

FP3 MULTIVARIABLE CALCULUS, SUMMER 2014, Q2

$$g(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24 = 0$$

(i) $\frac{\partial g}{\partial x} = 2x + 6z - 4y, \frac{\partial g}{\partial y} = 6y + 2z - 4x, \frac{\partial g}{\partial z} = 4z + 2y + 6x$

(ii) at P(2, 6, -2), $\nabla g = \begin{pmatrix} 2(2) + 6(-2) - 4(6) \\ 6(6) + 2(-2) - 4(2) \\ 4(-2) + 2(6) + 6(2) \end{pmatrix} = \begin{pmatrix} 4 & -12 & -24 \\ 36 & -4 & -8 \\ -8 & 12 & 12 \end{pmatrix} = \begin{pmatrix} -32 \\ 24 \\ 16 \end{pmatrix}$

∇g is in direction $\begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix}$

NORMAL LINE is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix}$

(iii) $h = \delta g \approx \frac{\partial g}{\partial x} \delta x + \frac{\partial g}{\partial y} \delta y + \frac{\partial g}{\partial z} \delta z$
 $= -32(-4\lambda) + 24(3\lambda) + 16(2\lambda)$
 $= 232\lambda$

$$\Rightarrow \lambda = \frac{h}{232}$$

distance is $\lambda \sqrt{(-4)^2 + 3^2 + 2^2} = \lambda \sqrt{4^2 + 3^2 + 2^2}$
 $= \lambda \sqrt{29}$
 $= \frac{|h| \sqrt{29}}{232}$

(iv) normal line is \parallel to y axis when $\nabla g = k \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 2x + 6z - 4y \\ 6y + 2z - 4x \\ 4z + 2y + 6x \end{pmatrix} = k \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} ① \quad 2x + 6z - 4y &= 0 \\ 6y + 2z - 4x &= 0 \\ ② \quad 12x + 8z + 4y &= 0 \end{aligned}$$

$$\begin{aligned} ① + ②: \quad 14x + 14y &= 0 \\ y &= -x \\ \text{subst. in } ①: \quad 2x + 6z + 4x &= 0 \\ z &= -2x \end{aligned}$$

$$\begin{aligned} \text{bst. in } g: \quad x^2 + 3(-x)^2 + 2(-x)^2 + 2(-x)(-x) + 6x(-x) - 4x(-x) - 24 &= 0 \\ x^2 + 3x^2 + 2x^2 + 2x^2 - 6x^2 + 4x^2 - 24 &= 0 \\ 6x^2 - 24 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

\therefore points are $(2, -2, -2)$ and $(-2, 2, 2)$

(v) $10x - y + 2z = 6 \Rightarrow \nabla g$ is a multiple of $\begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 2x + 6z - 4y \\ 6y + 2z - 4x \\ 4z + 2y + 6x \end{pmatrix} = \mu \begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$$

$$\begin{aligned} ①: 2x + 6z - 4y &= 10\mu & ① \\ ②: 12x + 8z + 4y &= 4\mu & ④ \\ ③: 14x + 14z &= 14\mu & ⑤ \end{aligned}$$

* from here, it is most efficient to use MATRICES on your CALCULATOR.

$$3 \times ③: \quad 18x + 6y + 12z = 6\mu \quad ⑥$$

$$\text{from } ⑤ \text{ and } ⑥, \quad 14\mu =$$

$$②: \quad -4x + 6y + 2z = -\mu$$

$$14x + 14z = 44x + 20z$$

$$⑥ - ②: \quad 22x + 10z = 7\mu \quad ⑦$$

$$-6z = 30x$$

$$⑦ \times 2: \quad 44x + 20z = 14\mu \quad ⑧$$

$$z = -5x$$

$$\text{subst. for } z \text{ in } ①: \quad 2x + 6(-5x) - 4y = 10\mu \Rightarrow -28x - 4y = 10\mu$$

$$\text{in } ②: \quad 6y + 2(-5x) - 4x = -\mu \Rightarrow 6y - 14x = -\mu$$

$$-60y + 140x = 10\mu$$

$$\therefore 10\mu = -28x - 4y = -60y + 140x$$

$$56y = 168x \Rightarrow y = 3x$$

$$\text{subst for } y \text{ & } z \text{ in plane: } 10x - 3x + 2(-5x) = 6$$

$$-3x = 6$$

$$x = -2$$

\therefore point of contact is (-2, -6, 10)

* USING CALCULATOR for (v) to find inverse matrix.

$$\begin{pmatrix} 2 & -4 & 6 \\ -4 & 6 & 2 \\ 6 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mu \begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -4 & 6 \\ -4 & 6 & 2 \\ 6 & 2 & 4 \end{pmatrix}^{-1} \mu \begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$$

$$= \mu \begin{pmatrix} -\frac{1}{4} \\ -\frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

$$\text{subst into plane: } \mu(10(-\frac{1}{4}) - (-\frac{3}{4}) + 2(\frac{5}{4})) = 6$$

$$\mu = \frac{6 \times 4}{-10 + 3 + 10}$$

$$\mu = 8$$

\therefore point is (-2, -6, 10)