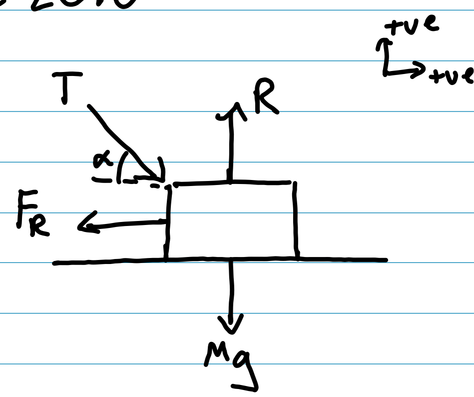
M1 June 2016

1) i)



ii) $\rightarrow$

Resolve $\quad T\cos\alpha - F_R = Ma$

$$40\cos 60 - F_R = 5 \times 1.5$$

$$F_R = 40\cos 60 - 5 \times 1.5 = 12.5\,N$$

2 i)

$$s = ut + \tfrac{1}{2}at^2$$

At A, $t=2$: $\quad 12 = u \times 2 + \tfrac{1}{2}a(2)^2$

① $12 = 2u + 2a$

$t=6$: $\quad 12 = \cancel{2u}\, 6u + \tfrac{1}{2}a(6)^2$

② $12 = 6u + 18a$

3×① $36 = 6u + 6a$

$-24 = 12a \Rightarrow a = -2$

$12 = 2u + 2(-2)$

$2u = 16$

$\underline{u = 8} \quad \underline{a = -2}$

ii) At B, $v=0$

$s = ?$
$u = 8$
$v = 0$
$a = -2$
$\cancel{t}$

$$v^2 = u^2 + 2as$$

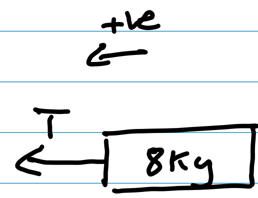
$$0 = 8^2 + 2(-2)s$$

$$4s = 64$$

$$s = 16$$

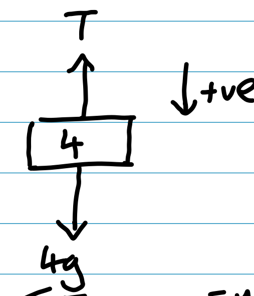
So $AB = 16 - 12 = 4m$

3 i)



$\Sigma F = ma$

$T = 8a$



$\Sigma F = ma$

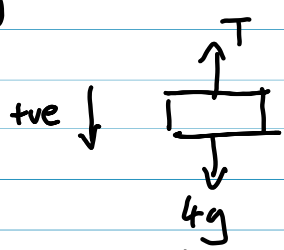
$4g - T = 4a$

ii) $4g - 8a = 4a$

$4 \times 9.8 = 12a$

$a = \frac{49}{15} = 3.27 \text{ ms}^{-2}$

iii)

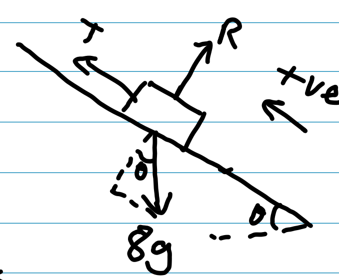


$\Sigma F = ma$

$4g - T = 0$

$T = 4 \times 9.8$

$= 39.2$



~~down~~ Up slope

$\Sigma F = ma$

$T - 8g\sin\theta = 0$

$39.2 - 8 \times 9.8 \sin\theta = 0$

$\sin\theta = \frac{39.2}{8 \times 9.8} = \frac{1}{2}$

$\theta = 30°$

4 i) $\underline{v} = \frac{d\underline{r}}{dt}$ $\quad \underline{r} = \begin{pmatrix} 2t \\ 6t-4t^2 \end{pmatrix}$

$$\underline{v} = \begin{pmatrix} \frac{d(2t)}{dt} \\ \frac{d(6t-4t^2)}{dt} \end{pmatrix} = \begin{pmatrix} 2 \\ 6-8t \end{pmatrix} = \underset{\substack{\text{when}\\ t=0}}{\begin{pmatrix} 2 \\ 6 \end{pmatrix}}$$

$\underline{u} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $\quad \underline{a} = \frac{d\underline{u}}{dt} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}$

ii) At $t=2$ $\underline{v} = \begin{pmatrix} 2 \\ 6-8(2) \end{pmatrix} = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$

$$|\underline{v}| = \sqrt{2^2+(-10)^2} = \sqrt{104} = 10.2\,ms^{-1}$$

iii) $x = 2t$ $\quad y = 6t - 4t^2$

$t = \frac{x}{2}$ $\quad y = 6\left(\frac{x}{2}\right) - 4\left(\frac{x}{2}\right)^2$

$y = 3x - x^2$ as required.

5

	Horizontal (x)	Vertical (y)
initial pos	0	0
$\underline{a}$	0	-9.8
$\underline{u}$	15	8
$\underline{v}$	15	$8-9.8t$
$\underline{r}$	$15t$	$8t-4.9t^2$

Find highest point $\Rightarrow v_y = 0$ $\quad 8 - 9.8t = 0$

$$t = \frac{8}{9.8} = \frac{40}{49} = 0.816$$

when $t = 0.\cancel{82}\,816$ then $r_y = 8(\cancel{0.82}\,0.816) - 4.9(\cancel{0.82}\,0.816)^2$

$= 3.27m$

So since the highest the stone goes is 3.27m it will not reach the height of the pigeon at 4m.

When $\cancel{s}$ $r_x = 22.5$

$$22.5 = 15t$$
$$t = 1.5\text{ sec}$$

when $t = 1.5$

$$r_y = 8(1.5) - 4.9(1.5)^2 = 0.975$$

The window is between 0.8m and 2m and

$$0.8 < 0.975 < 2\text{m}$$

So he hits his window.

Section B

6 i) $g = 10\text{ms}^{-2}$

$$d = 5(8.0)^2 = 320\text{m}$$

This is not consistent with records as $320 > 200 > 150$

Piran has used $s = ut + \frac{1}{2}at^2$ with $s = d$, $u = 0$, $a = 10$, $t = T$

So $d = 0 \times T + \frac{1}{2} \times 10 \times T^2$

$$d = 5T^2$$

ii) displacement is area under graph

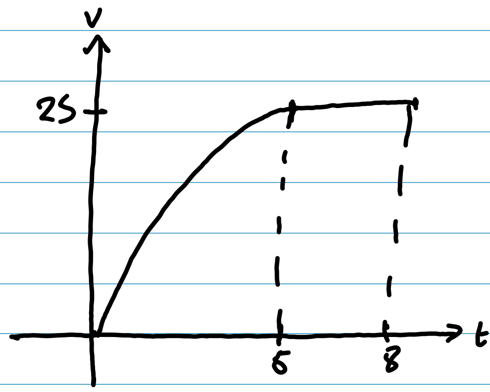
$$= \frac{5 \times 50}{2} + 3 \times 50 = 275\text{m}$$

This is still not within 150 to 200m so not consistent.

iii) Both assume an initial constant acceleration of 10ms^{-2}

Model B is different as it ~~ass~~ takes 50ms^{-1} to be the maximum velocity of the stone. ~~after~~

iv)



Distance travelled $0 < t \leq 5$

$$= \int_0^5 10t - t^2 \, dt = \left[5t^2 - \tfrac{1}{3}t^3\right]_5^0$$

$$= 5(5)^2 - \tfrac{1}{3}(5)^3 = \frac{250}{3} = 83.3\text{m}$$

Distance travelled $5 < t \leq 8$

$$= 25 \times 3 = 75$$

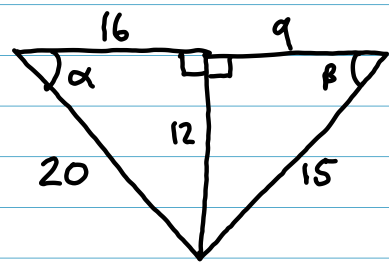
Total depth $= 83.3 + 75 = 158.3$ m which is consistent with local records as $150 < 158.3 < 200$

v) Similar as they both assume a constant speed for $5 \leq t \leq 8$

(or since $\frac{dv}{dt} = 10 - 2t$ at $t = 0$ ~~and~~ $a = \frac{dv}{dt} = 10$ they have initial acceleration of 10ms^{-2})

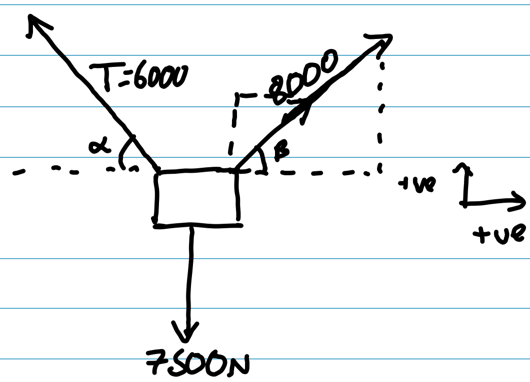
Different as model C does not have constant acceleration in $0 \leq t \leq 5$

7 i)



$\cos\alpha = \frac{16}{20} = \frac{4}{5}$ $\qquad$ $\cos\beta = \frac{9}{15} = \frac{3}{5}$

ii)



Resolve

$\Sigma F = ma$

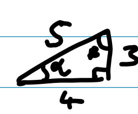
horiz $\longrightarrow$

$8000\cos\beta - T\cos\alpha = ma$

$8000 \times \frac{3}{5} - 6000 \times \frac{4}{5} = 0$ $\quad$ hence $a = 0$ in horizontal directon

Vert

$\sin\alpha = \frac{3}{5}$ $\qquad$ $\sin\beta = \frac{4}{5}$



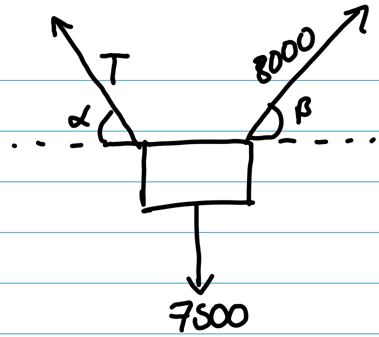
$7500 = m \times 9.8$
$\Rightarrow m = 765.3$

$\uparrow$

$T\sin\alpha + 8000\sin\beta - 7500 = ma$

$6000 \times \frac{3}{5} + 8000 \times \frac{4}{5} - 7500 = 765.3\,a$

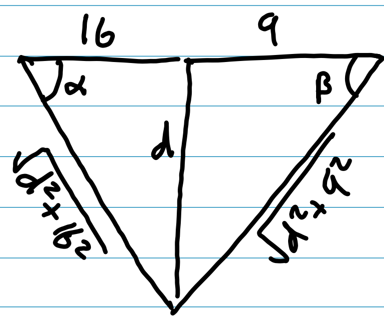
$a = \frac{2500}{765.3} = 3.27\,ms^{-2}$ upwards iii)



horizontal component kept in equilibrium so

$$8000\cos\beta - T\cos\alpha = 0$$

$$T = \frac{8000\cos\beta}{\cos\alpha} \quad \text{as required}$$



$$\cos\alpha = \frac{16}{\sqrt{d^2+16^2}}$$

$$\cos\beta = \frac{9}{\sqrt{d^2+9^2}}$$

So $$T = \frac{8000\left(\frac{9}{\sqrt{d^2+9^2}}\right)}{\frac{16}{\sqrt{d^2+16^2}}} \frac{\times\sqrt{d^2+16^2}}{\times\sqrt{d^2+16^2}} = \frac{8000\times 9\sqrt{d^2+16^2}}{16\sqrt{d^2+9^2}}$$

$$= \frac{4500\sqrt{d^2+256}}{\sqrt{d^2+81}}$$

iv) When $d = 6.75$ $\quad T = \frac{4500\sqrt{6.75^2+256}}{\sqrt{6.75^2+81}} = 6946.2$

Resolve vertically

$$\uparrow \quad T\sin\alpha + 8000\sin\beta - 7500 = ma$$

$\sin\alpha = \frac{6.75}{\sqrt{6.75^2+16^2}}$ $\quad \sin\beta = \frac{6.75}{\sqrt{6.75^2+9^2}}$ $\quad m = \frac{7500}{9.8}$

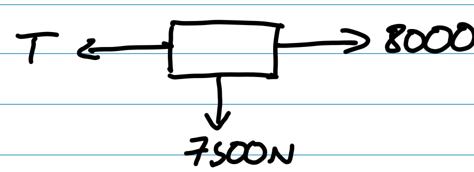
$$a = \frac{T\sin\alpha + 8000\sin\beta - 7500}{m}$$

$$= \frac{\dfrac{4500\sqrt{6.75^2+256}}{\sqrt{6.75^2+81}} \times \dfrac{6.75}{\sqrt{6.75^2+16^2}} + 8000 \times \dfrac{6.75}{\sqrt{6.75^2+9^2}} - 7500}{\dfrac{7500}{9.8}}$$

$$= \frac{2700 + 4800 - 7500}{\dfrac{7500}{9.8}} = 0$$

= acceleration when $d = 6.75$

v) At P the bomb would theoretically have the following force diagram:



It could not be in equilibrium, as there are no vertical forces to counteract force due to gravity ~~force~~ (the weight)

If the bomb was at P and ~~the~~ tensions equil it would have vertical acceleration downwards. (which would equal $g = 9.8ms^{-2}$)