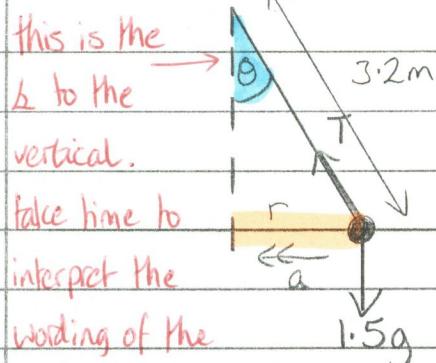


M3 Summer 2013

1(a)



this is the
angle to the
vertical.
Take time to
interpret the
working of the
question correctly to
draw the diagram correctly.

$$\omega = 2.5 \text{ rad s}^{-1}$$

$$a = r\omega^2$$

$$r = 3.2 \sin \theta$$

$$(i) \text{ res} \rightarrow, F = ma$$

$$T \sin \theta = 1.5 (3.2) \sin \theta (2.5)^2$$

$$T = 30 \text{ N}$$

$$(ii) \text{ res} \uparrow \text{ in equil'm } T \cos \theta = 1.5g$$

$$\cos \theta = \frac{1.5(9.8)}{30}$$

$$\theta = \arccos \left(\frac{1.5(9.8)}{30} \right)$$

$$\angle \text{ to vertical} = 60.7^\circ \text{ 3.s.f.}$$

$$(b) (i) u^2 = \frac{4kd^2}{3m}$$

$$\text{or } [k] = \begin{bmatrix} 2 \\ L \end{bmatrix}$$

$$\left[\frac{u^2 m}{d^2} \right] = [k]$$

show plenty of
working for a

$$= \frac{MLT^{-2}}{L}$$

$$= \frac{(LT^{-1})^2 M}{L^2}$$

"show that"

$$= MT^{-2}$$

$$[k] = \underline{MT^{-2}}$$

question

need to make

this clear, that
dimensions are

$$(ii) \text{ LHS } [u] = LT^{-1}$$

$$\text{RHS } \left[\frac{4kd^2}{3m} \right] = \left(\frac{MT^{-2} L^2}{M} \right)^{1/2}$$

SAME on both
sides.

$$[\text{LHS}] = [\text{RHS}]$$

same dimensions $\rightarrow = \underline{LT^{-1}}$ \therefore consistent

$$(iii) t = Ak^\alpha d^\beta m^\gamma$$

$$T = (MT^{-2})^\alpha L^\beta M^\gamma$$

$$T = M^{\alpha+\gamma} T^{-2\alpha} L^\beta$$

equating powers of M: $0 = \alpha + \gamma$

$$L: 0 = \beta$$

$$T: 1 = -2\alpha$$

$$-\frac{1}{2} = \alpha$$

$$\underline{\underline{\alpha = -\frac{1}{2}, \beta = 0, \gamma = \frac{1}{2}}}$$

$$\Rightarrow \gamma = \frac{1}{2}$$

1(b)(w) $m = 5$, $d = 0.7$, $k = 60$ when string is double d,
 $E.P.E. = \frac{1}{2} kx^2$ $x = d$
 $u = \sqrt{\frac{4kd^2}{3m}}$
 $= \sqrt{\frac{4(60)(0.7)^2}{3(5)}}$
 $= 2.8 \text{ ms}^{-1}$

$= \frac{1}{2}(60)(0.7)^2$
 $= 14.7$

give units in answers

$$KE \text{ at start} = KE \text{ at end} + E.P.E. \text{ at end}$$

$$\frac{1}{2}(5)(2.8)^2 = \frac{1}{2}(5)v^2 + 14.7$$

$$v^2 = \frac{1}{5}(2.5(2.8)^2 - 14.5)$$

$$v = 1.4 \text{ ms}^{-1} \text{ speed when double nat. length}$$

2(i) $KE_{\text{at bottom}} = KE_{\text{at } \theta^\circ} + GPE_{\text{at } \theta^\circ}$

$$\frac{1}{2}(0.25)(8.4)^2 = \frac{1}{2}(0.25)v^2 + 0.25(9.8)(a - a\cos\theta)$$

$$v^2 = 8\left(\frac{8.4^2}{8} - \frac{9.8}{4}a(1 - \cos\theta)\right)$$

$$v^2 = 70.56 - 19.6a(1 - \cos\theta)$$

$$h = a - a\cos\theta$$

res towards centre, $F = ma$, $a = \frac{v^2}{r} = \frac{v^2}{a}$ ← radius

iii be careful to avoid confusion between a for acceleration and the radius a in this question!!!

$$T - 0.25g \cos\theta = \frac{0.25}{a}(70.56 - 19.6a(1 - \cos\theta))$$

show plenty of

working in a

"show that" question

$$T = \frac{0.25(70.56)}{a} + 0.25(9.8 + 19.6)\cos\theta - 0.25(9.8)$$

$$T = \frac{17.64}{a} + 7.35\cos\theta - 4.9$$

$$2(ii) \quad a = 0.9, \quad T = \frac{17.64}{0.9} - 4.9 + 7.35 \cos\theta$$

$$= 14.7 + 7.35 \cos\theta$$

max. when $\cos\theta = 1 \Rightarrow \theta = 0$

$$T = 14.7 + 7.35$$

$$= 22.05 \text{ N}$$

min. when $\cos\theta = -1 \Rightarrow \theta = \pi$

$$T = 14.7 - 7.35$$

$$= 7.35 \text{ N}$$

T > 0 at all times, $\therefore P$
moves in a complete circle.

(iii) For largest possible circle, $T = 0$ at $\theta = \pi$ at "top" of the circle

$$\Rightarrow 0 = \frac{17.64}{a} + 7.35 \cos\pi - 4.9$$

$$7.35 + 4.9 = \frac{17.64}{a}$$

$$a = \frac{17.64}{7.35 + 4.9}$$

$$= 1.44 \text{ m} \quad \text{largest value of } a$$

(iv) $a = 1.6, \quad T = \frac{17.64}{1.6} - 4.9 + 7.35 \cos\theta$

slack when $T = 0 = 6.125 + 7.35 \cos\theta$

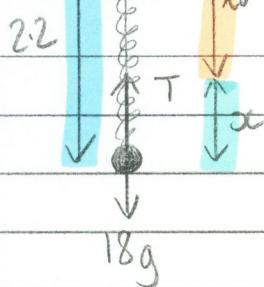
$$\cos\theta = -\frac{6.125}{7.35}$$

sub into

$$v^2 = 70.56 - 19.6(1.6)\left(1 + \frac{6.125}{7.35}\right)$$

$$v = 3.61 \text{ ms}^{-1} \quad \text{speed when string becomes slack}$$

$$3.(i) \quad A = 686 \quad \text{in equil'm , res } \uparrow$$



$$T = 18g$$

$$\frac{686(2.2) - 686}{l_0} = 18(9.8)$$

$$T = kx$$

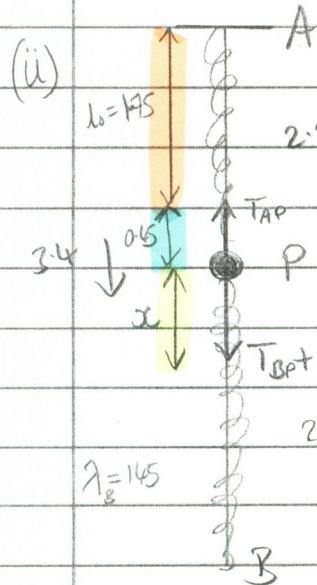
$$= \frac{686}{l_0}(2.2 - l_0)$$

$$\frac{686(2.2)}{l_0} = 686 + 18(9.8)$$

$$l_0 = \frac{686(2.2)}{686 + 18(9.8)}$$

$$\text{natural length} = \underline{1.75 \text{ m}}$$

✓ a clear diagram helps you find this expression



$$\text{for AP, } T_{AP} = k(x + 0.45)$$

$$= \frac{686}{1.75}(0.45 + x)$$

$$= \underline{176.4 + 392x \text{ N}}$$

$$\text{for BP, } T_{BP} = kx$$

$$= \frac{145}{2.5}x$$

$$= \underline{58x \text{ N}}$$

compression as it is a negative extension

$$(iii) \downarrow \text{ve} \quad F = ma \quad T_{BP} + 18g - T_{AP} = 18 \frac{d^2x}{dt^2}$$

$$-58x + 18(9.8) - (176.4 + 392x) = 18 \frac{d^2x}{dt^2}$$

To get eqn of the form acc'n = $-w^2x$

use $F = ma$ in the direction of motion $-25x = \frac{d^2x}{dt^2}$

$$(iv) \quad \text{acc'n} = -w^2x \Rightarrow w = 5$$

$$\text{period} = \frac{2\pi}{5}$$

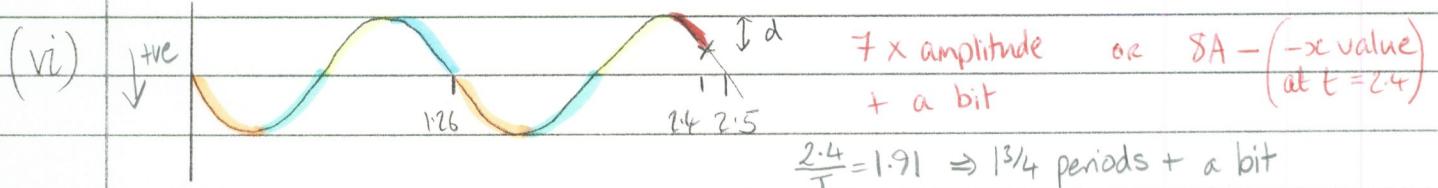
$$\text{max speed} = Aw$$

$$3.4 = 5A$$

$$= \underline{1.26 \text{ s}} \text{ (3.s.f.)}$$

$$A = \frac{3.4}{5} = \underline{0.68 \text{ m}} \text{ amplitude}$$

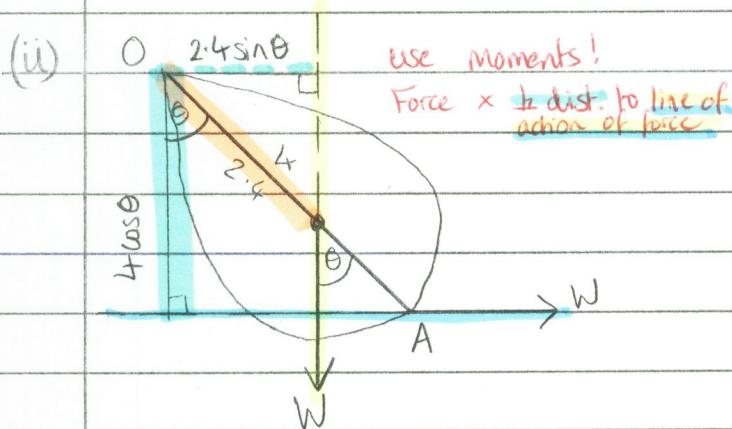
3.(v) motion is $x = 0.68 \sin 5t$ ↓ disp. +ve
 $v = \frac{dx}{dt} = 3.4 \cos 5t$ be careful with +ve / -ve.
if +ve eqn is $x = -0.68 \sin 5t$
 $= 3.4 \cos(5(2.4))$
 $= 2.87 \text{ ms}^{-1}$ (3 s.f.) remember to state
the direction
vertically downwards



dist. is $7A + d$
 $= 7(0.68) + (0.68 + 0.68 \sin(5(2.4)))$
 $= 5.08 \text{ m}$ (3 s.f.) total dist. travelled.

4.(a) (i) $\bar{x} = \frac{\int y^2 x \, dx}{\int y^2 \, dx}$ learn (or be able to quickly derive) this formula
 $= \frac{51.2}{64/3}$
 $= 2.4$

(i) $y = x \sqrt{4-x}$
 $y^2 = x^2(4-x)$
 $= 4x^2 - x^3$
 $\int_0^4 4x^2 - x^3 \, dx$
 $= \left[\frac{4}{3}x^3 - \frac{1}{4}x^4 \right]_0^4$



in equil'm $M(O): \Sigma = 0$

$$\begin{aligned} &= \frac{4}{3}(4)^3 - \frac{1}{4}(4)^4 - 0 \\ &= \frac{64}{3} \\ \int_0^4 y^2 x \, dx &= \int_0^4 4x^2 - x^3 \, dx \\ &= \left[x^4 - \frac{1}{5}x^5 \right]_0^4 \\ &= 4^4 - \frac{1}{5}(4)^5 - 0 \\ &= 51.2 \end{aligned}$$

$4W \cos \theta = 2.4W \sin \theta$

$\frac{4}{2.4} = \tan \theta$

$\theta = \arctan\left(\frac{4}{2.4}\right)$

$= 59.0^\circ$ + to vertical

LEARN this (round x or round y axis)

$$4.(b) M(\bar{x}) = \begin{pmatrix} \int \frac{x^2}{2} dy \\ \int xy dy \end{pmatrix} \quad \text{where } M \text{ is area } \int x dy$$

$$y = (x-2)^3$$

be very careful
with the
algebra manipulation
here!!

$$\sqrt[3]{y} + 2 = x$$

$$(y^{1/3} + 2)^2 = x^2$$

$$x^2 = y^{2/3} + 4y^{1/3} + 4$$

$$\int x dy = \int_0^8 y^{1/3} + 2 dy$$

$$= \left[\frac{3}{4} y^{4/3} + 2y \right]_0^8$$

$$= \frac{3}{4}(16) + 2(8) - 0$$

$$= 28 = M$$

$$\int \frac{1}{2} x^2 dy = \frac{1}{2} \int_0^8 y^{2/3} + 4y^{1/3} + 4 dy$$

$$= \frac{1}{2} \left[\frac{3}{5} y^{5/3} + 3y^{4/3} + 4y \right]_0^8$$

$$= \frac{1}{2} \left(\frac{3}{5}(32) + 3(16) + 4(8) \right) - 0$$

$$= 49.6$$

$$\int xy dy = \int_0^8 y^{4/3} + 2y dy$$

$$= \left[\frac{3}{7} y^{7/3} + y^2 \right]_0^8$$

$$= \left(\frac{3}{7}(128) + 8^2 \right) - 0$$

$$= \frac{832}{7}$$

$$\bar{x} = \frac{\int x dy}{M}$$

$$= \frac{49.6}{28}$$

$$= \frac{62}{35} = 1.77 \text{ (3s.f.)}$$

$$\bar{y} = \frac{\int xy dy}{M}$$

$$= \frac{832/7}{28}$$

$$= \frac{208}{49} = 4.24 \text{ (3s.f.)}$$

give final answer

as coords

(,)

\therefore coords of C of M: $\underline{\underline{\left(\frac{62}{35}, \frac{208}{49} \right)}}$ or $\underline{\underline{(1.77, 4.24)}}$
to 3s.f.