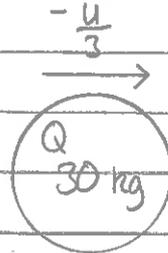
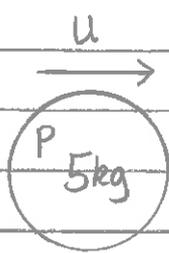


M2 Summer 2014

1.(a)(i)

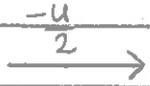
velocity before:

+ve
direction →



be extra careful
with signs ~ I choose
to draw arrows for vel'y
all in same direction, then
-ve sign indicates object
travelling in opposite
direction

velocity after:



by cons. of mmtm

$$5u - \frac{u}{3}(30) = -\frac{5u}{2} + 30v$$

$$v = \frac{1}{30}(5u - 10u + \frac{5}{2}u)$$

vel'y of Q after collision = $-\frac{1}{12}u \text{ ms}^{-1}$ ∴ $\frac{1}{12}u \text{ ms}^{-1}$ in
original direction
of Q.

coeff. of rest.

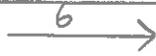
$$e = \frac{\frac{u}{2} - \frac{1}{12}u}{u - -\frac{u}{3}}$$

$$= \frac{5/12}{4/3} = \frac{5}{12} \times \frac{3}{4}$$

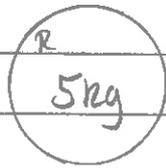
$$= \frac{5}{16}$$

(ii) impulse of P on Q = $M_Q(V_Q - U_Q)$ ← change in momentum of Q
= $-M_P(V_P - U_P)$ ← same as -ve change in mmtm of P
= $-5(-\frac{u}{2} - u)$
= $\frac{15}{2}u \text{ Ns}$

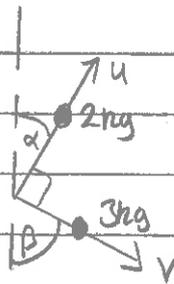
(b) (i)



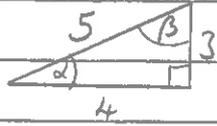
before



after



$$\cos \alpha = \frac{4}{5}$$



$$\sin \alpha = \frac{3}{5}$$

$$\cos \beta = \frac{3}{5}$$

$$\sin \beta = \frac{4}{5}$$

We don't need to know

α and β , only \sin and \cos of α & β

Use EXACT values

by cons. of mmtm.
$$5 \begin{pmatrix} 6 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} u \sin \alpha \\ u \cos \alpha \end{pmatrix} + 3 \begin{pmatrix} v \sin \beta \\ -v \cos \beta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 30 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{16}{5} u \\ \frac{8}{5} u \end{pmatrix} + \begin{pmatrix} \frac{12}{5} v \\ -\frac{9}{5} v \end{pmatrix}$$

from \uparrow

$$0 = 8u - 9v$$

$$v = \frac{8u}{9} \quad (1)$$

from \rightarrow $150 = 6u + 12v$

$$25 = u + 2v \quad (2)$$

subst. (1) into (2):

$$25 = u + 2\left(\frac{8u}{9}\right)$$

$$u = \frac{25}{1 + \frac{16}{9}}$$

$$u = \underline{\underline{9 \text{ ms}^{-1}}}$$

subst. in (1) $v = \frac{8}{9}(9)$

$$v = \underline{\underline{8 \text{ ms}^{-1}}}$$

(ii)

increase in KE = final - initial

$$= \frac{1}{2}(2)(9^2) + \frac{1}{2}(3)(8^2) - \frac{1}{2}(5)(6^2)$$

$$= \underline{\underline{87 \text{ J}}}$$

2(i)	box A	base	4 sides	let each face have mass f
	$5f$	f	$4f$	← MASS
	\bar{z}	0	$\frac{1}{2}a$	← z-word

$$5f\bar{z} = 0 + 4\left(\frac{1}{2}\right)fa$$

$$\bar{z} = \frac{2a}{5}$$

(ii)	object B	object A	WT and UV	VW	
	$8f$	$5f$	$2f$	f	mass
	\bar{x}	0	0	0	← (by symmetry)
	\bar{y}	$\frac{1}{2}a$	$\frac{1}{2}a$	a	
	\bar{z}	$\frac{2}{5}a$	a	a	

∴ x-word is 0

$$8\bar{y} = 5\left(\frac{1}{2}\right)a + 2\left(\frac{1}{2}\right)a + a$$

$$\bar{y} = \frac{1}{8}\left(\frac{5}{2} + 2\right)a$$

$$= \frac{9a}{16} \quad \text{QED}$$

$$8\bar{z} = 5\left(\frac{2}{5}\right)a + 2a + a$$

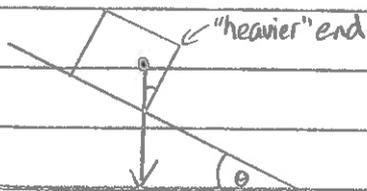
$$\bar{z} = \frac{1}{8}(5a)$$

$$= \frac{5a}{8}$$

∴ words of CoM are

$$\left(0, \frac{9}{16}a, \frac{5}{8}a\right)$$

(iii) take time to "see" which way the object is sitting

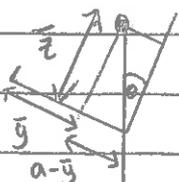


$$\tan \theta = \frac{a - \bar{y}}{\bar{z}}$$

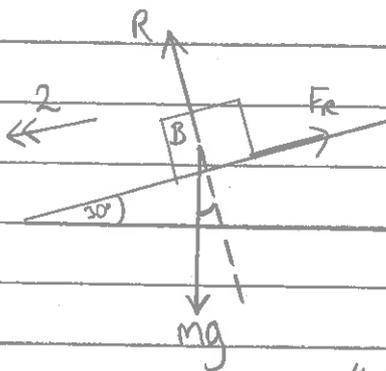
$$\theta = \arctan\left(\frac{\frac{7}{16}a}{\frac{5}{8}a}\right)$$

$$= \arctan\left(\frac{7}{10}\right)$$

$$= \underline{\underline{35.0^\circ}} \quad (3 \text{ s.f.})$$



(iv)



in motion, $\therefore F_r = \mu R$

res \perp to plane, $R = mg \cos 30^\circ$
 $= \frac{\sqrt{3}}{2} mg$

\parallel to plane, $F = ma$

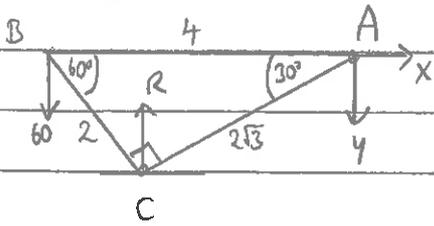
$$mg \sin 30^\circ - F_r = m(2)$$

$$\frac{1}{2} mg - \frac{\sqrt{3}}{2} mg \mu = 2m$$

$$\mu = \frac{-2 + 4.9}{\frac{\sqrt{3}}{2} g}$$

$$= \underline{\underline{0.342}} \text{ (3 s.f.)}$$

3 (a)(i) M(A):



$$2\sqrt{3} \cos 30^\circ R = 60(4)$$

$$R = \frac{240}{3}$$

$$= \underline{80\text{N}} \quad \text{QED}$$

$$\frac{2\sqrt{3} \times \sqrt{3}}{2} = 3$$

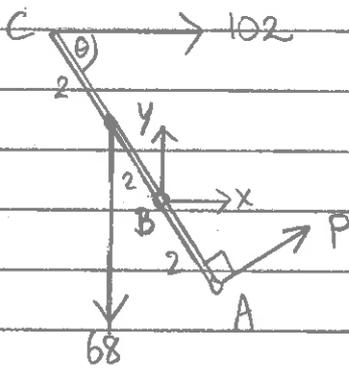
res ↑ $Y + 60 = R$

in equil'm $Y = 80 - 60$
 $= \underline{20\text{N}}$

res → in equil'm $\underline{X = 0}$

(ii) & (iii) on separate sheet.

(b)



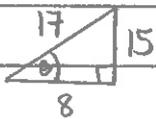
M(B) in equil'm

$$2P + 68 \times 2 \cos \theta = 102 \times 4 \sin \theta$$

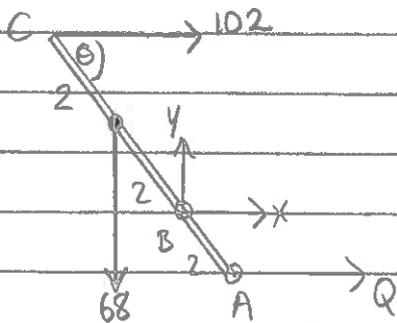
$$2P = 408 \left(\frac{15}{17}\right) - 136 \left(\frac{8}{17}\right) \quad (1)$$

$$P = \underline{148\text{N}}$$

$$\sin \theta = \frac{15}{17}, \quad \cos \theta = \frac{8}{17}$$



Pythagorean triple



M(B) in equil'm

$$2 \sin \theta Q + 68 \times 2 \cos \theta = 102 \times 4 \sin \theta$$

by comparing
to eqn (1)

$$Q = \frac{P}{\sin \theta}$$

$$= 148 \times \frac{17}{15}$$

$$= \underline{168\text{N}} \quad (3\text{s.f.})$$

force by hinge



magnitude of force is $\sqrt{68^2 + (148(\frac{17}{15}) + 102)^2}$
 $= \underline{278\text{N}} \quad (3\text{s.f.})$

3(a)(i)
(iii)

$$\text{At B res } \uparrow 60 + T_{BC} \sin 60^\circ = 0$$

"special Δs"

in equil'm

$$T_{BC} = \frac{-60}{\frac{\sqrt{3}}{2}}$$

$$= -40\sqrt{3}$$

$$\text{res } \rightarrow T_{AB} + T_{BC} \cos 60^\circ = 0$$

in equil'm

$$T_{AB} = 40\sqrt{3} \times \frac{1}{2}$$

$$= 20\sqrt{3}$$

$$\text{At A res } \uparrow 20 + T_{AC} \sin 30^\circ = 0$$

in equil'm

$$T_{AC} = \frac{-20}{\frac{1}{2}}$$

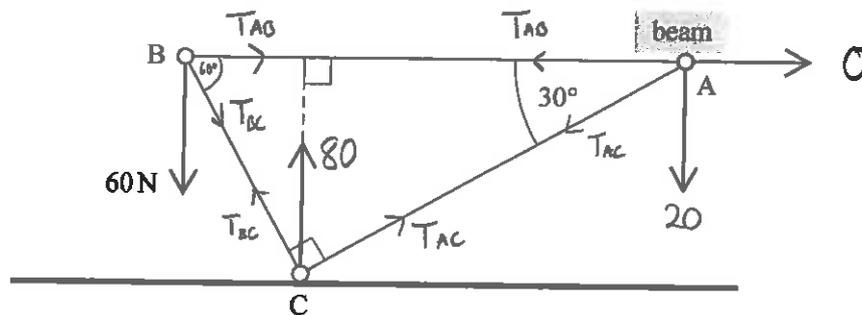
$$= -40$$

∴ rod AB in TENSION, $20\sqrt{3}$ N

rod BC in THRUST, $40\sqrt{3}$ N

rod AC in THRUST, 40 N

3(a)(ii) A spare copy of this diagram can be found on page 13



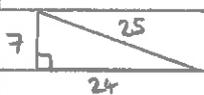
4 (i) GPE required to reach B is $mgh = 10(9.8)(14)$
 $= 1372 \text{ J}$

Initial KE is $\frac{1}{2}mv^2 = \frac{1}{2}(10)(16.6)^2 = 1377.8 \text{ J}$

AB is smooth. Initial KE > GPE at B, \therefore object still has energy at B, so continues moving beyond B.

(ii) energy at B + loss in GPE = Work done + KE at D
 against res.

$1377.8 - 1372 + 10(9.8)(7) = 14(25) + \text{KE}$



$\frac{1}{2}(10)v_D^2 = \frac{1709}{5}$

$v_D = \sqrt{\frac{1709}{25}}$

$= \underline{\underline{8.27 \text{ ms}^{-1}}}$ (3 s.f.)

(iii) vert. motion, loss in GPE = KE at ground

$10(9.8)(7) = \frac{1}{2}(10)v_g^2$

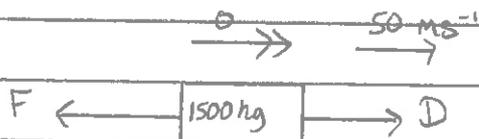
$\sqrt{14g} = v_g$

leaves ground with $v = \frac{1}{2}\sqrt{14g}$

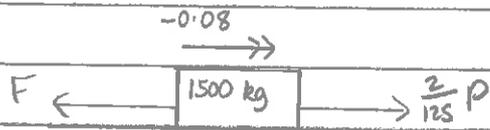
at greatest height $\frac{1}{2}(10)\left(\frac{1}{4}(14g)\right) = mgh$

$h = \frac{14}{8}$
 $= \underline{\underline{1.75 \text{ m}}}$

(b)



initially $D = F$, Power = $50F$
 $P = 50F$ ①



power = $0.8(50)F$ new driving force
is $\frac{0.8P}{50} = \frac{2}{125} P$

$$F = ma \quad \frac{2}{125} P - F = 1500(-0.08) \quad \text{②}$$

$$\text{subst. ① into ②: } \frac{2}{125}(50)F - F = -120$$

$$F = \underline{\underline{600 \text{ N}}}$$

$$P = 50(600)$$

$$= \underline{\underline{30\,000 \text{ W}}}$$

$$= \underline{\underline{30 \text{ kW}}}$$

(by subst. in ①)