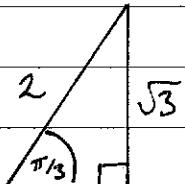


# C3 Summer 2014

$$\begin{aligned}
 1 & \int_0^{\pi/6} 1 - \sin 3x \, dx & \frac{d}{dx} \left( \frac{1}{3} \cos 3x \right) = 3 \sin 3x \times \frac{1}{3} \\
 &= \left[ x + \frac{1}{3} \cos 3x \right]_0^{\pi/6} & \cos \frac{\pi}{2} = 0 \\
 &= \left( \frac{\pi}{6} + \frac{1}{3} \cos \frac{\pi}{2} \right) - \left( 0 + \frac{1}{3} \cos 0 \right) & \cos 0 = 1 \\
 &= \frac{\pi}{6} + 0 - \left( \frac{1}{3} \right) \\
 &= \underline{\underline{\frac{\pi}{6} - \frac{1}{3}}}
 \end{aligned}$$

$$\begin{aligned}
 2 & y = \ln(1 - \cos 2x) \\
 \frac{dy}{dx} &= \frac{2 \sin 2x}{1 - \cos 2x} \\
 &= \frac{2 \sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} \quad \text{at pt. where } x = \frac{\pi}{6}
 \end{aligned}$$



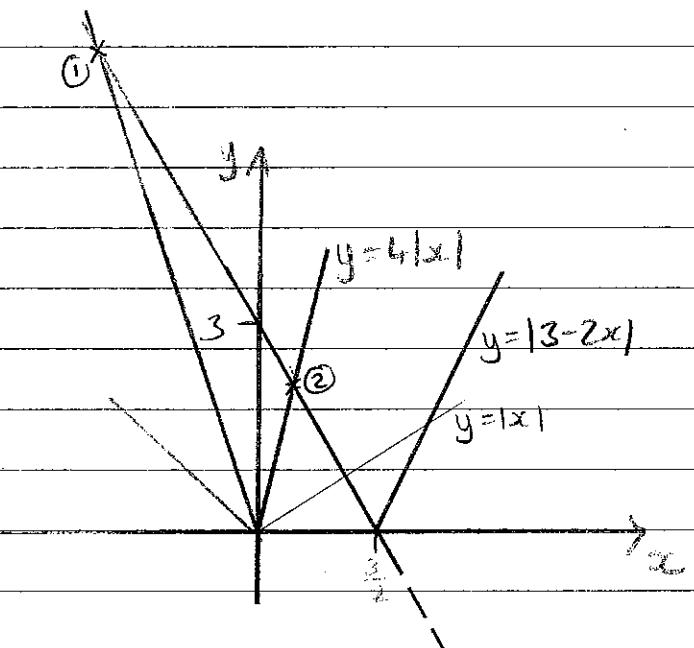
$$\begin{aligned}
 &= \frac{2 \left( \frac{\sqrt{3}}{2} \right)}{1 - \frac{1}{2}} \\
 &= \underline{\underline{2\sqrt{3}}}
 \end{aligned}$$

$$3 \quad |3 - 2x| = 4|x|$$

$$\begin{aligned}
 ① \quad 3 - 2x &= 4(-x) \\
 3 &= -2x \\
 -\frac{3}{2} &= x
 \end{aligned}$$

$$② \quad 3 - 2x = 4x$$

$$\begin{aligned}
 3 &= 6x \quad \therefore x = \frac{1}{2}, x = -\frac{3}{2} \\
 \frac{1}{2} &= x
 \end{aligned}$$



4  $f(x) = a + \cos bx$

$f(x)$  is a translation (2) and stretch s.f. 2 // to  $x$ -axis  
of  $\cos x$  curve.  $\therefore a = 2, b = \frac{1}{2}$

$$y = 2 + \cos \frac{1}{2}x$$

$$x = 2 + \cos \frac{1}{2}y$$

$$x - 2 = \cos \frac{1}{2}y$$

$$\arccos(x-2) = \frac{1}{2}y$$

$$2\arccos(x-2) = y$$

$$\therefore f^{-1}(x) = 2\arccos(x-2), 1 \leq x \leq 3$$

DOMAIN:  $1 \leq x \leq 3$ , RANGE:  $0 \leq f^{-1}(x) \leq 2\pi$

5

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 10$$

$$\frac{dV}{dr} = 4\pi r^2 \quad \frac{dr}{dt} = \frac{dr}{dV} \frac{dV}{dt}$$

$$= \frac{1}{4\pi r^2} (10)$$

$$\text{subst. } r = 8 \quad = \frac{10}{4(64)\pi}$$

$$\frac{dr}{dt} = \frac{5}{128\pi}$$

$$= \underline{\underline{0.0124 \text{ cm s}^{-1}} \text{ (3.s.f.)}}$$

6

$$V = ae^{-kt}$$

$$(i) V = 20000e^{-0.2t} \quad \text{when new, } t=0, V=20000$$

$$\text{after one year, } t=1, V = 20000e^{-0.2}$$

$$\text{amount lost is } 20000 - 20000e^{-0.2}$$

$$= 3625.39\dots$$

$\therefore$  value lost is £3600 (to nearest £100)

$$(ii) \text{ When } t=0, V = 15000 e^0 = Ae^{-kt} \\ \therefore A = 15000$$

$$\text{When } t=1, V = 13000 = 15000e^{-k} \\ 15000 - 2000 = 13000$$

$$\frac{13}{15} = \frac{1}{e^k}$$

$$e^k = \frac{15}{13}$$

$$k = \ln \frac{15}{13} \\ = 0.1431\dots \\ = \underline{\underline{0.143 \text{ (3 s.f.)}}}$$

Cars have same value when

$$20000 e^{-0.2t} = 15000 e^{-\ln(\frac{15}{13})t}$$

$$\frac{20000}{15000} = \frac{e^{-\ln(\frac{15}{13})t}}{e^{-0.2t}}$$

$$\frac{4}{3} = e^{0.2t - \ln(\frac{15}{13})t}$$

$$\ln \frac{4}{3} = t(0.2 - \ln \frac{15}{13})$$

$$t = \frac{\ln \frac{4}{3}}{0.2 + \ln \frac{15}{13}} \\ = 5.055\dots$$

$\therefore$  same value after just over 5 years

(i) Let  $m=25, n=27$ .  $m$  and  $n$  are consecutive odd numbers, but neither is prime (since  $25=5\times 5$  and  $27=3\times 9$ )  
 $\therefore$  DISPROVED by COUNTEREXAMPLE.

(ii) Let  $m=2k, n=2k+2$ . Then  $mn = 2k(2k+2)$

$$k \in \mathbb{Z} \qquad \qquad \qquad = 4k^2 + 4k$$

$k$  and  $k+1$  are consecutive integers,  $\therefore$   $= 4k(k+1)$

one of them is even (and other is odd)  $= 4 \times \text{even} \times \text{odd} \qquad a, b \in \mathbb{Z}$

$\therefore$  PRODUCT IS DIVISIBLE BY 8, proved by DEDUCTION.  $= 4 \times 2a \times 2b+1$

$$= 8a(2b+1)$$

$$8 \quad f(x) = \frac{x}{\sqrt{2+x^2}} \quad \text{for odd functions, } f(-x) = -f(x)$$

$$(i) \quad f(-x) = \frac{-x}{\sqrt{2+(-x)^2}}$$

$$= -\frac{x}{\sqrt{2+x^2}}$$

$$= -f(x) \therefore f(x) \text{ is odd.}$$

$f(x)$  has ROTATIONAL SYMMETRY OF ORDER 2, ABOUT THE ORIGIN.

$$(ii) \quad f'(x) = \frac{(2+x^2)^{1/2} - x^2(2+x^2)^{-1/2}}{2+x^2} \quad \text{QUOTIENT RULE}$$

$$= \frac{(2+x^2)^{1/2}}{(2+x^2)^{1/2}} \times \frac{(2+x^2)^{1/2} - x^2(2+x^2)^{-1/2}}{2+x^2}$$

$$= \frac{2+x^2 - x^2(2+x^2)^{-1/2}}{(2+x^2)^{3/2}}$$

$$= \frac{2}{(2+x^2)^{3/2}}$$

at origin,  $x=0$ ,  $\therefore$  grad is  $\frac{2}{(2+0)^{3/2}}$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$(iii) \quad \int_0^1 \frac{x}{\sqrt{2+x^2}} dx$$

by subst.  $u = 2+x^2$

$$\frac{du}{dx} = 2x$$

$$= \frac{1}{2} \int_2^3 \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{2} \int_2^3 u^{-1/2} du$$

$$\frac{1}{2} x du = dx$$

$$\text{LIMITS: } x=0, u=2+0=2$$

$$x=1, u=2+1=3$$

$$= \frac{1}{2} [2u^{1/2}]_2^3$$

$$= \underline{\underline{\sqrt{3} - \sqrt{2}}}$$

$$(iv) (A) y = \frac{x}{\sqrt{2+x^2}}$$

$$\begin{aligned} 1 &= \frac{2+x^2}{x^2} \\ &= \frac{2}{x^2} + 1 \end{aligned}$$

$$(B) \frac{1}{y^2} = \frac{2}{x^2} + 1$$

$$\begin{aligned} y^{-2} &= 2x^{-2} + 1 \\ -2y^{-3} \frac{dy}{dx} &= -4x^{-3} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{4+y^3}{2x^3} \\ &= \frac{2y^3}{x^3} \end{aligned}$$

At the origin  $x=0$ , so the DENOMINATOR of the expression would be 0, so this expression is not defined at the origin.

$$9 \quad y = xe^{-2x}$$

$$(i) \text{ At } P, \quad mx = xe^{-2x} \quad x \neq 0$$

$$m = e^{-2x}$$

$$m = \frac{1}{e^{2x}}$$

$$e^{2x} = \frac{1}{m}$$

$$2x = \ln \frac{1}{m}$$

$$\underline{\underline{x = -\frac{1}{2} \ln m}}$$

PRODUCT RULE

$$(ii) \quad y = xe^{-2x} \quad \frac{dy}{dx} = -2xe^{-2x} + e^{-2x}$$

$$\begin{aligned} u &= x & v &= e^{-2x} \\ u' &= 1 & v' &= -2e^{-2x} \end{aligned}$$

$$= e^{-2x} (1-2x)$$

$$= e^{\ln m} (1+\ln m)$$

$$\begin{aligned} \text{at } P \quad x &= -\frac{1}{2} \ln m \\ &= m(1+\ln m) \end{aligned}$$

$$\begin{aligned}
 & \text{(iii) grad. of } OP = - \text{ grad. of tangent} \\
 & m = -m(1 + \ln m) \quad m \neq 0 \\
 & 1 = -1 - \ln m \quad x\text{-coord at P is } -\frac{1}{2} \ln m \\
 & \ln m = -2 \quad = -\frac{1}{2} \ln e^{-2} = \ln e \\
 & m = e^{-2} \quad P \text{ is pt. } \left(1, \frac{1}{e^2}\right) \quad = 1 \quad \text{and } y = m(x)
 \end{aligned}$$

(iv) area required is given by integral - area of  $\Delta$

$$\begin{aligned}
 \text{AREA} &= \int_0^1 x e^{-2x} dx - \Delta \\
 &= \left[ -\frac{1}{2} x e^{-2x} \right]_0^1 + \frac{1}{2} \int_0^1 e^{-2x} dx - \Delta \\
 &= \left[ -\frac{1}{2} x e^{-2x} \right]_0^1 + \frac{1}{2} \left[ -\frac{1}{2} e^{-2x} \right]_0^1 - \Delta \\
 &= \left[ -\frac{1}{2} e^{-2x} \left( x + \frac{1}{2} \right) \right]_0^1 - \Delta \\
 &= -\frac{1}{2} e^{-2} \left( \frac{3}{2} \right) - \left( -\frac{1}{2} \left( \frac{1}{2} \right) \right) - \Delta \\
 &= -\frac{3}{4} e^{-2} + \frac{1}{4} - \frac{1}{2} e^{-2} \\
 &= \frac{1}{4} \left( 1 - \frac{5}{e^2} \right)
 \end{aligned}$$