

S2 Summer 2013

$$1.(i) \text{ PMCC} = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}} \quad \text{LEARN THIS!}$$

$$\bar{x} = \frac{\sum x}{n} \quad \bar{y} = \frac{\sum y}{n}$$

$$= 43.62 \quad = 55.15$$

$$= \frac{60}{60}$$

$$= 0.727 \quad = \frac{1103}{1200}$$

$$= 40.66 - 60(0.727)\left(\frac{1103}{1200}\right)$$

$$= \frac{\sqrt{(32.68 - 60(0.727)^2)(51.44 - 60(\frac{1103}{1200})^2)}}{60}$$

$$= 0.665 \quad (3 \text{ s.f.})$$

5 easy marks. Use the correct formula! Take your time and use exact values

$$H_0: \rho = 0$$

$H_1: \rho > 0$ +ve corr. between FEVI before & after the drug course.

where ρ is the PMCC in the population

make sure you define ρ

$0.665 > 0.2144 \therefore$ result is in the critical region.

The result is SIGNIFICANT. Reject H_0 .

There is sufficient evidence to suggest a positive correlation between FEVI before and after the course.

Conclusion must be non-assertive and in context.

(ii) We assume the underlying population has a bivariate normal distribution. This assumption appears to be valid since the scatter diagram has points mostly falling within an ellipse shape.

LEARN THESE!

(iv) The significance level is the probability of wrongly rejecting H_0 (when it is in fact true).

(v) If $(0.55, 1.00)$ had been recorded as $(1.00, 0.55)$ we would get (x values larger than they should be, y values smaller)

$$n=60, \sum x = 43.62 + 0.45, \sum y = 55.15 - 0.45, \sum xy = 40.66$$

<u>(same)</u>	<u>= 44.07</u>	<u>= 54.70</u>
		<u>(same)</u>

$$\sum x^2 = 32.68 + 1^2 - 0.55^2, \sum y^2 = 51.44 - 1^2 + 0.55^2$$

$$= 33.3775 \quad = 50.7425$$

2(i) $X \sim \text{Bin}(10, 0.03)$ $\rightarrow X$ is no. of people with red hair

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - (0.97)^{10} \\ &= \underline{0.263} \quad (\text{3 s.f.}) \end{aligned}$$

It's a good idea to write down the dist'n even if you don't have to

(ii) $X \sim \text{Bin}(60, 0.03)$ approx. with a Poisson(np) as we have a large n , small p and $\lambda = np$ not too large.
Mean of the Poisson dist'n is $= 60(0.03)$
 $= \underline{1.8}$

there are 3 conditions

$$\begin{aligned} (\text{iii}) \quad (\text{A}) \quad P(X=2) &= \frac{e^{-1.8}}{2} \frac{1.8^2}{2} \\ &= \underline{0.268} \quad (\text{3 s.f.}) \end{aligned}$$

for approximating, refer to

- n
- P
- np

$$(\text{B}) \quad P(X > 2) = 1 - P(X \leq 2)$$

$$1 \ 2 \ 3 \ 1 = 1 - 0.7306$$

part of a number line
helps get the correct
inequality

$$= \underline{0.2694} \quad (\text{from tables})$$

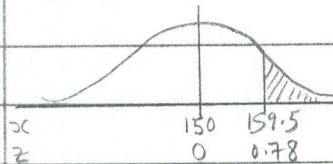
(iv) It would not be appropriate to use a Normal approx., as while we have the required large n , our p is too close to 0, making np too small. (Poisson is far preferable for $np < 10$, Normal when $np > 10$)

(v) $X \sim \text{Bin}(5000, 0.03)$ X is no. of people with red hair.

BINOMIAL DISTRIBUTION

continuity correction because
Bin. is discrete, approx'd with Normal (cont.)

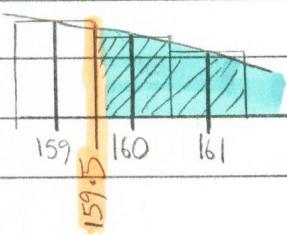
(vi) Approx with $X \sim N(150, 145.5)$



$$\begin{aligned} P(X \geq 160) &= P(Z > \frac{159.5 - 150}{\sqrt{145.5}}) \\ &= P(Z > 0.7876) \end{aligned}$$

$$\begin{aligned} \mu &= np \\ &= 5000(0.03) \\ &= 150 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= npq \\ &= 150(0.97) \\ &= 145.5 \end{aligned}$$

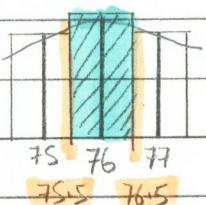


$$\begin{aligned} &= \underline{0.215} \quad (\text{3 s.f.}) \end{aligned}$$

cont. var. as scores reported as integers (discrete)

$$3.(i) X \sim N(76, 12^2)$$

X is score on English exam.



$$P(X=76) = P\left(\frac{75.5-76}{12} < Z < \frac{76.5-76}{12}\right)$$

$$= P(-0.0416 < Z < 0.0416)$$

$$= \Phi(0.0416) - \Phi(-0.0416)$$

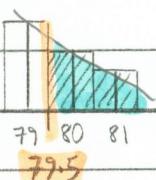
$$= 0.51662 - 0.48338$$

$$= 0.0332 \quad (3 \text{ s.f.})$$

using $Z = \pm \frac{1}{24}$ on calculator

use P button on calculator, or tables.

$$(ii) P(X \geq 80) = P\left(Z > \frac{79.5-76}{12}\right)$$



$$= P(Z > \frac{7}{12})$$

$$= 0.385 \quad (3 \text{ s.f.})$$

$$(iii) X \sim \text{Bin}(3, 0.38527)$$

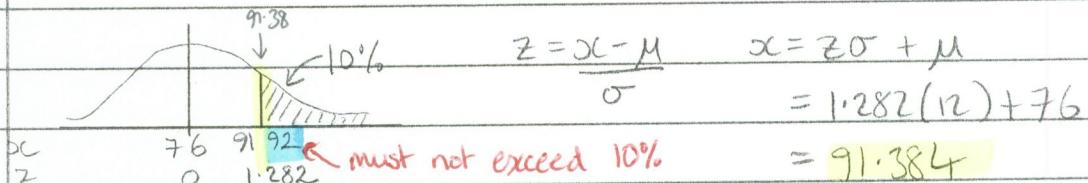
X is no with rep. mark ≥ 80

$$P(X=1) = {}^3C_1 (0.38527)^1 (0.61473)^2$$

use $n \cdot p^r q^{n-r}$ when it's prob $X = \dots$

$$= 0.437 \quad (3 \text{ s.f.})$$

(iv)



$$Z = \frac{X - \mu}{\sigma}$$

$$x = z\sigma + \mu$$

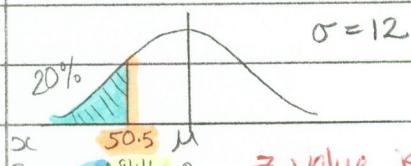
$$= 1.282(12) + 76$$

$$= 91.384$$

$$\Phi^{-1}(0.9)$$

\therefore lowest reported mark for A* is 92 marks.

(v)



$$\mu = \bar{x} - 2\sigma$$

$$= 50.5 - (-0.8416)(12)$$

$$= 60.6 \quad (3 \text{ s.f.})$$

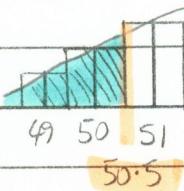
Z value is -ve

because it is less than

the mean. (look up $\Phi^{-1}(0.8)$)

then by symmetry, we want

- that value)



artist

4.(i)

	M	R	D	C	tot.	
sex	M	8	25	18	19	70
	F	18	35	10	17	80
	tot.	26	60	28	36	150

Show the calculations for one cell on each table

Eg for Monet n Male

$$f_e = \frac{26 \times 70}{150}$$

$$= 12.13$$

EXPECTED M R D C

$$\begin{array}{l} M \\ F \end{array} \quad \begin{array}{l} 12.13 \\ 13.87 \end{array} \quad \begin{array}{l} 28 \\ 32 \end{array} \quad \begin{array}{l} 13.07 \\ 14.93 \end{array} \quad \begin{array}{l} 16.8 \\ 19.2 \end{array}$$

EXPECTED χ^2 CONT'N

χ^2 CONT'N M R D C

M	7.4081	0.3814	1.8626	0.2881
F	1.2321	0.2813	1.6298	0.2521

Eg for Monet n Male

$$(f_o - f_e)^2 = (8 - 12.13)^2$$

$$f_e = 12.13$$

$$= 1.4081$$

(must show this table!!)

$$D = (2-1)(4-1)$$

= 3 ← state this clearly.

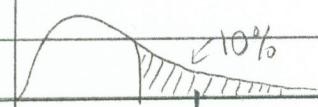
Must be in context!

H_0 : There is NO ASSOCIATION between sex and artist preferred.

H_1 : There is some ASSOCIATION between sex and artist preferred.

CRITICAL value, 10% level, $D=3$ is 6.251

don't need a diagram, but it helps make it clear.



$$\text{result } \chi^2 = 7.28 > 6.251$$

∴ result is SIGNIFICANT Reject H_0 . There is

SUFFICIENT evidence to suggest that there is some association between sex and artist preferred.

non-assertive, in context.

(ii) For Monet, χ^2 cont's of 1.4 & 1.2 (quite large) indicate fewer males and more females than would be expected.

For Degas, χ^2 cont's of 1.8 & 1.6 (quite large) indicate more males and fewer females than would be expected.

For both Renoir & Cézanne, small χ^2 cont's (around 0.3) indicate that preferences of both sexes are about what we'd expect if there were no association. Read the question carefully → answer for each artist. Refer to the contributions to χ^2 (even if they don't ask you to!)