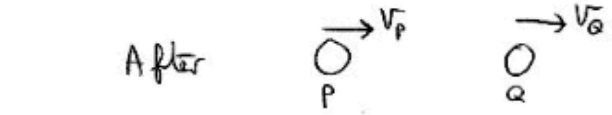
M2 June 2006 Worked Solutions

1/ Momentum: $2\times4 + 6\times(-2) = 8\times v$ **Str Maths M2 06S**

$\therefore \quad v = -0.5\,ms^{-1}$

Kinetic energy lost $= \frac{1}{2}\times2\times4^2 + \frac{1}{2}\times6\times2^2 - \frac{1}{2}\times8\times(0.5)^2$

$= 27\,J$

ii) After 

Restitution: $v_Q - v_P = 6e = 4$ ①

Momentum $8 - 12 = 2v_P + 6v_Q$

$\Rightarrow \quad -2 = v_P + 3v_Q$ ②

① + ② $\Rightarrow \quad 2 = 4v_Q \quad \therefore v_Q = 0.5\,ms^{-1}$

① and above $\Rightarrow \quad v_P = -3.5\,ms^{-1}$

b) Horizontal: no impulse $\Rightarrow \quad u\cos(\arcsin\frac{12}{13}) = v\cos(\arcsin\frac{3}{5})$

as $u = 26 \Rightarrow$

$$v = \frac{26\cos(\arcsin\frac{12}{13})}{\cos(\arcsin\frac{3}{5})} = \frac{26\times\frac{5}{13}}{\frac{4}{5}}$$

$v = 12.5\,ms^{-1}$

Perpendicular to the plane

$$e = \frac{v\sin(\arcsin\frac{3}{5})}{26\sin(\arcsin\frac{12}{13})}$$

$$e = \frac{12.5\times\frac{3}{5}}{26\times\frac{12}{13}}$$

$$e = \frac{5}{16}$$

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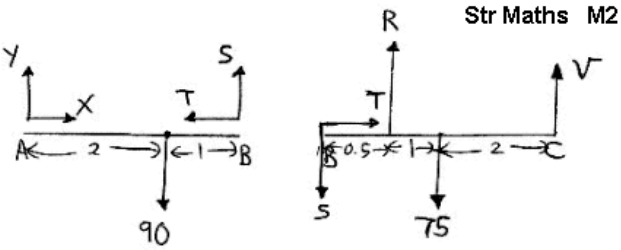
2/



Rod AB: Moments about A : $2 \times 90 - 3S = 0$

$\therefore S = 60N$ upwards.

Rod BC: Resolve horizontally $\Rightarrow T = 0\,N$

Moments about R in rod BC

$$0.5\,S + 3V = 1 \times 75$$

$$\therefore V = \frac{75 - 60 \times 0.5}{3}$$

$$\therefore V = 15N$$

ii) Moments about A

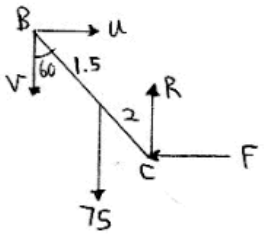
$$2\cos 30 \times 90 = 3\sin 60\,V + 3\cos 60\,U$$

$$90\sqrt{3} = \frac{3\sqrt{3}}{2}V + \frac{3U}{2}$$

$$180\sqrt{3} = 3\sqrt{3}V + 3U$$

$$\therefore 60\sqrt{3} = U + V\sqrt{3}.$$

iii)



Moments about C

$$3.5\sin 60\,V + 2\sin 60 \times 75 = 3.5\cos 60\,U$$

$$7\,\frac{\sqrt{3}}{2}V + 300\,\frac{\sqrt{3}}{2} = \frac{7}{2}U$$

$$\therefore U - V\sqrt{3} = \frac{300\sqrt{3}}{7}$$

we had $U + V\sqrt{3} = 60\sqrt{3}$

add to get $2U = \frac{720\sqrt{3}}{7}$

$$U = \frac{360\sqrt{3}}{7}$$

$$\therefore V = \frac{60}{7}$$

On BC resolve horizontally $\Rightarrow F = U = \frac{360\sqrt{3}}{7}N$

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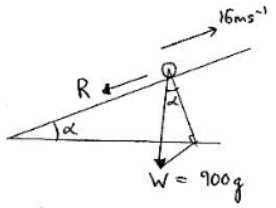
3/a)



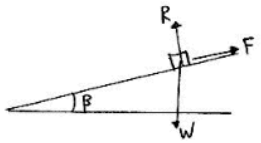
$P = Fv$

$\Rightarrow 20000 = (R + W\sin\alpha) \times 16$

$\frac{20000}{16} = R + 900 \times 9.8 \times 0.1$

$\therefore R = 358\,N$

b)



Resolve down the slope $W\sin\beta = F$ ①

Friction equation is $F \leq \mu R$ ②

$F = F_{max}$ ① & ② $\Rightarrow W\sin\beta \leq \mu R$

but $R = W\cos\beta \Rightarrow W\sin\beta \leq \mu W\cos\beta$

$\Rightarrow \mu \geq \tan\beta$

$\therefore \mu \geq \frac{5}{12}$

$\mu = 0.2$

Kinetic energy lost $= \frac{1}{2} \times 11 \times v^2$

PE gained $= 11 \times 9.8 \times 1.5\sin\beta$

Work done against Friction $= 0.2 \times (11 \times 9.8 \times \cos\beta) \times 1.5$

Work done against air $= 6\,J/m \times 1.5 = 9\,J$

$\therefore \frac{1}{2} \times 11 \times v^2 = 11 \times 9.8 \times 1.5 \times \sin\beta + 0.2 \times 11 \times 9.8 \times \cos\beta \times 1.5 + 9$

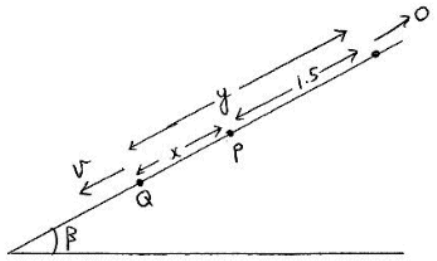
$5.5v^2 = 101.0446154$

$v^2 = 18.37174825$

$v = 4.286227741\ldots$

$v = 4.286\ ms^{-1}$

iii)



Loss of GPE $= 11g\,y\sin\beta$

Gain of KE $= \frac{1}{2} \times 11v^2$

Work done against friction $= 0.2 \times 11g\cos\beta \times y$

Work done against air $= 6y$

Then $11g\,y\sin\beta = \frac{1}{2} \times 11v^2 + 0.2 \times 11g\cos\beta\,y + 6y$

$\frac{5.5v^2}{}$

$\therefore y = 11g\sin\beta - 2.2g\cos\beta\,y - 6$

$y = 6.493869884\ldots$

so $x = y - 1.5 = 4.993869884\ldots$

$= 4.99m.$

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4i) $100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10\begin{pmatrix}5\\0\end{pmatrix} + 30\begin{pmatrix}10\\15\end{pmatrix} + 30\begin{pmatrix}20\\15\end{pmatrix} + 30\begin{pmatrix}25\\30\end{pmatrix}$

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$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}1700\\1800\end{pmatrix}$

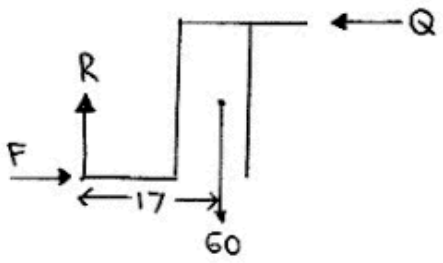
$\bar{x} = 17 \qquad \bar{y} = 18$

ii) $(17, 18, 20)$

iii) MOMENTS about D: $20P = 60 \times 3$

$P = 9$

iv)



v) Moment about OZ

$30 \times Q = 60 \times 17$

$Q = 34\,N$

Resolving horizontally $F = Q$

As $34 > 30$ it slips first.