

C2 January 2008 Paper

1. $y = 10x^4 + 12 \quad \frac{dy}{dx} = \underline{\underline{40x^3}}$

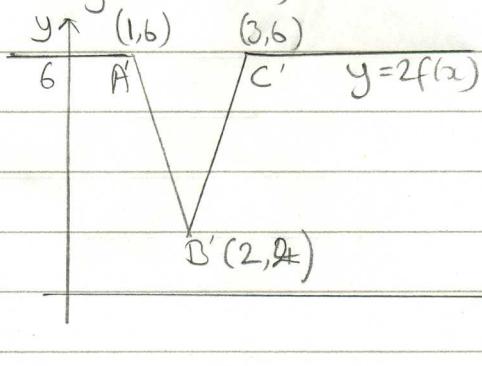
2. i) $n: 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11$
 $n^{\text{th}} \text{ term: } 1 \ 2 \ 3 \ 4 \ 5 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$
 every $(5k)^{\text{th}}$ term is a 5 so 45^{th} is a 5... $\begin{matrix} 46 & 47 & 48 \\ 1 & 2 & 3 \end{matrix}$
 48^{th} term = 3

ii) $S_{48} = 9(1+2+3+4+5) + 1+2+3 = 9(15) + 6 = \underline{\underline{141}}$

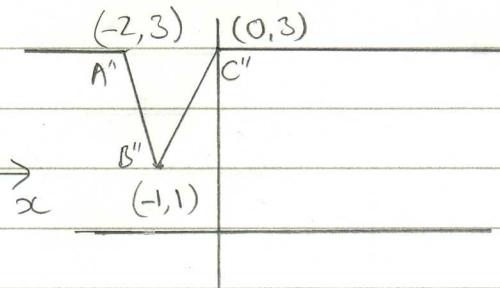
3. $\tan \theta = \frac{1}{2}$ θ acute $\text{hyp} = \sqrt{1^2+2^2} = \sqrt{5}$ by Pythag.

$$\cos \theta = \frac{2}{\sqrt{5}} \quad \cos^2 \theta = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$$

4. i) $y = 2f(x)$ stretch s.t. in y direction

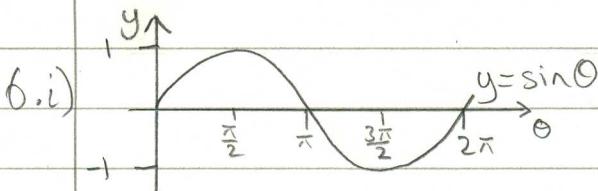


ii) $y = f(x+3)$ translation 3 units in the $-ve x$ direction



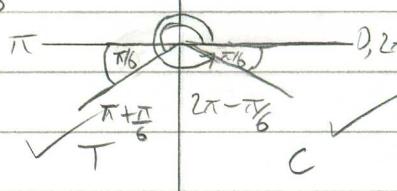
5. $\int 12x^5 + x^{13} + 7 \, dx = \frac{12x^6}{6} + \frac{3x^{4/3}}{4} + 7x + C$

$$= \underline{\underline{2x^6 + \frac{3}{4}x^{4/3} + 7x + C}}$$



ii) $2 \sin \theta = -1 \quad 0 \leq \theta \leq 2\pi$
 $\sin -ve \leftrightarrow \sin \theta = -\frac{1}{2} \quad S$
 $\sin^{-1}(-\frac{1}{2}) = \pi/6$

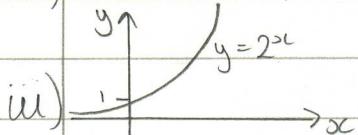
$$\theta = \underline{\underline{\frac{7\pi}{6}}, \frac{11\pi}{6}}$$



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7.i) $\sum_{n=2}^5 2^n = 2^2 + 2^3 + 2^4 + 2^5 = 4 + 8 + 16 + 32 = \underline{\underline{60}}$

ii) $2^n = \frac{1}{64} = \frac{1}{2^6} \therefore 2^{-6} = \frac{1}{64}, n = \underline{\underline{-6}}$



8. G.P. $a_n = ar^{n-1}$ $a_2 = 18 = ar \textcircled{1}$ $a_4 = 2 = ar^3 \textcircled{2}$ $r > 0$
 $\textcircled{2} \div \textcircled{1} \quad \frac{2}{18} = \frac{ar^3}{ar} \quad \frac{1}{9} = r^2 \quad r = +\frac{1}{3}$
 subst. in \textcircled{1} $18 = \frac{1}{3}a \quad a = \underline{\underline{54}}$
 $S_\infty = \frac{a}{1-r} = \frac{54}{1-\frac{1}{3}} = \frac{54}{\frac{2}{3}} = 54 \times \frac{3}{2} = \underline{\underline{81}}$

9. $\log_{10} y = 3x + 2$ i) $\log_{10} 500 = 3x + 2$
 $10^{3x+2} = 500$

ii) $\log_{10} y = -3 + 2$ $(3x+2)\log_{10} 10 = \log_{10} 500$
 $\log_{10} y = -1$ $3x+2 = \log_{10} 500$
 $10^{-1} = \underline{\underline{y}} = \frac{1}{10} = 0.1$ $x = \frac{1}{3}(\log_{10} 500 - 2)$
 $= \underline{\underline{0.23}} \quad (\text{2d.p.})$

iii) $\log_{10} y^4 = 4 \log_{10} y = 4(3x+2)$
 $= \underline{\underline{12x+8}}$

iv) $\log_{10} y = \frac{12x+8}{4} = 3x + 2$

$\underline{\underline{y}} = 10^{3x+2}$

B 10.i) $V = x^2 h = 120 \quad h = \frac{120}{x^2} = 120x^{-2}$

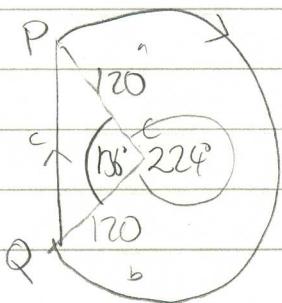
$A = 2x^2 + 4hx = 2x^2 + 4x(120x^{-2}) = 2x^2 + 480x^{-1}$
 base and top \nwarrow 4 side faces $= 2x^2 + \frac{480}{x}$

ii) $\frac{dA}{dx} = 4x - 480x^{-2} = 4x - \frac{480}{x^2}$

$\frac{d^2A}{dx^2} = 4 + 960x^{-3} = 4 + \frac{960}{x^3}$

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11ii)



$$\text{arc length} = r\theta = 120 \times 224 \times \frac{\pi}{180} = 149\frac{1}{2}\pi$$

$$= 469.144\pi \dots$$

$$QP = \sqrt{120^2 + 120^2 - 2(120)^2 \cos 136}$$

$$= 222.5241 \dots$$

$$\text{total length} = 469.144\pi + 222.5241 = \underline{691.67 \text{ m}} \quad (2 \text{ dp})$$

12ii)

$$f(x) = x^4$$

binomial coeffs. 1 4 6 4 1

$$(A) f(x+h) = (x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$(B) \frac{f(x+h) - f(x)}{h} = \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \underline{4x^3 + 6x^2h + 4xh^2 + h^3}$$

(C)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \underline{4x^3} \quad (\text{terms in } h \text{ are negligibly small})$$

(D)

this represents $f'(x)$, the derivative of x^4 .

it is the gradient of the tangent to the curve $y = x^4$

10. cont'd

iii) min A where $\frac{dA}{dx} = 0$ and $\frac{d^2A}{dx^2} > 0$

$$\frac{dA}{dx} = 4x - \frac{480}{x^2} = 0$$

$$4x = \frac{480}{x^2}$$

$$\frac{d^2A}{dx^2} = 4 + \frac{960}{x^3} > 0$$

$$4x^3 = 480$$

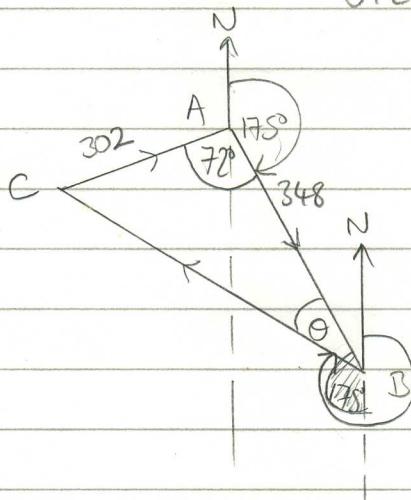
$$x = \sqrt[3]{120}$$

$$= \underline{\underline{4.932 \text{ cm}}} \quad (3 \text{ d.p.})$$

\therefore min.

$$A = 2\sqrt[3]{120^2} + \frac{480}{\sqrt[3]{120}} = \underline{\underline{145.97 \text{ cm}^2}} \quad (2 \text{ d.p.})$$

11.



$$BC = \sqrt{302^2 + 348^2 - 2(302)(348)\cos 72^\circ} \\ = 383.8685\dots$$

$$(A) \text{ total length} = 302 + 348 + 383.8685\dots \\ = \underline{\underline{1033.87 \text{ m}}} \quad (2 \text{ d.p.}) \\ = 1.034 \text{ km} \quad (3 \text{ d.p.})$$

bearing of C from B use sine rule

$$\frac{\sin \theta}{302} = \frac{\sin 72^\circ}{383.86} \quad \theta = \sin^{-1} \left(\frac{302 \sin 72^\circ}{383.86} \right)$$

$$\text{bearing is } 180 + 175 - 48 = \underline{\underline{307^\circ}} \quad = 48.43\dots$$

$$12(A) \quad y = x^4, \quad y = 8x \quad \text{at intersection} \quad x^4 = 8x \\ x^4 - 8x = 0$$

$$P \text{ has } y \text{ coord } 2^4 = 16$$

$$P \text{ is pt. } (2, 16)$$

$$x(x^3 - 8) = 0$$

$$x=0 \text{ or } x^3 = 8 \quad x = 2$$

$$\text{area of } \Delta = \frac{1}{2}bh = \frac{1}{2} \times 2 \times 16 = 16 \text{ square units.}$$

(B) area of bounded region = Δ area - area under curve

$$= 16 - \int_0^2 x^4 dx = 16 - \left[\frac{x^5}{5} \right]_0^2 = 16 - \left(\frac{32}{5} - 0 \right) \\ = 9\frac{3}{5} = \underline{\underline{9.6 \text{ square units}}}$$