

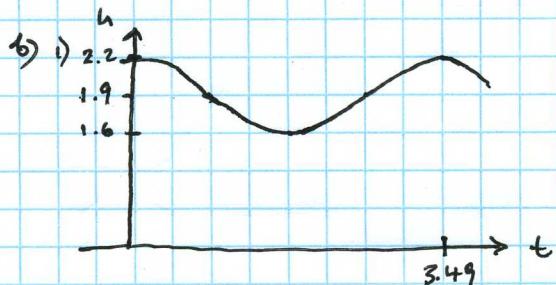
M3 JUNE 07 : SOLUTIONS

- ① a) i) $[Vel] = LT^{-1}$ $[Acc] = LT^{-2}$ $[F] = MLT^{-2}$ $[D] = ML^{-3}$ $[P] = ML^{-1}T^{-1}$
- ii) $P + \frac{1}{2}\rho v^2 + \rho gh = \text{const}$
method: $P, \frac{1}{2}\rho v^2, \rho gh$ must have same dimensions

(the constant can have any dimension)

$$[P] = \underline{ML^{-1}T^{-2}} \quad [\frac{1}{2}\rho v^2] = \underline{ML^{-3} \cdot (LT^{-1})^2} = \underline{ML^{-1}T^{-2}}$$

$$[\rho gh] = \underline{ML^{-3} \cdot LT^{-2} \cdot L} = \underline{ML^{-1}T^{-2}}$$



ii) $h = 1.9 + a \cos \omega t$

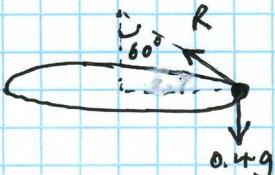
$$a = 0.3 \quad \frac{2\pi}{\omega} = 3.49 \Rightarrow \omega = 1.8$$

$$\therefore h = 1.9 + 0.3 \cos 1.8t$$

iii) $acc = -\omega^2 x \quad x = -0.2 \quad (0.2\text{m below centre})$

$$\therefore acc = 1.8^2 \times 0.2 = \underline{0.648 \text{ ms}^{-2}}$$

② i)

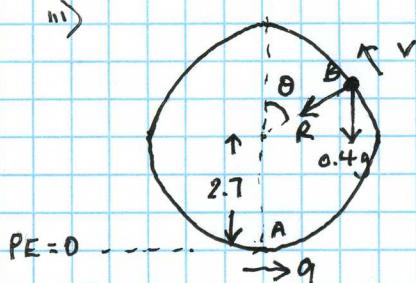


$$R(\uparrow) \quad R \cos 60 = 0.4g$$

$$\underline{R = 7.84 \text{ N}}$$

ii) $R(\leftarrow) \quad R \sin 60 = ma = \frac{mv^2}{r} = \frac{0.4v^2}{2.7 \sin 60} \Rightarrow v = \underline{6.3 \text{ ms}^{-1}}$

iii)



$$\text{energy at A} = \text{energy at B}$$

$$\Rightarrow \frac{1}{2} 0.4 \times g^2 = \frac{1}{2} 0.4v^2 + 0.4g(2.7 + 2.7 \cos \theta)$$

$$16.2 = 0.2v^2 + 10.584 + 10.584 \cos \theta$$

$$\underline{v^2 = 28.08 - 52.92 \cos \theta}$$

iv) $R(\leftarrow) \quad R + 0.4g \cos \theta = \frac{mv^2}{r} = \frac{0.4}{2.7} (28.08 - 52.92 \cos \theta)$
 $\Rightarrow \underline{R = 4.16 - 11.76 \cos \theta}$

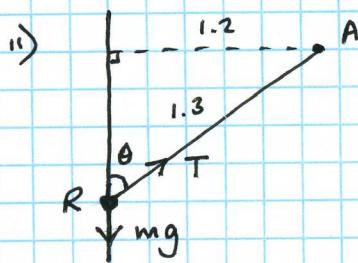
v) leaves surface when $R = 0 \Rightarrow \cos \theta = 4.16 / 11.76$

$$v^2 = 28.08 - 52.92 \times \frac{4.16}{11.76}$$

$$\underline{v = 3.06 \text{ ms}^{-1}}$$

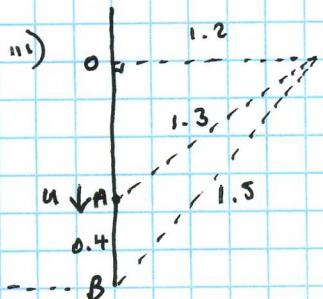
$$③ \quad i) \quad T = kx = 637 \times 0.1 = \underline{63.7 \text{ N}}$$

$$\text{EPE} = \frac{1}{2} kx^2 = \frac{1}{2} 637 \times 0.1^2 = \underline{3.185 \text{ J}}$$



$$\sin\theta = \frac{1.2}{1.3} = \frac{12}{13} \quad \therefore \cos\theta = \frac{5}{13}$$

$$R(\uparrow) \quad T \cos\theta = mg \Rightarrow m = 63.7 \times \frac{5}{13} \div 9.8 \\ \underline{m = 2.5 \text{ kg}}$$



$$OA = 0.5 \text{ m} \quad OB = 0.9 \text{ m} \quad (3+5 \Delta)$$

energy at A = energy at B

$$\text{PE} + \text{KE} + \text{EPE} = \text{EPE}$$

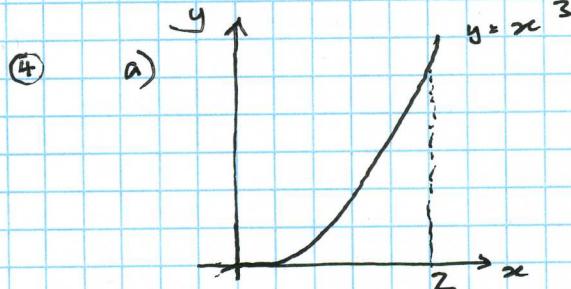
$$2.5 \times 9.8 \times 0.4 + \frac{1}{2} 2.5 u^2 + 3.185 = \frac{1}{2} 637 \times 0.3^2 \\ \Rightarrow \underline{u = 3.54}$$

iv) energy at B = energy at O

$$\text{EPE} = \text{PE} + \text{KE} \quad (\text{EPE} = 0)$$

$$\frac{1}{2} 637 \times 0.3^2 = 2.5 \times 9.8 \times 0.9 + \text{KE} \quad \text{KE} = 6.615 > 0$$

\therefore raised above O



$$\text{wt} \propto \text{area} \quad \text{Area} = \int_0^2 x^3 dx = \frac{4}{4}$$

$$M(Oy) \quad 4\bar{x} = \int_0^2 xy dx = \int_0^2 x^4 dx = \left[\frac{x^5}{5} \right]$$

$$\bar{x} = 6.4 \div 4 = \underline{1.6}$$

$$M(Ox) \quad 4\bar{y} = \int_0^2 \frac{1}{2} y^2 dx = 64/7$$

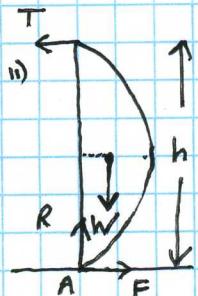
$$\bar{y} = \underline{16/7}$$

$$b) i) \text{wt} \propto \text{Vol} \quad \text{Vol} = \int_1^2 \pi y^2 dx = \pi \int_1^2 4-x^2 dx = \frac{5\pi}{3}$$

$$M(Oy) \quad \frac{5\pi}{3} \bar{x} = \int_1^2 \pi x y^2 dx = \frac{9\pi}{4} \quad \bar{x} = 1.35$$

$$x=1 \quad y = \sqrt{4-1^2} = \sqrt{3} \quad \therefore \underline{h = 2\sqrt{3}}$$

$$M(A) \quad T \times 2\sqrt{3} = 0.35 \text{ W}$$



$$R(\uparrow) \quad R = W \quad R(\leftrightarrow) \quad F = T = 0.101W \Rightarrow \min \mu = \frac{F}{R}$$

$$\min \mu = 0.10$$