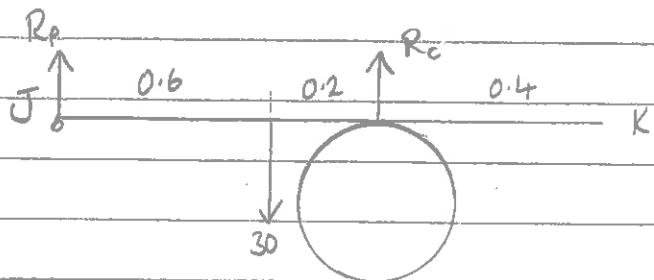


## M2 2015

1 (i)



in equil'm

$$M(J): 30(0.6) = R_c(0.8)$$

$$R_c = \frac{30(0.6)}{0.8}$$

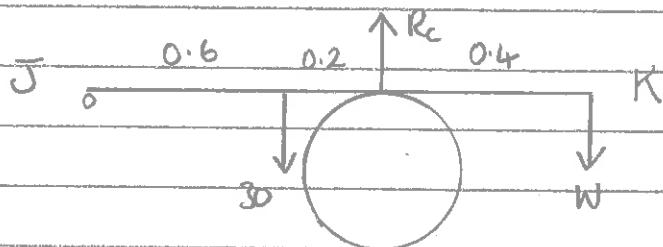
Force exerted by cylinder = 22.5N

$$\text{rest} \uparrow R_p + R_c = 30$$

$$R_p = 30 - 22.5$$

Force exerted by peg = 7.5N

(ii)



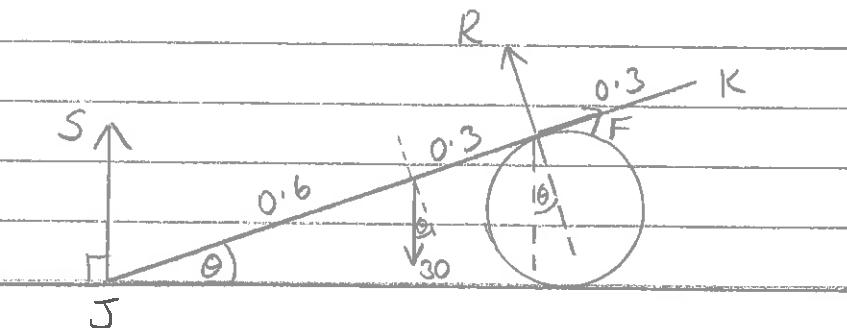
in equil'm, about to lose contact with J ( $R_p = 0$ )

$$M(\text{top of cylinder}): 0.4W = 0.2(30)$$

$$W = \frac{0.2(30)}{0.4}$$

Greatest value of  $W = \underline{15N}$

(iii)



(iii) in equil'm M (contact with cylinder):

cont'd

$$0.9 \cos\theta S = 30(0.3 \cos\theta)$$

$$S = \frac{30(0.3)}{0.9}$$

$$= \underline{\underline{10N}}$$

$$M(S): 0.9 R = 30(0.6 \cos\theta)$$

$$R = 20 \cos\theta N$$

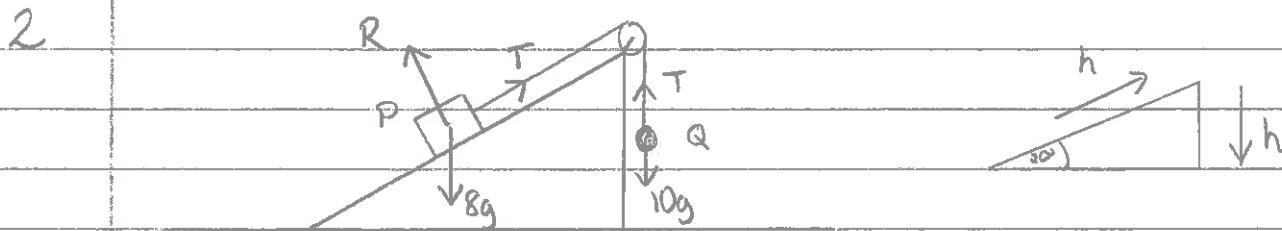
$$\text{res // to rod } F + S \sin\theta = 30 \sin\theta \quad (\text{subst. } S=10)$$

$$F = 30 \sin\theta - 10 \sin\theta$$

$$F = \underline{\underline{20 \sin\theta}}$$

(iv)  $F_r \leq \mu R$        $20 \sin\theta \leq \mu \times 20 \cos\theta$

$$\mu \geq \frac{20 \sin\theta}{20 \cos\theta} \quad \text{IDENTITY } \frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\mu \geq \underline{\underline{\tan\theta}}$$



Q descends h m  $\Rightarrow$  P rises by  $h \sin\theta = \frac{1}{2}h$  m

$\therefore Q$  loses  $mgh = 10gh J$ , P gains  $mgh = 8g(\frac{1}{2}h) = 4gh J$

$\therefore$  overall gain of  $(4 - 10)gh J = -6gh J$

system LOSES  $6gh J = 58.8hJ$

(ii)(A) loss in PE = gain in KE

$$6g(1.2) = \frac{1}{2}(8+10)v^2$$

$$v^2 = \frac{6(9.8)(1.2)}{9}$$

J J

$$v = \underline{2.8 \text{ ms}^{-1}}$$

(B) Q decelerates upwards then travels back down towards floor.

Q has speed  $1.05 \text{ ms}^{-1}$  when it is back at height  $0.3 \text{ m}$

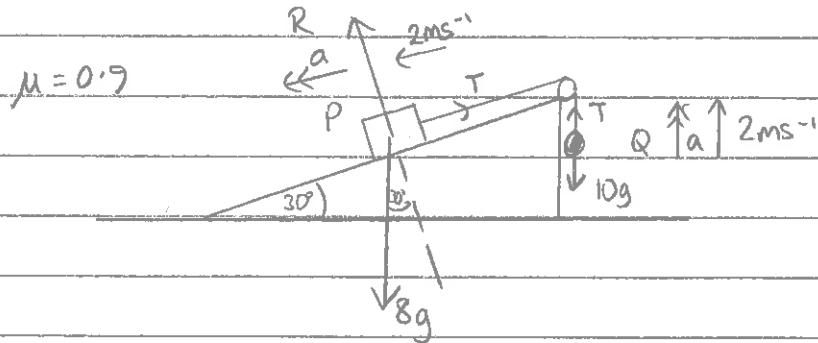
loss in PE = gain in KE

$$6g(0.3) = \frac{1}{2}(8+10)(v^2 - 1.05^2)$$

$$v^2 = \frac{6(9.8)(0.3) + 1.05^2}{9}$$

J J

$$v = \underline{1.75 \text{ ms}^{-1}}$$



res  $\perp$  to slope

$$\begin{aligned} R &= 8g \cos 30^\circ \\ &= \frac{\sqrt{3}}{2} \times 8g \end{aligned}$$

$$= 4\sqrt{3}g$$

(iii) system gains  $6gd$  J as P moves  $d \text{ m}$  down slope

gain in PE + Work done = loss in KE

$$6gd + F_r d = \frac{1}{2}(8+10)(2)^2$$

using  $F_r = \mu R$

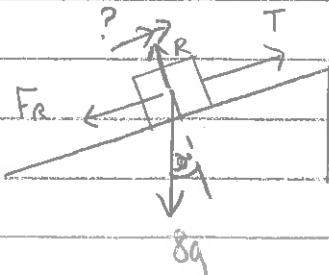
$$F_r = 0.9(4\sqrt{3}g)$$

$$d = 36$$

$$6(9.8) + 0.9(4\sqrt{3})(9.8)$$

$$\text{P moves } = \underline{0.300 \text{ m}} \quad (\text{3.s.f.})$$

(iv)



P will remain at rest if friction + component of weight down slope are greater than tension.

(iv)  $T = 10g$  (equilibrium of Q)

cont'd.

$$F_{\text{net}} + \text{weight component} = 0.9(4\sqrt{3})(9.8) + 8(9.8)\sin 30^\circ \\ = 100.3 \text{ (4s.f.)}$$

$$T = 10(9.8)$$

$$= 98$$

$$100.3 > 98$$

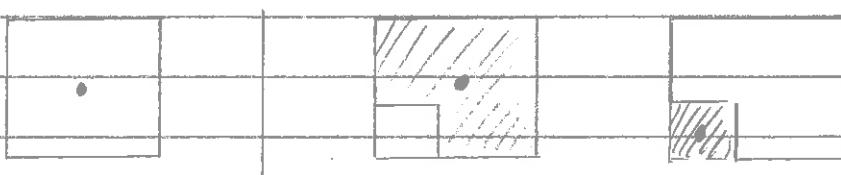
$\therefore P$  remains at rest, system is in equilibrium with both P and Q stationary.

(v) Power =  $FV$

$$= 0.9(4\sqrt{3})(9.8)(2)$$

$$= \underline{122 \text{ W}} \text{ (3s.f.)}$$

3 (i)



MASS & AREA	$a^2$	$a^2 - 1$	1
Cd. M. words	$\frac{a}{2}$	$\bar{x}$	$\frac{1}{2}$
	$\frac{a}{2}$	$\bar{y}$	$\frac{1}{2}$

$$\begin{pmatrix} M(\bar{x}) \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \sum m_i x_i \\ \sum m_i y_i \end{pmatrix} \quad \text{for } x: \quad a^2 \left( \frac{a}{2} \right) = \bar{x}(a^2 - 1) + 1 \left( \frac{1}{2} \right)$$

$$2\bar{x} = a^3 - 1$$

$$\text{let } f(a) = a^3 - 1$$

$$f(1) = 0 \therefore (a-1) \text{ is a factor}$$

$$\text{of } f(a).$$

$$x | a^2 + a + 1$$

$$a | a^3 + a^2 + a$$

$$-1 | -a^2 - a - 1$$

$$\therefore \frac{a^3 - 1}{a^2 - 1} = \frac{(a-1)}{(a-1)(a+1)} (a^2 + a + 1)$$

$$\bar{x} = \frac{a^3 - 1}{2(a^2 - 1)}$$

$$= \frac{a^2 + a + 1}{2(a+1)}$$

$= \bar{y}$  by symmetry

QED

(ii) Cof M lies on line  $y=x$  and on perimeter of shape,  
 $\therefore$  at  $(1,1)$

$$\bar{x} = 1 \Rightarrow \frac{a^2 + a + 1}{2(a+1)} = 1$$

$$a^2 + a + 1 = 2a + 2$$

$$a^2 - a - 1 = 0$$

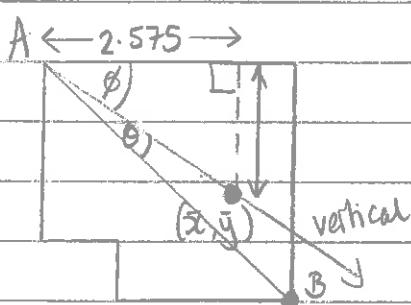
$$(a - \frac{1}{2})^2 - \frac{1}{4} = 1$$

$$(a - \frac{1}{2})^2 = \frac{5}{4}$$

$$a - \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$$

$$a = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$a > 0, \therefore a = \frac{1}{2}(1 + \sqrt{5}) \quad \text{QED}$$



mass	$\frac{4}{3}m$	$m$	$\frac{1}{3}m$
coords	$\bar{x}$	$\frac{4^2 + 4 + 1}{2(4+1)} = 2.1$	$\frac{4}{4}$
of Cof M	$\bar{y}$	$2.1$	$0$

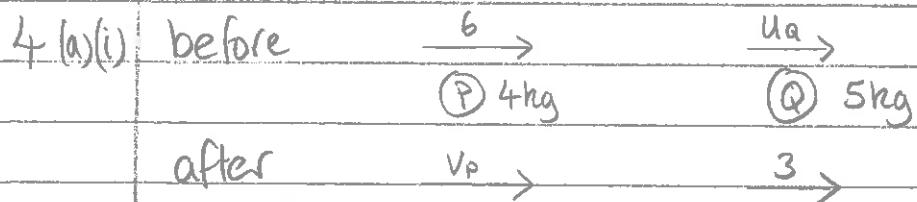
$$\frac{4m}{3} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 2.1 \\ 2.1 \end{pmatrix} m + \frac{1}{3} \begin{pmatrix} 4 \\ 0 \end{pmatrix} m$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{3}{4} \begin{pmatrix} 2.1 \\ 2.1 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2.575 \\ 1.575 \end{pmatrix}$$

$$\phi = \arctan \left( \frac{4 - 1.575}{2.575} \right) \quad \theta = 45^\circ - \phi$$

$$= 1.72^\circ \text{ (3 s.f.) } \Delta \text{ to the vertical}$$



$$\text{KE of } P \text{ before} = \frac{1}{2}(4)(6)^2 = 72 \text{ J}$$

$$\text{KE of } P \text{ after} = \frac{1}{2}(72) = \frac{1}{2}(4)v_P^2$$

velocity of Q  
↓

$$v_P = \pm \sqrt{\frac{1}{2}(\frac{1}{2})72} \\ = \pm 4$$

$4 > 3 \therefore P$  must be travelling at  $4 \text{ ms}^{-1}$  in the opposite direction to its original motion. QED.

(ii) impulse =  $MV - MU$  =  $FT$   
on P =  $4(6) - 4(-4)$

$$= \underline{40 \text{ Ns}} \quad FT = 40 \\ F = \underline{\frac{40}{15}}$$

$$\text{average force} = \underline{200 \text{ N}}$$

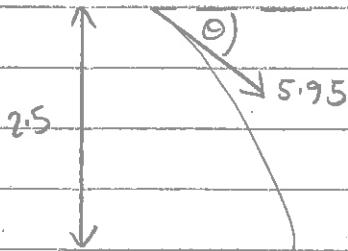
(iii) by cons. of mmbm.  $6(4) + 5U_Q = (-4)(4) + 5(3)$

$$U_Q = \frac{-16 + 15 - 24}{5} \\ = -5$$

Q is travelling at  $5 \text{ ms}^{-1}$  in the opposite direction to P before the collision (ie towards P)

$$e = \frac{3 - -4}{6 - -5} \\ = \underline{\frac{7}{11}}$$

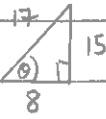
(b) (i)



$$c = \frac{4}{5}$$

$$\sin \theta = \frac{15}{17}$$

$$\cos \theta = \frac{8}{17}$$



vertically

$$s = 2.5$$

$$v^2 = u^2 + 2as$$

$\downarrow$  +ve

$$u = 5.95 \sin \theta = \frac{15}{17}(5.95)$$

$$v = \sqrt{(5.95 \left(\frac{15}{17}\right))^2 + 19.6(2.5)}$$

const. acc'n

$$v =$$

journey to

$$a = 9.8$$

$$= 8.75$$

surface)

$$t$$

Particle rebounds with vertical component of velocity

$$\frac{4}{5}(8.75) = 7 \text{ ms}^{-1}$$

KE at bounce = PE at greatest height h

$$\frac{1}{2} m (7)^2 = m (9.8)h$$

$$h = \frac{49}{2(9.8)}$$

$$= 2.5 \text{ m}$$

travelling downwards

$$v = u + at$$

$$t = \frac{v-u}{a}$$

$$= 8.75 - \frac{15}{17}(5.95)$$

$$\frac{9.8}{9.8}$$

$$= \frac{5}{14}$$

travelling upwards

$$t = \frac{v-u}{a}$$

$$= \frac{0-7}{-9.8}$$

$$= \frac{5}{7}$$

$$\therefore \text{total time} = \frac{5}{14} + \frac{5}{7} = \frac{15}{14}$$

horizontally

$$s = xc$$

$$u = 5.95 \cos \theta$$

$$v$$

$$a = 0$$

$$t = \frac{15}{4}$$

$$s = ut$$

$$x = 5.95 \left(\frac{8}{15}\right) \left(\frac{15}{14}\right)$$

$$= \frac{17}{14}$$

$$= 3 \text{ m} \text{ between the two points.}$$