

S2 June 2006 Worked Solutions

Q1

(i) $X \sim \text{Bi}(8, 0.1)$ distribution of no. of breakfasts ordered
Str Maths 04 S2-06S

$$\begin{aligned} P(X=1) &= {}^8C_1 (0.1)(0.9)^7 \\ &= 0.38263 \dots \\ &= 0.383 \text{ (3sf)} \end{aligned}$$

(ii) $X \sim \text{Bi}(30, 0.1)$ n is large, p is small.
 \therefore Approximated by $X \sim \text{Po}(30 \times 0.1)$
 $\sim \text{Po}(3)$

$$\begin{aligned} \text{(a)} P(X=6) &= e^{-3} \frac{3^6}{6!} \\ &= 0.0504 \dots \\ &= 0.0504 \text{ (3sf)} \end{aligned}$$

$$\begin{aligned} \text{(b)} P(X \geq 8) &= 1 - P(X \leq 7) \quad \text{discrete distribution} \\ &= 1 - 0.9881 \\ &= 0.0119 \quad \text{(from tables)} \end{aligned}$$

(iii) The Poisson distribution is appropriate for approximating the Binomial distribution when n is large and p is small.

$$\begin{aligned} \text{(iv)} \mu &= np = 120 \times 0.1 = 12 & \sigma^2 &= npq = 120 \times 0.1 \times 0.9 = 10.8 \\ & & & \sigma &= \sqrt{10.8} \end{aligned}$$

\therefore approximated by $N(12, 10.8)$

$$\begin{aligned} \text{(v)} P(X > 15) &= 1 - \Phi\left(\frac{15.5 - 12}{\sqrt{10.8}}\right) \quad \text{continuity correction} \\ &= 1 - \Phi(1.065) \\ &= 1 - 0.8566 \\ &= 0.1434 \end{aligned}$$

(vi) Let n be minimum number of breakfasts to be carried

$$\begin{aligned} P(X \leq n) &= \Phi\left(\frac{n-12}{\sqrt{10.8}}\right) \\ 0.99 &= \Phi\left(\frac{n-12}{\sqrt{10.8}}\right) \\ 2.326 &= \frac{n-12}{\sqrt{10.8}} \end{aligned}$$

$$\begin{aligned} n &= 19.644 \dots \\ \therefore \text{Need to carry 20 meals.} \end{aligned}$$

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Q2

Boys (B1)
Str Maths 04 S2-06S

2. Head circumference $\sim N(49.7, 1.6^2)$

$$\begin{aligned} \text{a) A) } P(H > 51.5) &= 1 - P(H \leq 51.5) \\ &= 1 - \Phi\left(\frac{51.5 - 49.7}{1.6}\right) \\ &= 1 - \Phi(1.125) \\ &= 0.8696 \end{aligned}$$

$$\begin{aligned} \text{B) } P(48.0 \leq H \leq 51.5) &= P(H \leq 51.5) - P(H \leq 48.0) \\ &= \Phi\left(\frac{51.5 - 49.7}{1.6}\right) - \Phi\left(\frac{48.0 - 49.7}{1.6}\right) \\ &= \Phi(1.125) - \Phi(-1.0625) \\ &= 0.8696 - (1 - 0.856) \\ &= 0.7256 \end{aligned}$$

$$\begin{aligned} \text{ii) } P(\text{1 out of 4 have } 48.0 \leq H \leq 51.5) \\ &= {}^4C_1 (0.7256)^1 (0.2744)^3 \\ &= 0.05996 \dots \\ &= 0.0600 \text{ (3sf)} \end{aligned}$$

iii) $GH \sim N(\mu, \sigma^2)$ Head circumference of girls.

$$P(GH < 49.0) = 0.6$$

$$P(GH < 47.5) = 0.3$$

$$\therefore \Phi\left(\frac{49.0 - \mu}{\sigma}\right) = 0.6 \quad \text{①}$$

$$\Phi\left(\frac{47.5 - \mu}{\sigma}\right) = 0.3 \quad \text{②}$$

$$\text{①} \Rightarrow 49.0 - \mu = \Phi^{-1}(0.6)\sigma = 0.25335\sigma$$

$$\text{②} \Rightarrow 47.5 - \mu = \Phi^{-1}(0.3)\sigma = -0.52445\sigma$$

eliminate μ .

$$\text{①} - \text{②} \Rightarrow 1.5 = 0.77775\sigma$$

$$\sigma = 1.9287 \dots$$

$$= 1.929 \text{ (to 3dp)}$$

$$\text{sub } \sigma = 1.9287 \dots \text{ in ①}$$

$$\mu = 48.511444$$

So mean is 48.5 cm and $\sigma = 1.9$ cm (to 1dp)

iv) $n = 10$, mean 50.45 cm, organic diet.

$$H_0: \mu = 49.7 \text{ cm vs } H_1: \mu > 49.7 \text{ cm}$$

$$\text{Under } H_0: H \sim N(49.7, 1.6^2)$$

$$H \sim N(49.7, 1.6^2/10)$$

$$\begin{aligned} P(H \geq 50.45) &= 1 - P(H < 50.45) \\ &= 1 - \Phi\left(\frac{50.45 - 49.7}{1.6/\sqrt{10}}\right) \\ &= 1 - \Phi(1.482) \\ &= 1 - 0.9309 \\ &= 0.0691 < 0.1 \end{aligned}$$

\therefore Reject null hypothesis, suggesting that, at the 10% level, boys fed on an organic diet have a head circumference greater than 49.7 cm.

S2 June 2006 Worked Solutions

Q3

$$3. \sum x = 4715 \quad \sum y = 13175 \quad \sum x^2 = 2237725$$

Str Maths 04 S2-06S

$$\sum y^2 = 17455825 \quad \sum xy = 6235575 \quad n=10$$

$$(i) r = \frac{\sum xy - \bar{x}\bar{y}}{\sqrt{(\sum x^2 - \bar{x}^2)(\sum y^2 - \bar{y}^2)}}$$

$$= \frac{6235575 - 471.5 \times 1317.5}{10}$$

$$\sqrt{\left(\frac{2237725 - 471.5^2}{10}\right)\left(\frac{17455825 - 1317.5^2}{10}\right)}$$

$$= \frac{9425}{4}$$

$$= \frac{\sqrt{5841} \times \sqrt{39105}}{4}$$

$$= 0.6232...$$

$$= 0.623 \quad (3sf)$$

$$(ii) H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

ρ - population correlation

two tailed as no direction specified

$n=10$

$$5\% \text{ sig level } \text{critical value} = 0.6319$$

There is no reason to doubt the null hypothesis at the 5% level as $0.623 < 0.6319$
Thus suggesting that there is no relationship between length and circumference

(iii) (A) A 5% significance level - type I error set at 5% is happy to have a 5% chance of rejecting the null hypothesis when in fact the null hypothesis is true.

(B) An advantage of a 1% sig level is that it is more stringent - P(type I error) reduced dramatically. A disadvantage is that the P(type II error is increased) is more likely to accept the null hypothesis when the null hypothesis is in fact false.

(iv) The student is manipulating the data - ie choosing a conclusion that she/he would like to reach. A better approach would be to combine the results of both studies (providing same conditions e.g. same person does measurements) - this would increase the sample size and hence confidence about conclusion reached.

S2 June 2006 Worked Solutions

Q4

H_0 : no association between musical preference and age;
 H_1 : some association between musical preference and age;

Observed		Musical preference			Row totals
		Pop	Classical	Jazz	
Age group	Under 25	57	15	12	84
	25 – 50	43	21	21	85
	Over 50	22	32	27	81
Column totals		122	68	60	250

Expected		Musical preference			Row totals
		Pop	Classical	Jazz	
Age group	Under 25	40.992	22.848	20.160	84
	25 – 50	41.480	23.120	20.400	85
	Over 50	39.528	22.032	19.440	81
Column totals		122	68	60	250

Contributions		Musical preference			
		Pop	Classical	Jazz	
Age group	Under 25	6.25	2.70	3.30	
	25 – 50	0.06	0.19	0.02	
	Over 50	7.77	4.51	2.94	

$$\chi^2 = 27.74$$

Refer to χ^2_4

Critical value at 5% level = 9.488

Result is significant

There is some association between age group and musical preference.

NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1

Note you were asked specifically to give a table with the contribution to the test statistics.

Note the marking scheme required a detailed discussion of the contribution

(ii)	<p>The values of 6.25 and 7.77 show that under 25's have a strong positive association with pop whereas over 50's have a strong negative association with pop.</p> <p>The values of 4.51 and 2.94 show that over 50's have a reasonably strong positive association with both classical and jazz.</p> <p>The values of 2.70 and 3.30 show that under 25's have a reasonably strong negative associations with both classical and jazz.</p> <p>The 25-50 group's preferences differ very little from the overall preferences.</p>	<p>B1, B1 for specific reference to a value from the table of contributions followed by an appropriate comment B1, B1 (as above for second value) B1, B1 (as above for third value)</p>	6
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