

M2 June 2006 Worked Solutions

i) Momentum: $2 \times 4 + 6 \times (-2) = 8 \times v$ Str Maths M2 06S
 $\therefore v = -0.5 \text{ ms}^{-1}$

kinetic energy lost $= \frac{1}{2} \times 2 \times 4^2 + \frac{1}{2} \times 6 \times 2^2 - \frac{1}{2} \times 8 \times (0.5)^2$
 $= 27 \text{ J}$

ii) After 

Restitution: $v_Q - v_P = 6e = 4$ (1)

Momentum $8 - 12 = 2v_P + 6v_Q$
 $\Rightarrow -2 = v_P + 3v_Q$ (2)

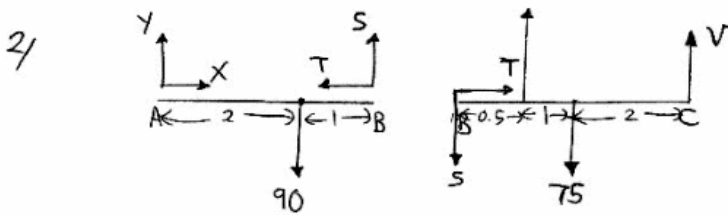
(1) + (2) $\Rightarrow 2 = 4v_Q \therefore v_Q = 0.5 \text{ ms}^{-1}$
 (1) and above $\Rightarrow v_P = -3.5 \text{ ms}^{-1}$

b) Horizontal: no impulse $\Rightarrow u \cos(\arcsin \frac{12}{13}) = v \cos(\arcsin \frac{2}{5})$
 as $u = 26 \Rightarrow v = \frac{26 \cos(\arcsin \frac{12}{13})}{\cos(\arcsin \frac{2}{5})} = \frac{26 \times \frac{5}{13}}{\frac{4}{5}}$
 $v = 12.5 \text{ ms}^{-1}$

Perpendicular to the plane $e = \frac{v \sin(\arcsin \frac{2}{5})}{26 \sin(\arcsin \frac{12}{13})}$
 $e = \frac{12.5 \times \frac{2}{5}}{26 \times \frac{12}{13}}$
 $e = \frac{5}{16}$

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Rod AB: Moments about A : $2 \times 90 - 3S = 0$
 $\therefore S = 60\text{N}$ upwards.

Rod BC: Resolve horizontally $\Rightarrow T = 0\text{N}$

Moments about R in rod BC

$$0.5S + 3V = 1 \times 75$$

$$\therefore V = \frac{75 - 60 \times 0.5}{3}$$

$$\therefore V = 15\text{N}$$

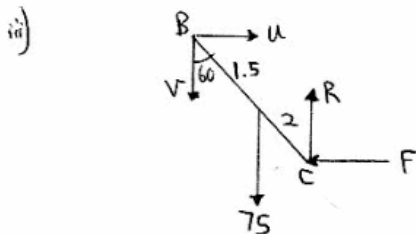
ii) Moments about A

$$2 \cos 30^\circ \times 90 = 3 \sin 60^\circ V + 3 \cos 60^\circ U$$

$$90\sqrt{3} = \frac{3\sqrt{3}}{2} V + \frac{3U}{2}$$

$$180\sqrt{3} = 3\sqrt{3} V + 3U$$

$$\therefore 60\sqrt{3} = U + V\sqrt{3}$$



Moments about C

$$3.5 \sin 60^\circ V + 2 \sin 60^\circ \times 75 = 3.5 \cos 60^\circ U$$

$$7 \frac{\sqrt{3}}{2} V + 300 \frac{\sqrt{3}}{2} = \frac{7}{2} U$$

$$\therefore U - V\sqrt{3} = \frac{300\sqrt{3}}{7}$$

we had $U + V\sqrt{3} = 60\sqrt{3}$

add to get $2U = \frac{720\sqrt{3}}{7}$

$$U = \frac{360\sqrt{3}}{7}$$

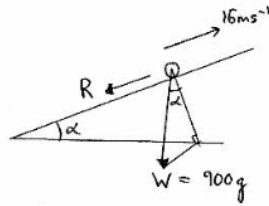
$$\therefore V = \frac{60}{7}$$

On BC resolve horizontally $\Rightarrow F = U = \frac{360\sqrt{3}}{7}\text{N}$

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3/a)



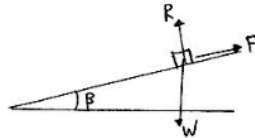
$$P = Fv$$

$$\Rightarrow 20000 = (R + W \sin \alpha) \times 16$$

$$\frac{20000}{16} = R + 900 \times 9.8 \times 0.1$$

$$\therefore R = 358 \text{ N}$$

b)



Resolve down the slope $W \sin \beta = F$ ①

Friction equation is $F \leq \mu R$ ②

$F = F_{\text{max}}$ ① & ② $\Rightarrow W \sin \beta \leq \mu R$

but $R = W \cos \beta \Rightarrow W \sin \beta \leq \mu W \cos \beta$

$\Rightarrow \mu \geq \frac{\sin \beta}{\cos \beta}$

$\therefore \mu \geq \frac{5}{12}$

$$\mu = 0.2$$

$$\text{Kinetic energy lost} = \frac{1}{2} \times 11 \times v^2$$

$$\text{PE gained} = 11 \times 9.8 \times 1.5 \sin \beta$$

$$\text{Work done against friction} = 0.2 \times (11 \times 9.8 \times \cos \beta) \times 1.5$$

$$\text{Work done against air} = 6 \text{ J} \times 1.5 = 9 \text{ J}$$

$$\therefore \frac{1}{2} \times 11 \times v^2 = 11 \times 9.8 \times 1.5 \sin \beta + 0.2 \times 11 \times 9.8 \times \cos \beta \times 1.5 + 9$$

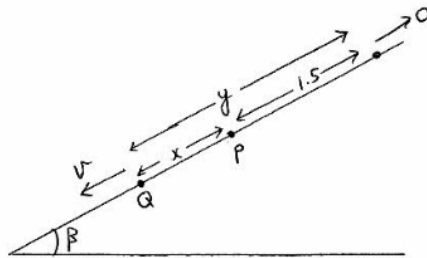
$$5.5 v^2 = 101.0446154$$

$$v^2 = 18.37174825$$

$$v = 4.286227741 \dots$$

$$v = 4.286 \text{ ms}^{-1}$$

iii)



$$\text{Loss of GPE} = 11g y \sin \beta$$

$$\text{Gain of KE} = \frac{1}{2} \times 11 v^2$$

$$\text{Work done against friction} = 0.2 \times 11g \cos \beta \times y$$

$$\text{Work done against air} = 6y$$

$$\text{Then } 11g y \sin \beta = \frac{1}{2} \times 11 v^2 + 0.2 \times 11g \cos \beta y + 6y$$

$$\therefore y = \frac{11g \sin \beta - 2.2g \cos \beta}{g} - 6$$

$$y = 6.493869884 \dots$$

$$\therefore x = y - 1.5 = 4.993869884 \dots$$

$$= 4.99 \text{ m}$$

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4i) $100 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 10 \begin{pmatrix} 5 \\ 0 \end{pmatrix} + 30 \begin{pmatrix} 10 \\ 15 \end{pmatrix} + 30 \begin{pmatrix} 20 \\ 15 \end{pmatrix} + 30 \begin{pmatrix} 25 \\ 30 \end{pmatrix}$

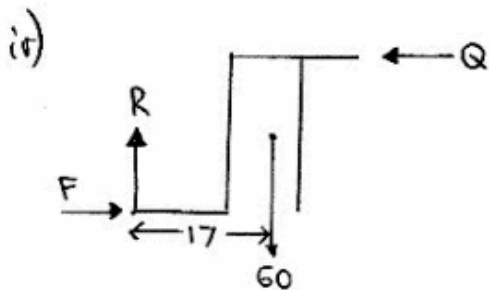
$100 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1700 \\ 1800 \end{pmatrix}$

$\bar{x} = 17$ $\bar{y} = 18$

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ii) $(17, 18, 20)$

iii) MOMENTS about D: $20P = 60 \times 3$
 $P = 9$



v) Moment about OZ

$$30 \times Q = 60 \times 17$$

$$Q = 34 \text{ N}$$

Resolving horizontally $F = Q$

As $34 > 30$ it slips first.