M1 JUNE 2010   WORKED SOLUTIONS
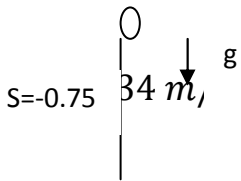
Q1 This is a 'uvast ' (suvat) problem with u v a s t

$$0 \quad ? \quad -9.8 \quad -0.75 \quad ?$$

Make a diagram to be sure of signs. Note we do not have v or t and need to find v so we use
$v^2 - u^2 = 2as$

S=-0.75 $34\ m$,

$v^2 = 2as$ as u=0 $v = \pm\sqrt{2as} = \pm\sqrt{2 \times (-9.8) \times (-0.75)} =$

As we are asked for speed (only magnitude) $\boxed{v = 3.834\ m/s}$

Q2   We are given the weight , i.e. mg=250N, also as we have equilibrium that means that
$\sum F = 0$. Remember a pulley changes direction of force but not magnitude, so the tension in
string between the pulley and the wall is same as mg=250N. At B we need to resolve forces,
make a diagram it will help.
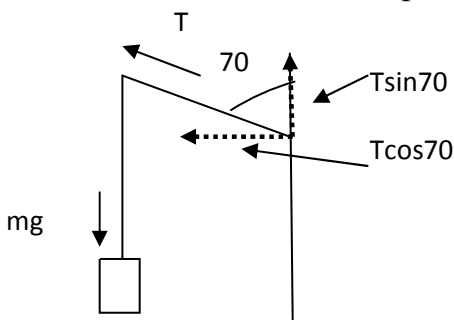
i)   Eq. of Motion // $T\sin70 = F_x$ $\boxed{F_x = 235N}$

Eq. of Motion $T\cos70 = F_y$ $\boxed{F_y = 85.3N}$

ii) In the new set up, the overall tension us the same but it now shared between the two part
of the rope.

So tension is $\boxed{T = \dfrac{250}{2} = 125N}$

Q3 We call the 3 forces $\overrightarrow{F_1} = \begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix}$, $\overrightarrow{F_2} = \begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix}$ and $\vec{F} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and the acc. $\vec{a} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$

The eq. of motion (Newton 2nd Law) in vector form gives $\sum \vec{F} = m\vec{a}$

So that $\begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix} + \begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4\begin{pmatrix} -4 \\ 8 \\ 16 \end{pmatrix} - \begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix}$

Hence $\vec{F} = \begin{pmatrix} -4+1-3 \\ 8-14+9 \\ 16+8-10 \end{pmatrix} \rightarrow \boxed{\vec{F} = \begin{pmatrix} -6 \\ 3 \\ 14 \end{pmatrix}}$ Note required in vector form (could be i,j,k)
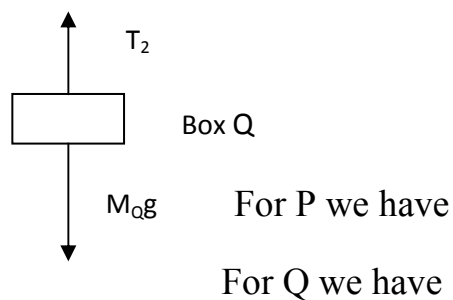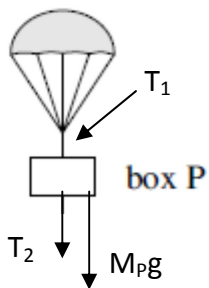
ii) We need to use "uvast " with vectors.

Use ☐ =

Speed = ———— — ☐

Q4 Note 'At one stage the boxes are slowing in their descent ' corresponds to 'The acceleration of the boxes is *a* upwards' as acc. in opposite direction to motion gives slowing down. Diagram does help, that's why they ask for it!!

T_1

box P

T_2 M_Pg

T_2

Box Q

M_Qg For P we have

For Q we have

We want   so eliminate *a* from sim. Eqs. Easiest is to divide both by masses and then equate them

———— ————

Now put numbers

———— ————

Simplify and rearrange

Note no need for g, cancels out   ☐

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Q5 i) If the basis vectors $\vec{i}, \vec{j}$ are in the east & north direction, then the angle θ of the vector $-4\vec{i} - 6\vec{j}$ is w.r.t. the east anti clockwise, so for the bearing we need θ, as we go from north full circle, then ¼ turn more to go on to east axis and then back θ.

As $\theta = \tan^{-1}\frac{-6}{-4}$ we get $\theta = 56.31$ from calculator but diagram show we need 2nd solution

$answer + 180$ hence $\theta = 236.31$ The <u>bearing is</u> therefore $360 - 236.31 + 90 = \boxed{213.7}$

ii) Remember that two vectors are in same direction, i.e. parallel, if the ratio of components is constant.

So with $\vec{v_1} = (-4 + 3k)\vec{i} + (-6 - 2k)\vec{j}$ rearranging given vector and $\vec{v_2} = 7\vec{i} + (-9)\vec{j}$

Be careful with signs! $\frac{-4+3k}{7} = \frac{-6-2k}{-9}$ hence $36 - 27k = -42 - 14k$

$$78 = 13k \rightarrow \boxed{k = 6}$$

Q6 Easiest in this case to put origin at 0, not at sea level as we need to find height above 0.

Remember

$y = y_0 + v_y t - \frac{1}{2}gt^2$ and as $y_0 = 0, v_y = 8m/s, g = 9.8m/s^2$ $\boxed{y = 8t - 4.9t^2}$

And $x = x_0 + v_x t$ with $x_0 = 0, v_x = 12$ $\boxed{x = 12t}$

When the ball hits the sea level, we have $y = -3.6m$ so we need to solve

$$-3.6 = 8t - 4.9t^2$$

$$4.9t^2 - 8t - 3.6 = 0$$

solve using quadratic formula gives t=2 or t=-0.372 . Clearly need to keep positive value t=2 and put this into formula for x we found above, $x = 12t$,

        gives distance from $\boxed{A = x = 12 \times 2 = 24m}$

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Q7 Note that the graph of the displacement is a curve, hence v is not constant, and we may need calculus.

  i) A) When $t=0$ $y=12$, then max is at $t=1$ when $y=16$,
    so the <u>greatest displacement above</u> is 16-12=<u>4 m</u>
  B) Distance is a scalar, means greatest either above or below point $t=0$, From diagram we see that a the minimum ($t=3.5$, $y=-4$) the displacement is largest and neg.   So the <u>greatest distance</u> is (-4)-12=<u>-16m</u>
  C) P is moving down, implies v is negative. So we need slope, $\frac{dy}{dt} < 0$, which we see can from diagram is the case between $t=1$ & $t=3.5$ i.e. we need $\boxed{1 < t < 3.5}$
  D) P is at rest when $v = \frac{dy}{dt} = 0$, i.e. at turning points <u>$t=1$ & $t=3.5$</u>

  ii) We are given that between $t=0$ & $t=3$ $y = -4t^2 + 8t + 12$, therefore
$$v = \frac{dy}{dt} = \frac{d(-4t^2 + 8t + 12)}{dt} \qquad \boxed{v = (-8t + 8)m/s}$$
 And $\boxed{a = \frac{dv}{dt} = -8 \; m/s^2}$

  iii) Note if the speed is 4m/s   then the velocity can be +4m/s or -4m/s, need to consider both cases.
 When $v = +4$ $4 = -8t + 8$ $\rightarrow$ $t = \frac{1}{2}$
 And $v = -4$ $-4 = -8t + 8$ $\rightarrow$ $t = \frac{3}{2}$
 This corresponds to the diagram, note the symmetry of t=0.5 and t=1.5.

  iv) More difficult part, but you should notice that as you are given the acceleration and need to find the displacement, you are doing the reverse of part iii) and will need to integrate the acc. twice. Also note acc. is constant hence displacement will be a quadratic.
 Call acceleration and velocity in this part $a_2 \; and \; v_2$

We know that $a_2 = \frac{dv_2}{dt} = 32$ so we can integrate a_2 to get $v_2 = \int 32 dt = 32t + C$

Now it becomes clear why the questions says *'There is no sudden change in velocity when t = 3.'* This allows us to find C.
Using from part ii) $v = (-8t + 8)m/s$ we get at $v = -16$ at $t = 3$.
So $v_2 = -16m/s$ also at $t=3$ which we put into $v_2 = 32t + C$ to get
    $-16 = 32 \times 3 + C \rightarrow C = -112$ so $v_2 = 32t - 112$
To find y we need to integrate as $y = \int v_2 dt$
$y = \int (32t - 112)dt = 32\frac{t^2}{2} - 112t + C = 16t^2 - 112t + C$ To find C we us value at
t=3 y=0 from graph: $0 = 16 \times 3^2 - 112 \times 3 + C \rightarrow C = 192$
    So finally we get to $\boxed{y = 16t^2 - 112t + 192}$

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Q7

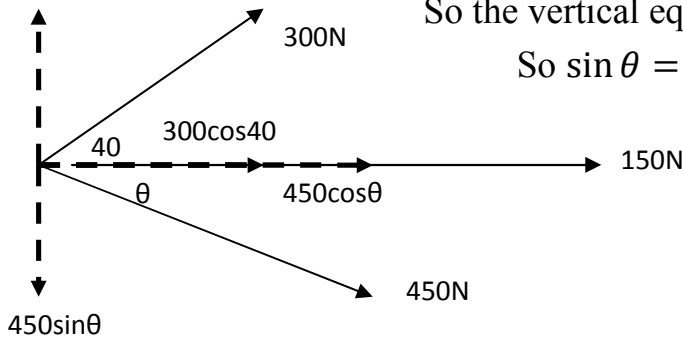i)   If friction is ignored only F=150N acting and using Newton's 2nd Law

$$F = ma \quad \rightarrow a = \frac{F}{m} \quad \text{gives} \quad a = \frac{150}{250} = \frac{3}{5} \quad \boxed{a = 0.6 \ m/s^2}$$

ii) If friction, F_R, is included then two forces acting and use $\sum F = ma$ and as no motion a=0

$$\text{Hence} \quad F + F_R = 0 \quad \rightarrow \boxed{F_R = -F = -150N}$$

iii) Make a diagram. Note there is no motion in vertical direction so $a_\perp = 0$

300sin40

300N

300cos40

40

θ

150N

450cosθ

450N

450sinθ

So the vertical eq. of motion gives $300 \sin 40 - 450 \sin \theta = 0$

So $\sin \theta = \frac{300 \sin 40}{450} = 0.4285..$ $\boxed{\theta = 25.37°}$

The resultant force will on be in horizontal direction

$$F_{res} = 300 \cos 40 + 450 \cos 25.37 + 150 \rightarrow \boxed{F_{res} = 786.4 \ N}$$

iv)

If friction, F_R, is included then new eq. of motion is $F_{res} - F_R = ma$

Hence $F_R = F_{res} - ma = 786.4 - 250a$ call it (A)

We still need to find a, so 'uvast ' (suvat) with u =0 v =? a=? s =1 t=2

So we use

$$s = ut + \frac{1}{2}at^2 \quad (u = 0) \quad a = \frac{2s}{t^2} = \frac{2 \times 1}{4} = \frac{1}{2}$$

Back into (A) gives $F_R = 786.4 - 250 \times \frac{1}{2} = 661.4$ So to 3s.f. $\boxed{F_R = 661 \ N}$

v) $F_R = 661 + 200 = 861 \ N$ then equation of motion $F_{res} - F_R = ma$ becomes

$$786.4 - 861.4 = 250a \qquad \rightarrow \boxed{a = \frac{-3}{10} m/s^2}$$

Again we use 'uvast ' (suvat) with u =18.5 v =? a=-0.3 s =1.65 t=?

So we use $v^2 - u^2 = 2as$ giving $v = \sqrt{u^2 + 2as} = \sqrt{18.5^2 + 2 \times (-0.3) \times 1.65}$

$$\boxed{v = 1.5 \ m/s}$$

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There were more candidates this series who seemed unable to engage properly with several questions; sometimes it was lack of familiarity with the technical language of the topics (eg see the comments on Q3(ii) below).

The work of many candidates would benefit from the use of clear, accurate diagrams not only when considering forces but also when attempting kinematics problems. Q1 is a good example of the need for clear diagrams even when dealing with apparently simple problems in kinematics. Such diagrams should include the direction taken to be positive; if this is done in Q1, then when substitution is made into $v^2 = u^2 + 2as$, it is much easier to see that a and s are either both positive or are both negative.