

## Section A (36 marks)

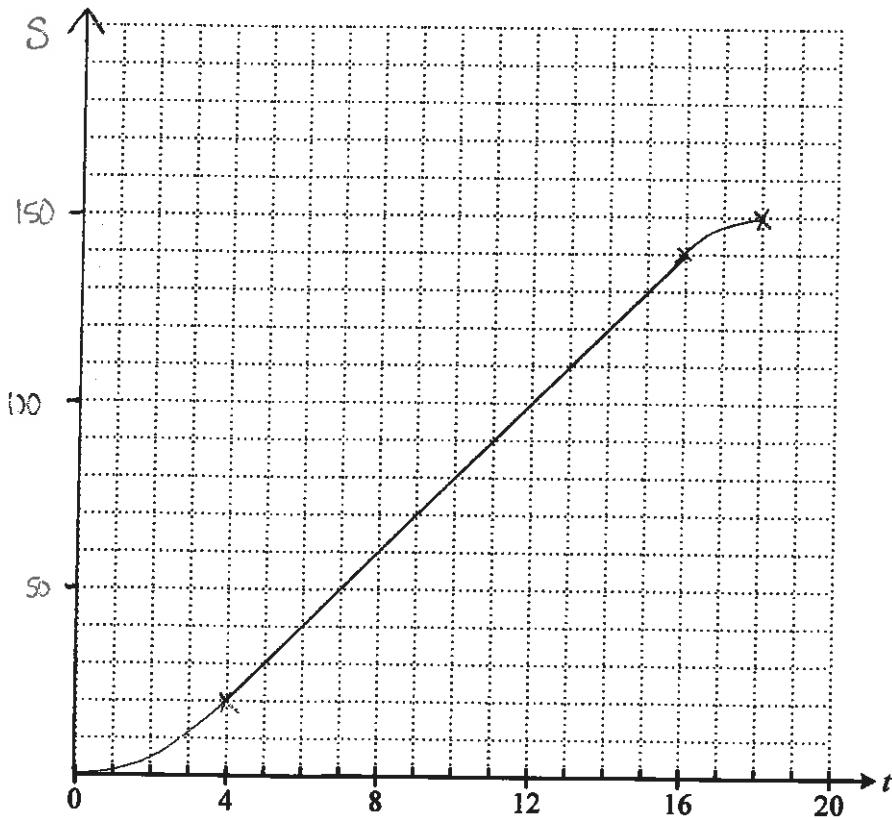
1(i)

$S = \text{area under graph}$

$$\text{when } t = 4, S = \frac{1}{2}(4)(10) \\ = \underline{\underline{20 \text{ m}}}$$

$$\text{when } t = 18, S = \frac{1}{2}(18 + (16-4))(10) \\ = \underline{\underline{150 \text{ m}}}$$

1(ii)



$$0 \leq t \leq 4 \quad S = \int \frac{5}{2}t \, dt \quad 4 \leq t \leq 16 \quad 16 \leq t \leq 20$$

$$= \frac{5}{4}t^2 \quad S = \int 10 \, dt \quad S = \int \dots -t \, dt$$

$$\checkmark \quad = 10t \quad / \quad = \dots -t^2$$

~VC ~

$$2(i) \quad \underline{P} = \begin{pmatrix} 12 \\ -5 \end{pmatrix} \rightarrow \underline{q} = \begin{pmatrix} 16 \\ 1.5 \end{pmatrix} \quad \underline{P} + \underline{q} = \begin{pmatrix} 12 \\ -5 \end{pmatrix} + \begin{pmatrix} 16 \\ 1.5 \end{pmatrix} \\ = 28\underline{i} - 3.5\underline{j}$$

$$\frac{28}{8} = \frac{7}{2} \quad \frac{7}{2} \times -1 = 3.5 \quad \underline{\underline{=}}$$

$\therefore \underline{P} + \underline{q} = \frac{7}{2} (8\underline{i} - \underline{j})$  so  $\underline{P} + \underline{q}$  is a scalar multiple of  $8\underline{i} - \underline{j}$ ,  $\therefore \parallel$  to it.

$$(ii) \quad 3\underline{P} + 10\underline{q} = 3 \begin{pmatrix} 12 \\ -5 \end{pmatrix} + 10 \begin{pmatrix} 16 \\ 1.5 \end{pmatrix}$$

$$= \begin{pmatrix} 36 \\ -15 \end{pmatrix} + \begin{pmatrix} 160 \\ 15 \end{pmatrix}$$

$$= 196\underline{i} + 0\underline{j} \quad \text{as there is no vertical component, } 3\underline{P} + 10\underline{q} \text{ acts in the horiz. direction.}$$

$$(iii) \quad k\underline{P} + 3\underline{q} = k \begin{pmatrix} 12 \\ -5 \end{pmatrix} + 3 \begin{pmatrix} 16 \\ 1.5 \end{pmatrix}$$

$$= \begin{pmatrix} 12k \\ -5k \end{pmatrix} + \begin{pmatrix} 48 \\ 4.5 \end{pmatrix} + \begin{pmatrix} 0 \\ -W \end{pmatrix}$$

↑ +ve, weight  
acts ↓, ∴ -W  
and W = mg

$$\text{For equil'm, } 12k + 48 = 0$$

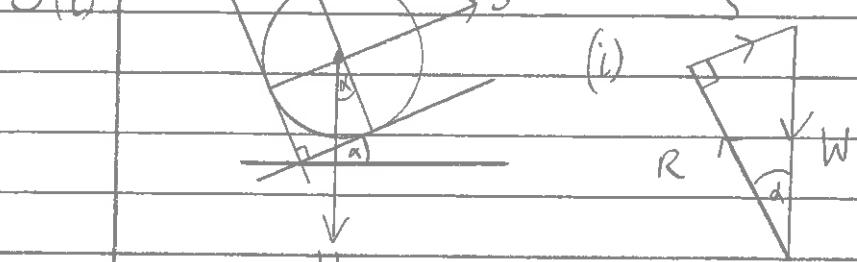
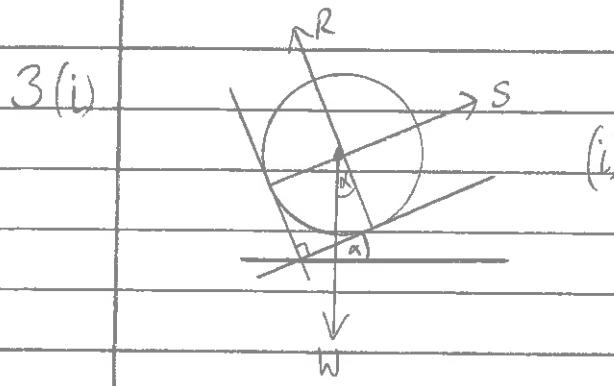
$$\Rightarrow k = \frac{-48}{12}$$

-4 QED

$$\text{vertical equil'm} \Rightarrow -5(-4) + 4.5 - mg = 0$$

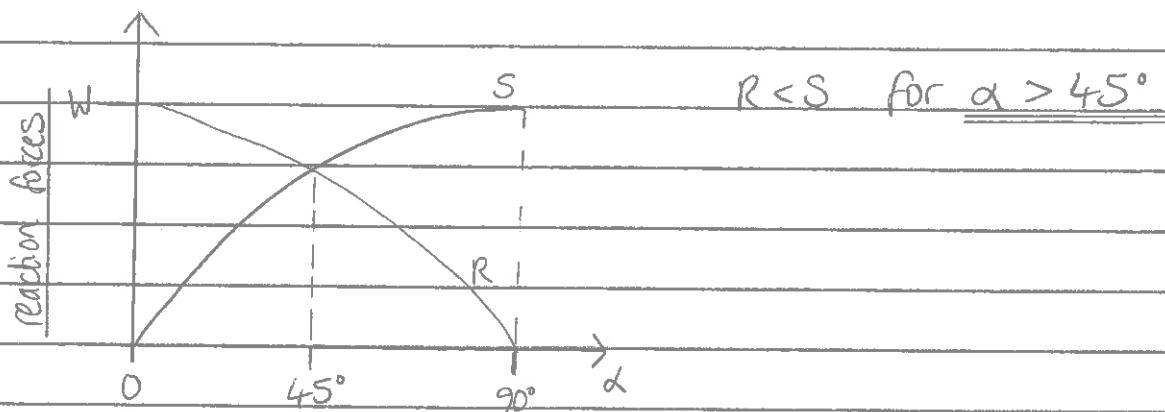
$$\begin{aligned} mg &= 20 + 4.5 \\ &= 24.5 \text{ N} \end{aligned}$$

$$\therefore \text{mass of particle is } \frac{24.5}{9.8} = \underline{\underline{2.5 \text{ kg}}} \text{ (3.s.f.)}$$



$$(ii) \cos \alpha = \frac{R}{W} \Rightarrow R = W \cos \alpha, \sin \alpha = \frac{S}{W} \Rightarrow S = W \sin \alpha$$

(iii)



4.  $20 \angle 30^\circ$   $\uparrow +ve$   $\rightarrow s =$   $\uparrow s = -75$

$$u = 20 \cos 30^\circ$$

$$u = 20 \sin 30^\circ = 10$$

$$v =$$

$$v =$$

SHIP

$$a = 0$$

$$a = -10$$

90m

$$t =$$

$$t =$$

$$\text{vert. motion } s = ut + \frac{1}{2}at^2$$

$$-75 = 10t - 5t^2$$

$$t^2 - 2t + 15 = 0$$

$$(t-5)(t+3) = 0$$

$$t = -3 \text{ or } 5, \text{ but } t > 0 \therefore t = 5 \text{ seconds QED}$$

$$\text{horiz. motion } s = ut + 0$$

$$= 20 \frac{\sqrt{3}}{2} (5)$$

$$= 86.6 \text{ m (3.s.f.)}$$

ship is 90m away.  $90 - 86.6 < 5 \therefore$  cannon ball hits water less than 5m from the ship

(ii) Yes it would travel further. If  $g=9.8$ , the calculation involves a smaller vertical acceleration, so it would take longer to hit the sea. In that extra time, it would travel further from the cliff in the horizontal direction.

$$5.(i) v = 37500(4t - t^2)$$

Reaches moon when  $t = 0$        $0 = 37500(4t - t^2)$   
 $4t = t^2$   
 $t = \underline{4 \text{ hrs QED}}$

(ii)  $s = \int v dt$       at  $t = 0, s = 0$   
 $= 37500\left(2t^2 - \frac{t^3}{3}\right) + C$        $\therefore C = 0$

$$s = \underline{37500\left(2t^2 - \frac{t^3}{3}\right)}$$

subst.  $t = 4$    dist. to moon =  $37500\left(2(4)^2 - \frac{4^3}{3}\right)$   
 $= \underline{400000 \text{ km}}$

(iii) greatest speed when  $a=0$

$$a = \frac{dv}{dt} = 37500(4 - 2t)$$

$$37500(4 - 2t) = 0$$

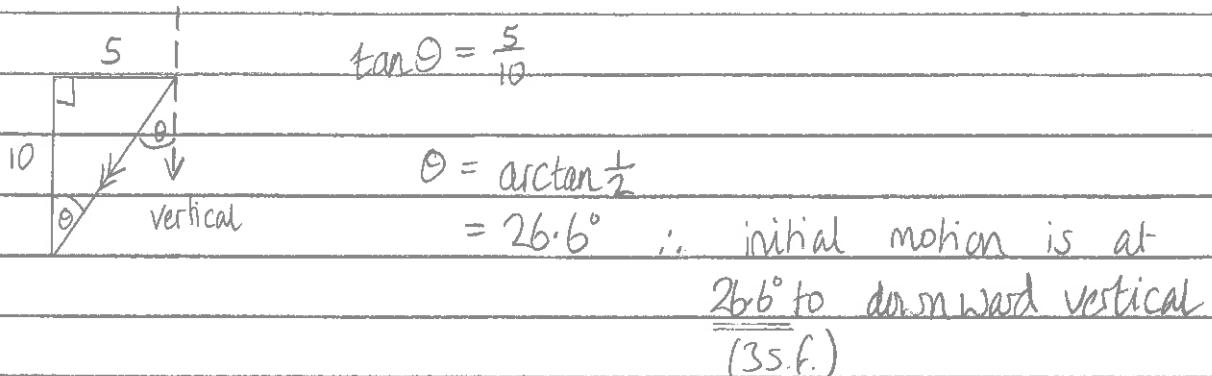
$$4 = 2t$$

$$t = 2 \text{ hours}$$

sub into  $v$ :

$$\begin{aligned} v_{\text{max}} &= 37500(4/2 - 2^2) \\ &= \underline{150000 \text{ km per hr.}} \end{aligned}$$

$$6 \text{ (i)} \quad \underline{v} = \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} \quad \text{speed} = |\underline{v}| = \sqrt{5^2 + 10^2} \\ = 11.2 \text{ ms}^{-1}$$



$$\text{(ii) Her weight, } mg, \text{ is a force } \begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix}$$

Air resistance, acting upwards, would be  $\begin{pmatrix} 0 \\ 0 \\ 880 \end{pmatrix}$

A force provided by her power unit in a horizontal direction could be  $\begin{pmatrix} 50 \\ -20 \\ 0 \end{pmatrix}$ .

$$\text{(iii) } \underline{F} = m\underline{a} \quad \underline{a} = \frac{1}{100} \left( \begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 880 \end{pmatrix} + \begin{pmatrix} 50 \\ -20 \\ 0 \end{pmatrix} \right)$$

$$= \frac{1}{100} \begin{pmatrix} 50 \\ -20 \\ 100 \end{pmatrix} \quad \text{magnitude is } |a|$$

$$= \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{5} \\ 1 \end{pmatrix} \text{ ms}^{-2} \quad \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{5}\right)^2 + 1^2}$$

$$= 1.14 \text{ ms}^{-2} \quad (3 \text{ s.f.})$$

$$\text{(iv) } \underline{v} = \int \underline{a} dt = \begin{pmatrix} \frac{1}{2}t + C_x \\ -\frac{1}{5}t + C_y \\ -t + C_z \end{pmatrix} \quad \text{using initial velocity } C_x = -5, C_y = 0, C_z = -10$$

$$\therefore \underline{v} = \begin{pmatrix} \frac{1}{2}t - 5 \\ -\frac{1}{5}t \\ -t - 10 \end{pmatrix} \text{ ms}^{-1}$$

$$\underline{c} = \int \underline{v} dt = \begin{pmatrix} \frac{1}{4}t^2 - 5t + C_x \\ -\frac{1}{10}t^2 + C_y \\ -\frac{1}{2}t^2 - 10t + C_z \end{pmatrix} \quad \text{using initial position}$$

$$C_x = -75, C_y = 90, C_z = 750$$

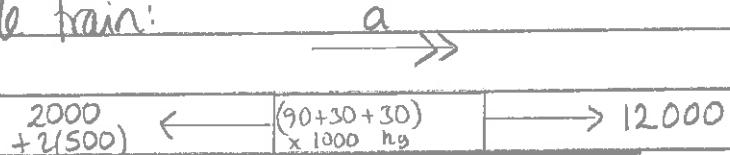
$$\therefore \underline{c} = \begin{pmatrix} \frac{1}{4}t^2 - 5t - 75 \\ -\frac{1}{10}t^2 + 90 \\ -\frac{1}{2}t^2 - 10t + 750 \end{pmatrix} \text{ M}$$

Subst.  $t = 30$ ,  $\underline{c} = \begin{pmatrix} \frac{1}{4}(30)^2 - 5(30) - 75 \\ -\frac{1}{10}(30)^2 + 90 \\ -\frac{1}{2}(30)^2 - 10(30) + 750 \end{pmatrix}$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \therefore \text{when } t = 30 \text{ she's at the origin.}$$

(v) at  $t = 30$ , vert. component of  $\underline{v}$  is  $40 \text{ ms}^{-1}$  downwards.  
 This would not allow her to land "gently on the ground"  
 as it is very fast!

7(i) whole train:



$$F = ma$$

$$12000 - 3000 = 150000 a$$

$$a = \frac{9000}{150000}$$

$$= 0.06 \text{ ms}^{-2}$$

(ii) truck B:  $\xrightarrow{0.06}$

$$F = ma$$

$$T_{AB} - 500 = 30000 (0.06)$$

$$500 \leftarrow 30000 \text{ kg} \rightarrow T_{AB}$$

$$T_{AB} = 2300 \text{ N}$$

(iii) whole train:

	a	$F = ma$
7500	150 000 kg	$-7500 = 150\ 000\ a$

$$a = \frac{-7500}{150\ 000}$$

$$= -0.05$$

$s = ?$

$$u = 10 \quad v^2 = u^2 + 2as$$

$$v = 0 \quad 0 = 10^2 + 2(-0.05)s$$

$$a = -0.05 \quad s = \frac{100}{2(0.05)}$$

$$t = \underline{\underline{1000\ m}}$$

truck B:

-0.05	>>	$F = ma$
500	30 000 kg	$T_{AB} - 500 = 30\ 000 (-0.05)$

$$T_{AB} = -1000$$

∴ there is a 1000N THRUST in coupling AB.

(iv)

res // to plane, in equil'm

$$12000 = 3000 + 150000g \sin\alpha$$

$$\sin\alpha = \frac{9000}{150000g}$$

$$\alpha = \arcsin\left(\frac{9}{15098}\right)$$

$$= \underline{\underline{0.351^\circ}} \quad (3\ s.f.)$$

(v)

res // to plane, in equil'm

$$T_{AB} - 500 - 30000g \sin\alpha = 0$$

$$T_{AB} = 500 + 30000g \left(\frac{3}{50g}\right)$$

$$= \underline{\underline{2300\ N}}, \text{ same value as in (ii)}$$