

# M2 Summer 2013

1(a)(i) before  $\xrightarrow{3}$  impulse = change in mmtm  
 (4) FE =  $mv - mu$   
 $2(2) = 4v - 4(3)$   
 after  $\xrightarrow{v}$   $4v + 12 = 4v$

be clear about +ve/-ve direction  $v = \underline{\underline{13.5 \text{ ms}^{-1}}}$   
 either draw  $\leftarrow 3$  or  $\rightarrow 3$ , but use  $-3$  in the equation

(ii) before  $\xrightarrow{+ve}$  after by cons of mmtm.  
 $13.5$   $-3$   $4(13.5) + 2(-3) = 6v$   
 $\rightarrow$   $\rightarrow$   $v = \underline{\underline{8 \text{ ms}^{-1}}}$   
 P (4) Q (2) (6) S +ve  $\therefore$  towards R

(iii) before  $\xrightarrow{+ve}$  after so if vely turns out -ve, we know it's going in opp direction.  
 $8$   $v_1$   $5$   $v_2$   $v_2 - 5 = \frac{1}{4}(8 - v_1)$   
 $S(6)$   $R(4)$   $S(6)$   $R(4)$   $4v_2 - 20 = 8 - v_1$   
 $e = \frac{1}{4}$   $4v_2 + v_1 = 28$  (1)

by cons. of mmtm

$$6(8) + 4v_1 = 6(5) + 4v_2 \quad (1) - (2): 10 = 5v_1$$

$$18 = 4v_2 - 4v_1 \quad (2) \quad 2 = v_1$$

subst. in (1):  $v_2 = (28 - 2) \times \frac{1}{4} = 6.5$

$\therefore$  R moves at  $2 \text{ ms}^{-1}$  before and  $6.5 \text{ ms}^{-1}$  after the collision, both in the direction SR.

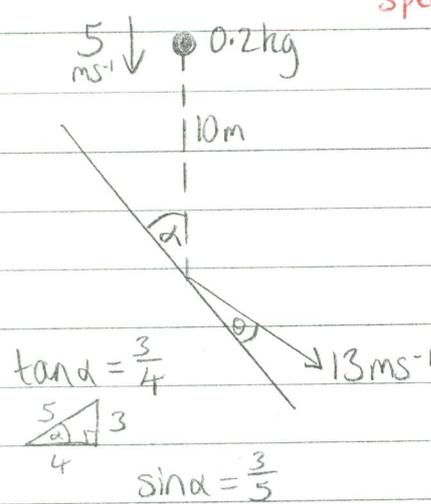
↑  
 must state direction, since velocity is a vector quantity.

take your time to get this correct way with correct +ve/-ve  
 +ve sep. if front > back } when drawn  
 +ve app. if back > front }  $\rightarrow \rightarrow \rightarrow$

take care when solving simultaneous equations - marks lost for slips here

speeds up over the 10m drop, so use suvat to find approach speed.

1 (b)(i)



journey from projection to collision with plane. ↓ +ve

$$s = 10 \quad v^2 = u^2 + 2as$$

$$u = 5 \quad = 5^2 + 2(10)(10)$$

$$v = \sqrt{225}$$

$$= 15 \text{ ms}^{-1}$$

$$a = 10$$

$$t$$

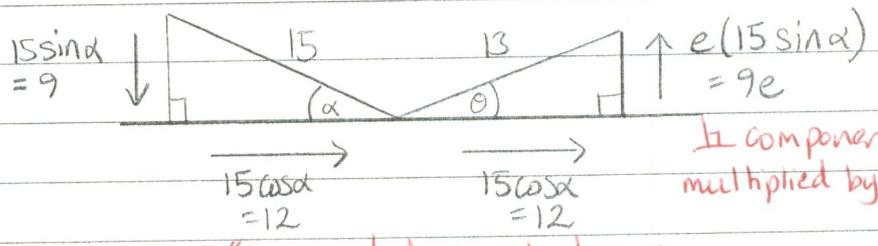
$\tan \alpha = \frac{3}{4}$

$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

All Δs are Pythagorean triples in this question 😊

DIAGRAM showing components of velocity // and ⊥ to plane



⊥ component multiplied by e

// component does not change

$\cos \theta = \frac{12}{13}$

by Pythag, vel'y ⊥ to plane is  $5 \text{ ms}^{-1} \therefore 5 = 9e$

Δ between motion & plane after =  $22.6^\circ$  (3s.f.) the collision

could use  $e = \frac{5}{9}$

$\tan \theta = e \tan \alpha$

(ii) change of mmtm = impulse

$$mv - mu = 0.2 \begin{pmatrix} 12 & -12 \\ 9 & -5 \end{pmatrix}$$

$$= 0.2 \begin{pmatrix} 0 \\ 14 \end{pmatrix} = \begin{pmatrix} 0 \\ 2.8 \end{pmatrix}$$

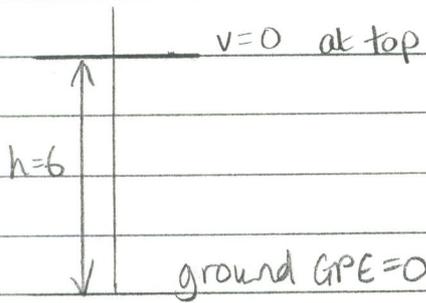
vector quantities!

⊥ to plane, one vel'y towards plane, one away, so must have different signs.

$|9 - (-5)| = |-9 - 5|$  gives same modulus (magnitude)

$\therefore$  |impulse| is  $2.8 \text{ Ns}$  acting ⊥ to the plane

2.(i)



Work done by lift = gain in GPE + Work done against res.

$$= mgh + Fd$$

$$= 800(9.8)(6) + 400(6)$$

$$= \underline{\underline{49440 \text{ J}}}$$

I think of the "=" as meaning "turns into"

average power =  $\frac{\text{Energy}}{\text{time}}$

$$= \frac{49440}{12}$$

$$= \underline{\underline{4120 \text{ W}}}$$

Take time to write an energy equation correctly, getting each term on the correct side (or with the appropriate +/- sign)

(ii)

Work done =  $Fd$  need to calculate the force first

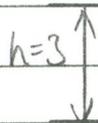
$$49440 = 6F$$

$$8240 = F$$

Power =  $Fv$

$$= 8240(0.55)$$

$$= \underline{\underline{4532 \text{ W}}}$$



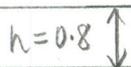
GPE = KE + Work done against res.

$$mgh = \frac{1}{2}mv^2 + Fd$$

$$800(9.8)(3) = \frac{1}{2}(800)v^2 + 400(3)$$

$$\frac{2(400(3) + 800(9.8)(3))}{800} = v^2$$

$$v = \underline{\underline{7.47 \text{ ms}^{-1}}} \text{ (3 s.f.)} \quad (\text{shared in calc A})$$



GPE + KE = KE + Work done against res. + Work done by brakes

$$mgh + \frac{1}{2}mv^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + Fd + \text{W.D.}$$

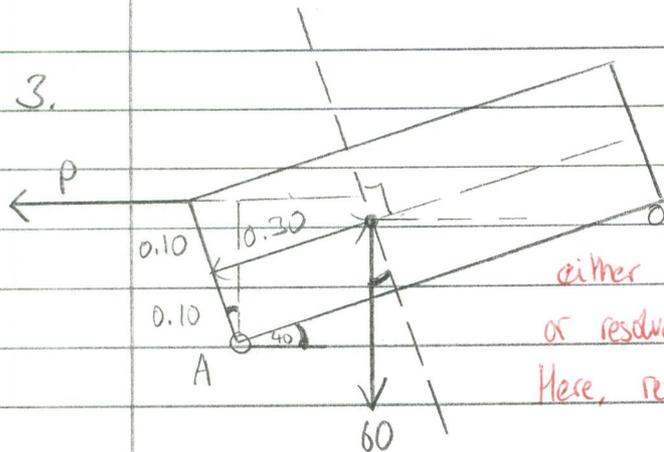
Work done by brakes =  $800(9.8)(0.8) + \frac{1}{2}(800)(7.47^2 - \left(\frac{7.47}{2}\right)^2) - 400(0.8)$

$$= \underline{\underline{22692 \text{ J}}}$$

Use EXACT value in calculations

read the question!

3.



(i) M(A): weight only

$$0.30(60 \cos 40^\circ) - 0.10(60 \sin 40^\circ) = 9.93207\dots$$

either find  $\perp$  dist. to line of action of force,  $= 9.93 \text{ N}$   
or resolve into two  $\perp$  components. (3s.f.)

Here, resolving was more efficient

[stored in calc B]

(ii) M(A): in equil'm  $\sum \tau = 0$

$$0.2 \cos 40^\circ P = 9.93$$

$$P = 9.93$$

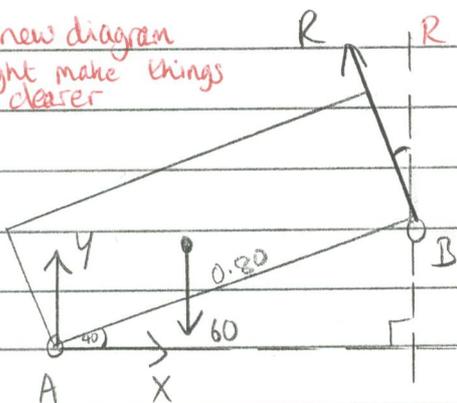
$$0.2 \cos 40^\circ$$

$$= \underline{\underline{64.8 \text{ N}}} \text{ (3s.f.)}$$

use answer (exact value) from (i)

(iii)

a new diagram might make things clearer



R is  $\perp$  to the surface resting on the peg.

let vert. comp't req'd be Y.

M(A): in equil'm  $\sum \tau = 0$

$$0.8 R = 9.93$$

$$R = 9.93$$

$$0.8$$

$$= 12.4$$

[stored in C]

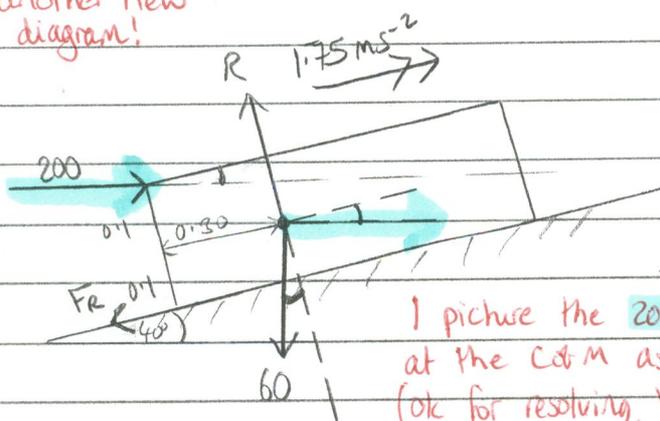
$$\text{res } \uparrow R \cos 40^\circ + Y = 60$$

$$Y = 60 - 12.4 \cos 40^\circ$$

$$= \underline{\underline{50.5 \text{ N}}} \text{ (3s.f.)}$$

(iv)

another new diagram!



res  $\perp$  to plane

$$R = 60 \cos 40^\circ + 200 \sin 40^\circ$$

$$= 174.5$$

[stored in D]

// to plane,  $F = ma$

$$200 \cos 40^\circ - 60 \sin 40^\circ - F_r = \frac{60}{g} (1.75)$$

$$103.9 = F_r$$

I picture the 200 force as acting at the C.M. as shown.

(ok for resolving, but couldn't do this taking moments) [store in E]

3(iv) in motion:  $F_a = \mu R$   
 can't

$$\mu = \frac{F_a}{R}$$

$$= \frac{103.9}{174.5}$$

$$= \underline{\underline{0.596}} \quad (3 \text{ s.f.})$$

watch out for -ve y-values.

a clear table helps

4(a) part of shape	whole shape				
mass or length	7.4	2.4	1.2	2.5	1.3
x coord of CoM	$\bar{x}$	1.2	2.4	$\frac{1}{2}(2.5)\cos\theta$	$1.2 + \frac{1}{2}(1.3)\cos\alpha$
y " " "	$\bar{y}$	0	-0.1	$-\frac{1}{2}(2.5)\sin\theta$	$\frac{1}{2}(1.3)\sin\alpha$

The  $\Delta$ s are all multiples of Pythagorean triples again!

Where  $\cos\theta = \frac{24}{25}$   $\cos\alpha = \frac{12}{13}$   
 $\sin\theta = \frac{7}{25}$   $\sin\alpha = \frac{5}{13}$   
 don't need  $\theta$ , just  $\sin\theta$  &  $\cos\theta$ , we EXACT values.

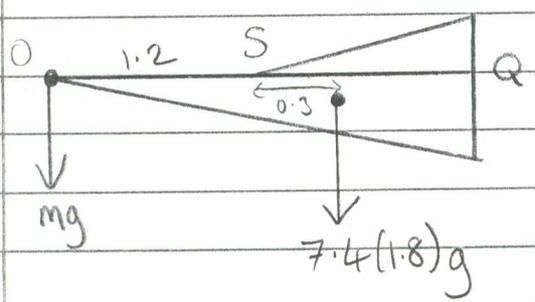
$$7.4 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 2.4 \begin{pmatrix} 1.2 \\ 0 \end{pmatrix} + 1.2 \begin{pmatrix} 2.4 \\ -0.1 \end{pmatrix} + 2.5 \begin{pmatrix} \frac{5}{7}(\frac{24}{25}) \\ -\frac{5}{7}(\frac{7}{25}) \end{pmatrix} + 1.3 \begin{pmatrix} 1.2 + \frac{12}{20}(\frac{12}{13}) \\ \frac{12}{20}(\frac{5}{13}) \end{pmatrix}$$

$$7.4 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 2.88 \\ 0 \end{pmatrix} + \begin{pmatrix} 2.88 \\ -0.12 \end{pmatrix} + \begin{pmatrix} 3 \\ -0.875 \end{pmatrix} + \begin{pmatrix} 2.34 \\ 0.325 \end{pmatrix}$$

these are all exact values

$$\bar{x} = \frac{2.88 + 2.88 + 3 + 2.34}{7.4} \quad \bar{y} = \frac{0 + -0.12 + -0.875 + 0.325}{7.4}$$

x-coord is = 1.5 and y-coord is =  $\frac{-67}{740} = \underline{\underline{-0.0905}}$  (3s.f.)



M(S): in equil'm  $\Sigma = 0$

$$1.2mg = 0.3(7.4)(1.8)g$$

$$m = \underline{\underline{1.48}} \text{ kg}$$

Alternatively, could use CoM method...

4. (i) If OR had tension  $T_{OR}$  and RQ had tension  $T_{RQ}$   
 we'd get  $\text{res} \rightarrow T_{RQ} \cos 45^\circ = 0$      $\text{res} \uparrow T_{OR} = T_{RQ} \cos 45^\circ$   
 show clearly WHY tensions are both zero. (resolving is easiest way to do this)     $T_{RQ} = 0$      $T_{OR} = 0$

(ii)  $\text{res} \uparrow Y = 120 + 60$      $\text{res} \rightarrow T = X$   
 in equil'm     $= 180\text{N}$     in equil'm

M(O):  $3T = 60(2) + 120(1)$     distances are  $\perp$  dist. to line of action of the force.  
 in equil'm     $T = \frac{2(120)}{3}$   
 $= 80$      $\therefore \underline{T = 80\text{N}, X = 80\text{N}, Y = 180\text{N}}$

(iii) at P,  $\text{res} \uparrow$ ,  $T_{OP} \sin \theta + 60 = 0$     make sure names of tensions "Top" etc. correspond to your diagram from (iii)  
 in equil'm     $T_{OP} = \frac{-60}{3/\sqrt{3}}$   
 $= -20\sqrt{3}$     -ve  $\therefore$  comp

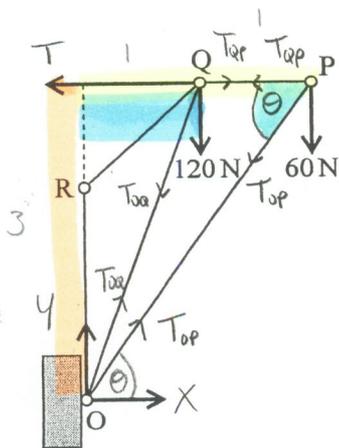
$\text{res} \rightarrow$ ,  $T_{OP} + T_{OP} \cos \theta = 0$   
 in equil'm     $T_{OP} = - -20\sqrt{3} \times \frac{2}{\sqrt{3}}$   
 $= 40$     +ve  $\therefore$  tens.

don't forget to state TENSION or COMPRESSION

OP is in compression,  $20\sqrt{3}\text{N}$   
PQ is in tension,  $40\text{N}$

4(b)(iii)

don't need forces in QR or OR as already shown these are zero in (i)



$\sin \theta = \frac{3}{\sqrt{13}}$     ← using SOHCAHTOA to get exact values (& Pythag.)  
 $\cos \theta = \frac{2}{\sqrt{13}}$

