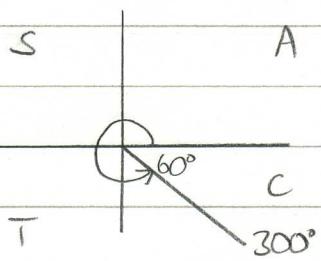
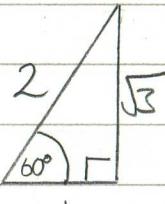


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1.i)

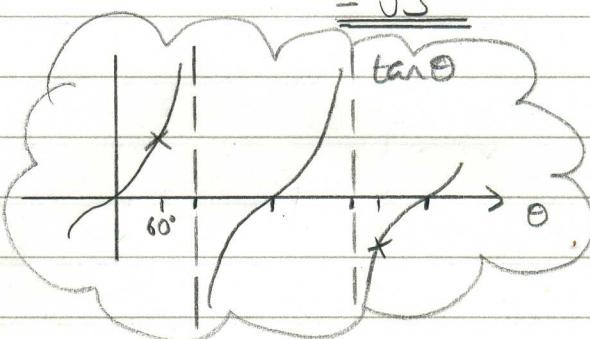


$$\tan 300^\circ = -\tan 60^\circ = -\sqrt{3}$$



ii)

$$300 \times \frac{\pi}{180} = \frac{5}{3}\pi^c$$



2.

$$y = 6x^{3/2} \quad \frac{dy}{dx} = \frac{3}{2} \times 6x^{1/2} = \underline{9x^{1/2}} = 9\sqrt{x}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2} \times 9x^{-1/2} = \underline{\frac{9}{2}x^{-1/2}} = \underline{\frac{9}{2\sqrt{x}}}$$

$$x = 36, \quad \frac{d^2y}{dx^2} = \frac{9}{2\sqrt{36}} = \frac{9}{2 \times 6} = \frac{9}{12} = \underline{\frac{3}{4}}$$

3. B is a stretch of A, s.f. 2 in the y-direction
 \therefore eqn of B is $2f(x)$

C is a translation of A 3 units in the x-direction
 \therefore eqn of C is $f(x-3)$

4. i) $t_{n+1} = 2t_n + 5, t_1 = 3$

$$t_2 = 2t_1 + 5 = 2 \times 3 + 5 = \underline{11}$$

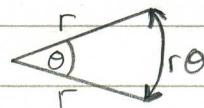
$$t_3 = 2t_2 + 5 = 2 \times 11 + 5 = \underline{27}$$

ii) $\sum_{k=1}^3 k(k+1) = 1 \times 2 + 2 \times 3 + 3 \times 4$
 $= 2 + 6 + 12$
 $= \underline{20}$

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$$5. \text{ area} = \frac{1}{2} r^2 \theta \quad \frac{2\text{area}}{r^2} = \theta = \frac{2 \times 9}{25} = \frac{18^\circ}{25} = 0.72^\circ$$

$$\text{per.} = r\theta + 2r \\ = 5 \times 0.72 + 10 \\ = 13.6 \text{ cm}$$



$$6.i) \log_a 1 = 0 \quad \text{since } a^0 = 1 \\ \log_a a = 1 \quad \text{since } a^1 = a$$

$$\log x^n = n \log x$$

$$\log \frac{a}{b} = \log a - \log b$$

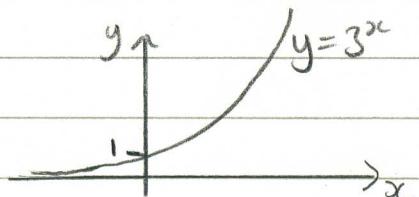
$$ii) \log_a x^{10} - 2 \log_a \left(\frac{x^3}{4} \right) = 10 \log_a x - \log_a \left(\frac{x^6}{16} \right)$$

$$= 10 \log_a x - 6 \log_a x + \log_a 16$$

$$= 4 \log_a x - \log_a 2^4$$

$$= 4 \log_a x - 4 \log_2$$

$$= \underline{\underline{4 \log_a (2x)}}$$



$$ii) 3^x = 20$$

$$\log 3^x = \log 20$$

$$x \log 3 = \log 20$$

$$x = \frac{\log 20}{\log 3}$$

$$= \underline{\underline{2.73}} \quad (2 \text{ d.p.})$$

$$8. \quad 2 \cos^2 \theta + 7 \sin \theta = 5$$

$$2(1 - \sin^2 \theta) + 7 \sin \theta - 5 = 0$$

$$2 - 2 \sin^2 \theta + 7 \sin \theta - 5 = 0$$

$$-2 \sin^2 \theta + 7 \sin \theta - 3 = 0$$

$$2 \sin^2 \theta - 7 \sin \theta + 3 = 0$$

use identity $\cos^2 \theta + \sin^2 \theta = 1$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\downarrow x(-1)$$

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8 ii) let $\sin \theta = s$

$$2s^2 - 7s + 3 = 0$$

$$2 \times 3 = 6 \quad -6 \times -1 = 6$$

$$2s^2 - s - 6s + 3 = 0$$

$$-6 + -1 = -7$$

$$s(2s-1) - 3(2s-1) = 0$$

$$(s-3)(2s-1) = 0$$

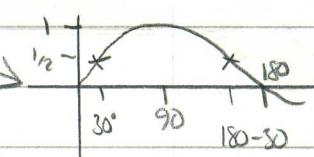
A

$$s-3=0 \quad \text{or} \quad 2s-1=0$$

$$\sin \theta = 3 \quad \text{or} \quad \sin \theta = \frac{1}{2}$$

C

$$\underline{\underline{\theta = 30^\circ, 150^\circ}}$$



B

$$y = 2x^3 - 9x^2 + 12x - 2 = f(x) \text{ at } x=3, \text{ grad} = 6(3^2) - 18(3) + 12 \\ = 54 - 54 + 12 \\ = 12$$

$$\text{at } x=3, y = 2(3^3) - 9(3^2) + 12(3) - 2 \quad \text{coords } (3, 7) \\ = 54 - 81 + 36 - 2 \\ = 7$$

$$m = 12$$

$$y - y_1 = m(x - x_1) \quad y - 7 = 12(x - 3) \quad \leftarrow \begin{matrix} \text{eqn of tangent} \\ \text{to curve at } x=3 \end{matrix}$$

$$\text{subst. } (-1, -4))$$

$$-41 - 7 = 12(-1 - 3)$$

$$-48 = -48 \quad \checkmark$$

$\therefore (-1, -4)$ is on the tangent to curve at $x=3$.

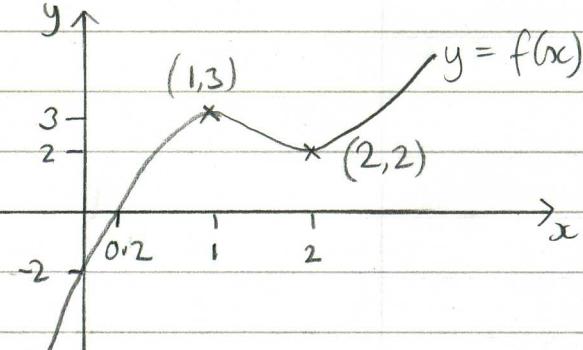
ii) at turning pts., $\frac{dy}{dx} = 0 = 6x^2 - 18x + 12$
 $= x^2 - 3x + 2$
 $= (x-2)(x-1)$
 $\Rightarrow x=2 \text{ or } x=1$

$$x=2, y = 2(2^3) - 9(2^2) + 12(2) - 2 = 16 - 36 + 24 - 2 = 2$$

$$x=1, y = 2(1^3) - 9(1^2) + 12(1) - 2 = 3$$

coords are (2, 2) and (1, 3)

$$x=0, y=-2$$

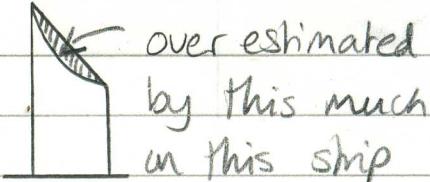


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10.i) $h = 10$

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 10 (28 + 22 + 2(19 + 14 + 11 + 12 + 16)) \\ &= 5(50 + 144) \\ &= \underline{\underline{970 \text{ sq. units}}} \quad [\text{represents } 970 \text{ m}] \end{aligned}$$

- ii) curve is concave so it dips below the tops of the trapezia, \therefore the real area is less than our estimate



To underestimate, use rectangles where the height is the lower of the two possible v values

$$\begin{aligned} \text{area} &\approx 10(19 + 14 + 11 + 11 + 12 + 16) \\ &= \underline{\underline{830 \text{ sq. units}}} \end{aligned}$$

iii) $v = 28 - t + 0.015t^2$ subst. $t = 10$, $v = 28 - 10 + 0.015(100)$
 $= 19.5 \text{ ms}^{-1}$

measured value of v at $t = 10$ was 19 ms^{-1}

diff. of 0.5 ms^{-1} 3% of 19 is $0.03 \times 19 = 0.57$

$0.5 < 0.57 \therefore$ diff. is less than 3% of measured.

iv) $\int_0^{60} (28 - t + 0.015t^2) dt = \left[28t - \frac{t^2}{2} + \frac{0.015t^3}{3} \right]_0^{60}$

$$= \left(28(60) - \frac{60^2}{2} + \frac{0.015(60)^3}{3} \right) - (0 - 0 + 0)$$

$$= 1680 - 1800 + 1080$$

$$= \underline{\underline{960 \text{ m}}}$$

(this is close to our trapezium rule estimate of 970m)

II. a) A.P.: 3, 5, 7, ... $a=3, d=2$ $a_n = a + (n-1)d$

i) $a_6 = 3 + 5 \times 2 = 13$

ii) $S_{10} = \frac{1}{2}(2a + (n-1)d), S_{10} = 5(6 + 18) = 120$

b) $P_n = \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^{n-1}$ i) $P_4 = \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^3 = \frac{125}{1296}$

ii) G.P. is $\frac{1}{6}, \frac{1}{6} \times \frac{5}{6}, \frac{1}{6} \times \left(\frac{5}{6}\right)^2, \dots$ $a = \frac{1}{6}; r = \frac{5}{6}$

$$S_{\infty} = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{5}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

iii) $P_n < 0.001$

$$\frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1} < 0.001$$

$$\left(\frac{5}{6}\right)^{n-1} < 0.006$$

$$\begin{aligned} (n-1)\log\left(\frac{5}{6}\right) &< \log 0.006 \quad \leftarrow \text{take logs} \\ n-1 &> \frac{\log 0.006}{\log\left(\frac{5}{6}\right)} \quad \leftarrow \text{!! notice that } \log\frac{5}{6} < 0 \\ n &> \frac{\log 0.006}{\log\left(\frac{5}{6}\right)} + 1 \end{aligned}$$

$$\frac{\log 0.006}{\log\left(\frac{5}{6}\right)} + 1 = 29.060\dots \text{ but } n \text{ is an integer}$$

\therefore least value of n is 30