.2 | 14.58 |
| **20–39** | 20 | 20 | 18 | 16.2 | 14.58 | 13.12 |
| **0–19** | 20 | 18 | 16.2 | 14.58 | 13.12 | 11.81 |
| **Total** | **71** | **78** | **76.6** | **71.18** | **64.7** | **58.23** |

**Table 6 Population profile of an imaginary country (in millions, to 4 significant figures)**

The figures in Table 6 show the total population rising quite sharply to a maximum and then reducing, initially rather slowly but then more quickly. Patterns like this are observed in many countries. In some, like Japan, the population has passed its maximum and is declining. In most, however, it is still increasing and consequently the population of the world as a whole is still increasing.

An important feature of Table 6 is that it is based on a low birth rate of 1.8 children per female. In many countries the birth rate is much higher than the stable level of 2 children per female.

**Birth rate and life expectancy**

Two key factors that determine the change in a country's population have been identified as its birth rate and its life expectancy. Data show that these are closely associated. Countries with high birth rates tend to have low life expectancy and those with low birth rates have high life expectancy. This is illustrated in Fig. 7 for all 221 countries; the data were drawn from the CIA World Factbook for 2009.

Notice that in Fig. 7 the birth rate is the number of births per 1000 of the population per year. It is thus a different measure from that used so far in this article which is mean births per female over her lifetime. So, for example, a country with a population of 80 million people and 1.2 million births per year has a birth rate of 15 births per 1000 per year.

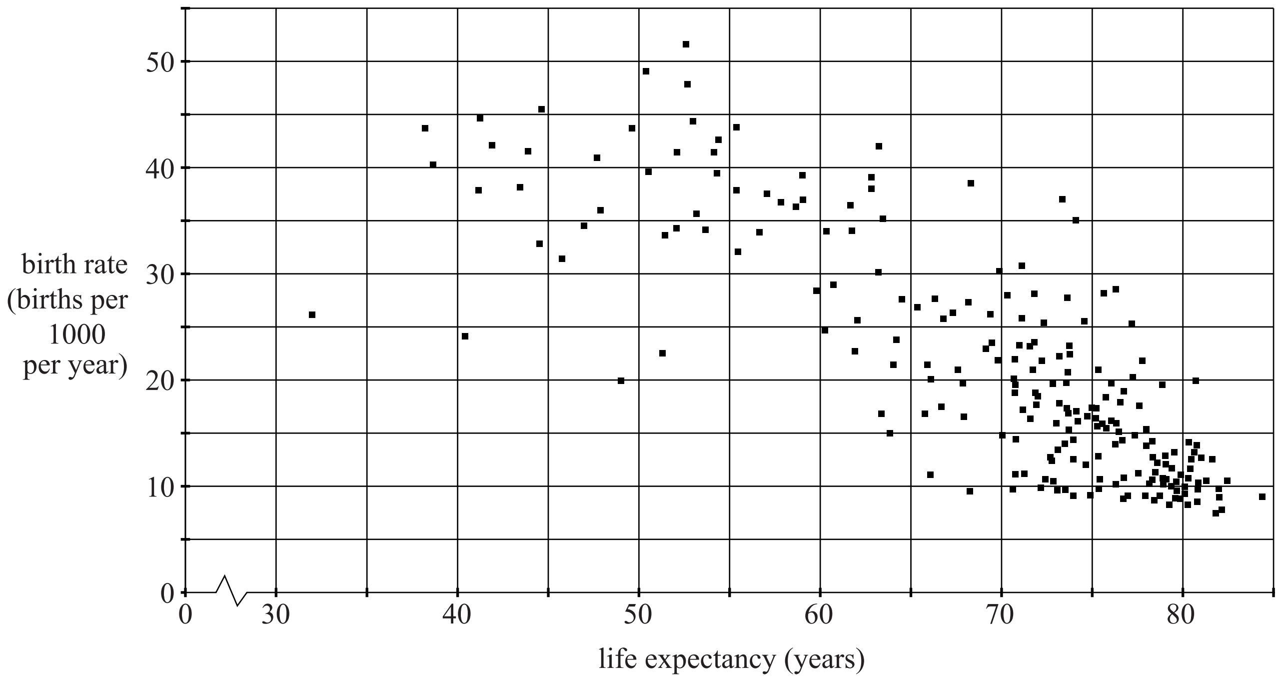



**Fig. 7 Scatter diagram showing birth rate against life expectancy for the countries of the world**

**Conclusion**

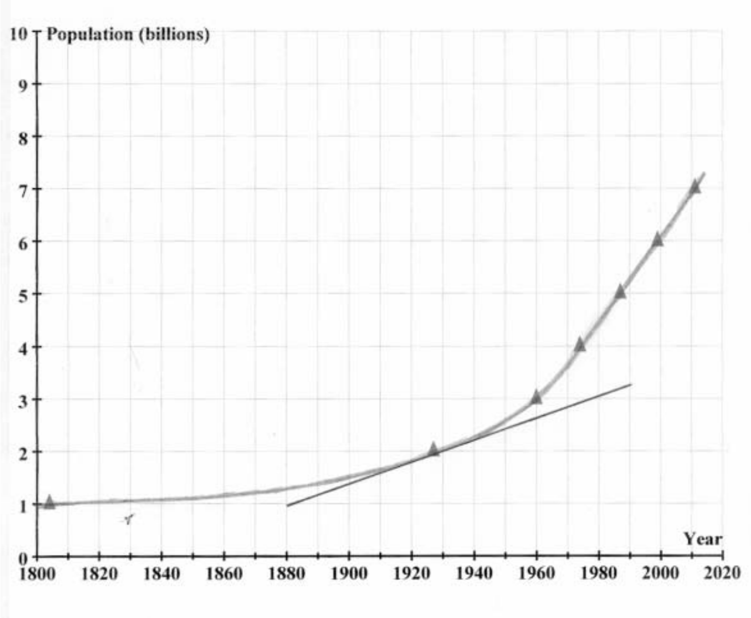
Table 8 gives the data for some selected countries in 2009.

| Country | Life expectancy | Birth rate |
|---|---|---|
| Japan | 82.12 | 7.64 |
| Sweden | 80.86 | 10.13 |
| Italy | 80.20 | 8.18 |
| UK | 79.01 | 10.65 |
| USA | 78.11 | 13.82 |
| Tunisia | 75.78 | 15.42 |
| Jamaica | 73.53 | 19.68 |
| China | 73.47 | 14.00 |
| Brazil | 71.99 | 18.43 |
| India | 69.89 | 21.76 |
| Bangladesh | 60.25 | 24.68 |
| Ghana | 59.85 | 28.58 |
| Uganda | 52.72 | 47.84 |
| Afghanistan | 44.64 | 45.46 |

**Table 8 Life expectancy and birth rates of selected countries (2009)**

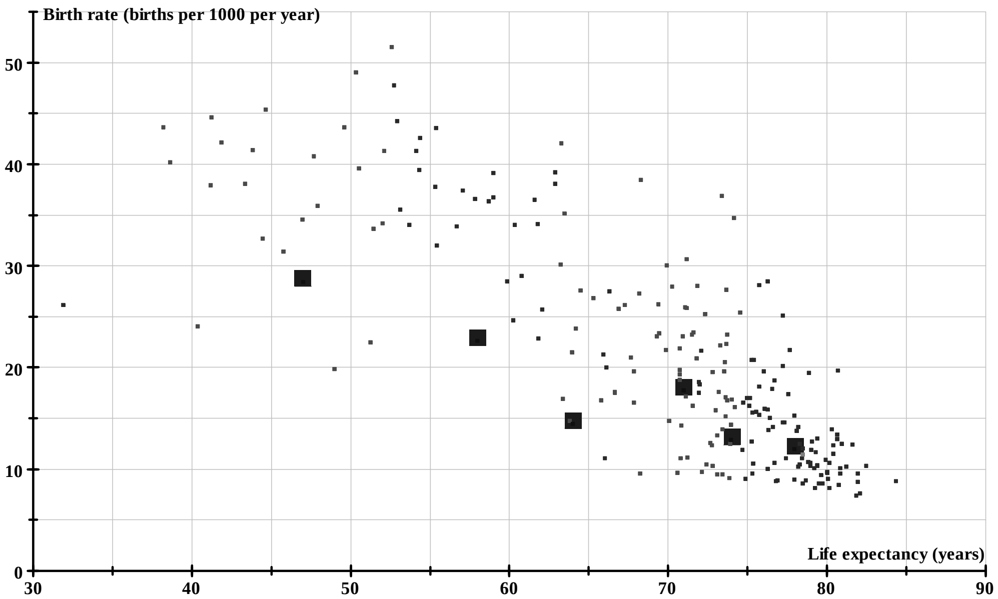
The data in Table 8 illustrate the observation that countries with low birth rate and high life expectancy tend to be those with developed economies. Studies over time indicate that as they develop, countries follow a path from high birth rate and low life expectancy to low birth rate and high life expectancy. So it is reasonable to expect that at some time in the future, the world's population will attain a maximum value and then start to decline.

When that maximum occurs, and how great the population then is, will depend on how quickly countries progress along that path. Consequently modelling the world's population requires an understanding of the factors involved. Then it will be possible to determine what can be done to match the population to the planet's resources.

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| **1** | | | Males 1.95 million, Females 2 million: Total 3.95 million | B1<br>**[1]** | accept 3.9-4 million<br>allow 4000 thousand oe |
| **2** | | (i) | <br>Gradient $0.018\times10^9$ giving 18 000 000 **(** people per year) | B1<br><br>M1<br>A1<br><br>**[3]** | curve and tangent drawn<br>do not accept a polygon<br>accept any reasonable **tangent** at the correct point (ie touches, not crosses)<br>(NB B0M1A1 is possible if a full curve is not drawn)<br>use of gradient (**<u>from tangent only</u>**)<br>accept 12-28,000,000<br>do not accept unreasonable accuracy eg no more than 3sf<br>(0.018 or 180 000, say, can score M1 A0)<br>without tangent is M0 A0 |
| | | (ii) | $\frac{0.018\times10^9}{2\times10^9}\times100\% = 0.9\%$ | M1<br>A1<br><br>**[2]** | allow follow through from previous part **for both marks**<br>ie (**their (i)**/ $2\times10^9$ ) × 100%,<br>for A mark do not allow more than 3sf<br>could get M1A1 from say 0.018/2 × 100% without having scored A1 in (i) |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 3 | | (i) | $\frac{dp}{dt} = kp$ <br> $\int \frac{dp}{p} = k\int dt$ <br> $\ln p = kt + c$ | M1 <br> A1 | separating variables correctly and intending to integrate <br> solving correctly, any form, need a constant |
| | | | When $t = 0, p = p_0 \Rightarrow c = \ln p_0$ <br> $\ln\left(\frac{p}{p_0}\right) = kt$ <br> $p = p_0 e^{kt}$ | A1 <br> **[3]** | **AG,** fully correct derivation of given result including **explicitly using** initial condition (condone $t = 0$, $p = 10^9 = \boldsymbol{p_0}$) <br> SC1 for <u>verifying</u> the given result correctly ie differentiating $p$=… and substituting for $p$ |
| | | (ii) | $p_0 = 10^9$ so the 1927 figures give $2\times10^9 = 10^9 \times e^{k\times(1927-1804)}$ <br> $\Rightarrow 123k = \ln 2$ <br> $\Rightarrow k = 0.00563...$ * | M1 <br> A1 <br> **[2]** | the equation must be correct (soi) <br> ($10^9$ could be cancelled) <br> cao **AG** so must **SHOW** enough, eg $k$ = ln2/123 or 0.005635…. |
| 4 | | | (table below) | B1 <br> B1 <br> B1 <br> **[3]** | 2030 column <br> 2050 column <br> 2070 column (need 2dp) <br> SCB2 for columns correct but no totals |

| Age group | 2010 | 2030 | 2050 | 2070 |
|---|---|---|---|---|
| **80⁺** | 1 | **2** | **2.4** | **2.4** |
| **60-79** | 10 | **12** | **12** | **12** |
| **40-59** | 20 | **20** | **20** | **22** |
| **20-39** | 20 | **20** | **22** | **24.2** |
| **0-19** | 20 | **22** | **24.2** | **26.62** |
| **Total** | **71** | **76** | **80.6** | **87.22** |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 5 | | (i) | The proportion of people in the 40-59 age group surviving into the 60-79 group is 80%. | B1<br>**[1]** | cao oe in fractions or decimals |
| | | (ii) | The proportion of those in the 60-79 group surviving in to the $80^+$ group is 40%. | B1<br>**[1]** | cao oe in fractions or decimals |
| 6 | | | The data show that, over the last 100 years, the birth rate in the UK has declined and the life expectancy has increased.<br><br>This pattern is consistent with the UK having a developing economy. | B1<br><br>B1<br>**[2]** | 1 mark for correct comments on changing (over time) birth rate and life expectancy. Not just negative correlation if no link (soi) to time<br>1 mark for linking to **development** |
| | | |  | | This diagram does not need to be seen but some candidates may use it to help in their explanations. |