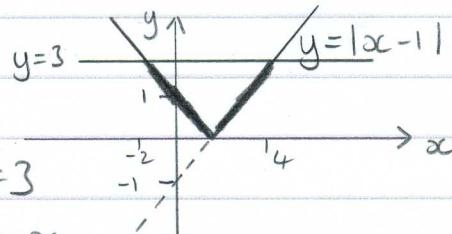


A C3 January 2009 WORKED SOLUTIONS

1. $|x-1| < 3$

solve $x-1 = 3$, $1-x = 3$
 $x = 4$ $-2 = x$



$\therefore \underline{-2 < x < 4}$

2. i) $\frac{d}{dx}(x \cos 2x) = \underline{-2x \sin 2x + \cos 2x}$

\uparrow

$\frac{d}{dx}(uv) = u'v + uv'$ ←

$u = x$	$v = \cos 2x$
$u' = 1$	$v' = -2 \sin 2x$

PRODUCT RULE

ii) $\int x \cos 2x \, dx$ BY PARTS: $u = x$ $v = \frac{1}{2} \sin 2x$
 $uv' = uv - \int u'v \quad u' = 1 \quad v' = \cos 2x$
 $= \frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$

$= \underline{\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + C}$

3. $f(x) = \frac{1}{2} \ln(x-1) \Rightarrow x = \frac{1}{2} \ln(y-1)$ where
 $y = f^{-1}(x)$

$2x = \ln(y-1)$
 $e^{2x} = y-1$
 $e^{2x+1} = y \quad \therefore \quad \underline{y = g(x) = f^{-1}(x)}$

4. $\int_0^2 (1+4x)^{1/2} \, dx$

$= \left[\frac{1}{6} \sqrt{(1+4x)^3} \right]_0^2$

$= \left(\frac{1}{6} \sqrt{9^3} \right) - \left(\frac{1}{6} \times 1 \right)$

$= \frac{25}{6}$

$= \underline{4^{1/3}}$

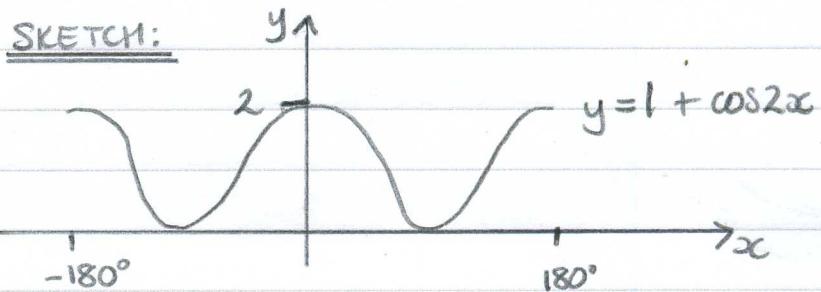
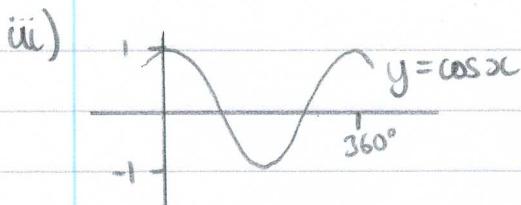
$\frac{d}{dx} \left(\frac{1}{6} (1+4x)^{3/2} \right) = \frac{3}{2} (1+4x)^{1/2} \times 4$

OR BY SUBST: $u = 1+4x$ LIMITS:

$\frac{du}{dx} = 4$
 $x=0, u=1$
 $x=2, u=9$

$\int_1^9 \frac{1}{4} u^{1/2} \, du = \left[\frac{1}{6} u^{3/2} \right]_1^9$
 $= \left(\frac{1}{6} \sqrt{9^3} \right) - \left(\frac{1}{6} \times 1 \right)$
 $= \underline{4^{1/3}}$

5. $f(x) = 1 + \cos 2x$ is a translation (°) and stretch s.f. $\frac{1}{2}$ // to x -axis of $y = \cos x$
 i) period is 180° ii) $y = \cos x$ has period 360°



6. i) BY COUNTER EXAMPLE: let $p=2, q=-3$
 then $p > q$ since $2 > -3$
 but $\frac{1}{2} \neq -\frac{1}{3}$

ii) CONDITION: $p > q > 0$

7. i) $x^{2/3} + y^{2/3} = 5$

DIFFERENTIATE IMPLICITLY
 (all terms w.r.t. x)

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \frac{dy}{dx} = 0$$

$$\frac{2}{3y^{1/3}} \frac{dy}{dx} = -\frac{2}{3x^{1/3}}$$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

ii) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$) SUBST. $x=1, y=8,$
 $\frac{dx}{dt} = 6$
 $= -\left(\frac{8}{1}\right)^{1/3} \times 6$
 $= -2 \times 6$
 $= -12$

$$\sqrt[3]{8} = 2$$

B

8.

$$y = x^2 - \frac{1}{8} \ln x \quad P \text{ has } x \text{ coord } 1 \Rightarrow y = 1 - \frac{1}{8} \ln 1 = 1$$

i) grad. of PR = $\frac{\frac{17}{8}}{1} = \frac{15}{8}$ P is pt. $(1, 1)$

ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ subst. $x=1$: $\frac{dy}{dx} = 2 - \frac{1}{8} = \frac{15}{8}$

\therefore grad. of tangent to curve at P is $\frac{15}{8}$, so PR is a tangent to

the curve as it goes thro' P . and has grad. $\frac{15}{8}$.

iii) At Q , $\frac{dy}{dx} = 0 = 2x - \frac{1}{8x}$

$$\frac{1}{8x} = 2x$$

$$0 = 16x^2 - 1$$

$$= (4x+1)(4x-1)$$

$$x = \pm \frac{1}{4} \quad \text{BUT } x \text{ is +ve} \therefore x = \frac{1}{4}$$

subst: $y = \left(\frac{1}{4}\right)^2 - \frac{1}{8} \ln\left(\frac{1}{4}\right)$

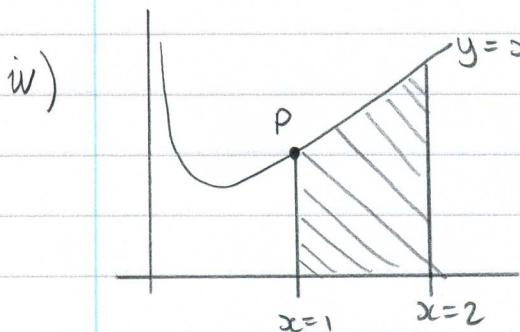
$$= \frac{1}{16} + \frac{1}{8} \ln 4 \quad \therefore Q \text{ is pt. } \left(\frac{1}{4}, \frac{1}{16} + \frac{1}{8} \ln 4\right)$$

iv) $\frac{d}{dx}(x \ln x - x) = x \cancel{\times \frac{1}{x}} + \ln x - 1$

PRODUCT RULE $\frac{d}{dx}(uv) = u'v + uv'$

$$u = x \quad v = \ln x$$

$$u' = 1 \quad v' = \frac{1}{x}$$



$$\text{AREA REQUIRED} = \int_1^2 x^2 - \frac{1}{8} \ln x \, dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{8}(x \ln x - x) \right]_1^2$$

$$= \left(\frac{8}{3} - \frac{1}{8}(2 \ln 2 - 2) \right) - \left(\frac{1}{3} - \frac{1}{8}(-1) \right)$$

$$= \frac{64}{24} - \frac{1}{4} \ln 2 + \frac{6}{24} - \frac{11}{24} = \underline{\underline{\frac{59}{24} - \frac{1}{4} \ln 2}}$$

$$9.i) f(x) = \frac{1}{\sqrt{2x-x^2}}$$

asymptote when denominator = 0

$$2x - x^2 = 0$$

$$x(2-x) = 0$$

$$x=0 \text{ or } x=2$$

$$\therefore \underline{a=2}$$

$$\underline{\text{domain: } 0 < x < 2}$$

$$ii) f(x) = (2x-x^2)^{-1/2}$$

$$f'(x) = -\frac{1}{2} (2x-x^2)^{-3/2} \times (2-2x) \quad \text{BY CHAIN RULE}$$

$$= -\frac{(1-x)}{(2x-x^2)^{3/2}}$$

$$\underline{\frac{dy}{dx} = \frac{x-1}{(2x-x^2)^{3/2}}}$$

$$iii) \text{ turning pt. where } \underline{\frac{dy}{dx} = 0} \Rightarrow x=1$$

$$\text{subst. } f(1) = \frac{1}{\sqrt{2(1)-1^2}}$$

$$= 1$$

$$\underline{\text{turning pt. is } (1, 1)}$$

$$\underline{\text{range: } f(x) \geq 1}$$

$$iii) g(x) = \frac{1}{\sqrt{1-x^2}} \quad (A) \text{ even functions have the property } f(-x) = f(x)$$

$$g(-x) = \frac{1}{\sqrt{1-(-x)^2}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

$$= g(x) \quad \therefore g(x) \text{ is an even function.}$$

$$(B) g(x-1) = \frac{1}{\sqrt{1-(x-1)^2}} = \frac{1}{\sqrt{1-(x^2-2x+1)}} = \frac{1}{\sqrt{2x-x^2}} = f(x)$$

(c) even functions have the y axis as a line of symmetry.
 $f(x)$ is a translation (δ) of an even function, so
it has the line $\underline{xc=1}$ as a line of symmetry.