

FPI Summer 2007 Q.7

$$\sum_{r=1}^n 3^{r-1} = \frac{3^n - 1}{2}$$

$$\begin{aligned} \text{Let } n=1 & \quad \text{LHS} = 3^{1-1} & \text{RHS} = \frac{3^1 - 1}{2} \\ & = 3^0 & = \frac{2}{2} \\ & = 1 & = 1 \end{aligned}$$

LHS=RHS \therefore true for $n=1$

Assume true for $n=k$, so $\sum_{r=1}^k 3^{r-1} = \frac{3^k - 1}{2}$

Sum to $k+1$ terms

First k terms + $(k+1)^{\text{th}}$ term

$$\frac{3^k - 1}{2} + 3^{k+1}$$

$$\frac{3^k - 1}{2} + \frac{2 \times 3^k}{2}$$

$$= \frac{1}{2}(3^k + 2 \times 3^k - 1)$$

$$= \frac{1}{2}(3^1 \times 3^k - 1)$$

$$= \frac{3^{k+1} - 1}{2}$$

formula for $(kn)^{\text{th}}$ term

$$\frac{3^{k+1} - 1}{2}$$

collect like terms
of 3^k , you have

one 3^k + two 3^k 's

giving 3 lots of 3^k ,
then add the indices

both give same result,

if it is true for $n=k$

then it is true for $n=k+1$

as it is true for $n=1$

it is true for all $n \geq 1$ by induction.