

C1 (Summer) 2014

$$\begin{aligned}
 1(i) \quad & \left(\frac{1}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{1}{27}}\right)^2 \\
 & = \frac{1}{3^2} \\
 & = \underline{\underline{\frac{1}{9}}}
 \end{aligned}
 \quad
 \begin{aligned}
 (ii) \quad & \frac{(4a^2c)^3}{32a^4c^7} = \frac{4^3 a^6 c^3}{32a^4 c^7} \\
 & = \frac{64a^{6-4}c^{3-7}}{32} \\
 & = \underline{\underline{\frac{2a^2}{c^4}}}
 \end{aligned}$$

2 $A(1, 5)$, $B(6, -1)$ M is $\left(\frac{1+6}{2}, \frac{5-1}{2}\right) = \left(\frac{7}{2}, 2\right)$

$$\begin{aligned}
 y &= 2x - 5 \text{ subst. } x = \frac{7}{2} \\
 &= 2\left(\frac{7}{2}\right) - 5
 \end{aligned}$$

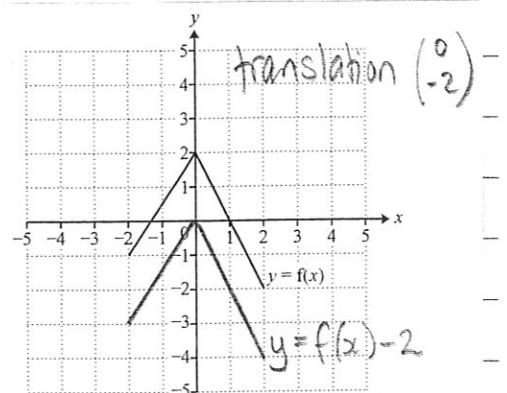
$= 2$ $\therefore \left(\frac{7}{2}, 2\right)$ lies on the line so the line $y = 2x - 5$ passes through M.

$$4(i) \quad (7 - 2\sqrt{3})^2 = (7 - 2\sqrt{3})(7 - 2\sqrt{3})$$

$$\begin{aligned}
 &= 49 - 28\sqrt{3} + 4(3) \\
 &= \underline{\underline{61 - 28\sqrt{3}}}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & \frac{20\sqrt{6}}{\sqrt{50}} = \frac{20\sqrt{2}\sqrt{3}}{\sqrt{25 \times 2}} \\
 &= \frac{20\sqrt{2}\sqrt{3}}{5\sqrt{2}} \\
 &= \underline{\underline{4\sqrt{3}}}
 \end{aligned}$$

3(i)



$$5 \quad 3(a+4) = ac + 5f$$

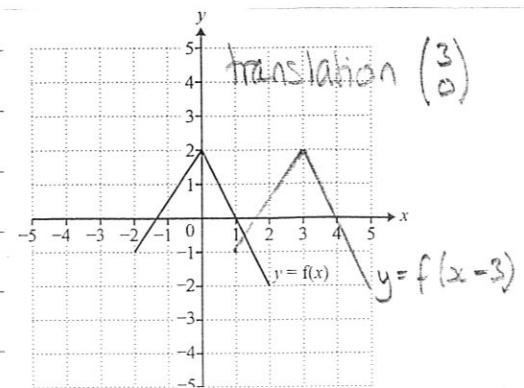
$$3a + 12 = ac + 5f$$

$$3a - ac = 5f - 12$$

$$a(3 - c) = 5f - 12$$

$$a = \underline{\underline{\frac{5f - 12}{3 - c}}}$$

(ii)

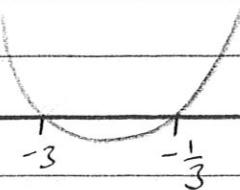


6 $3x^2 + 10x + 3 > 0$

$$3x^2 + 9x + 2x + 3 > 0$$

$$3x(x+3) + (2x+3) > 0$$

$$(3x+1)(x+3) > 0$$



$$x < -3, \quad x > -\frac{1}{3}$$

7 $(5+2x)^7$ x^4 term is ${}^7C_4(2x)^4(5)^3$

$$\begin{aligned} {}^7C_4 &= {}^7C_3 = \frac{7}{1} \times \frac{6}{2} \times \frac{5}{3} \\ &= 35 \end{aligned} \quad 2^4 = 16 \quad 5^3 = 125$$

$$2 \times 8 \times 125 = 2 \times 1000 = 2000$$

∴ coeff. of x^4 term is $35 \times 2000 = \underline{\underline{70000}}$

8 $f(x) = 4x^3 + kx + 6$ $f(2)$ gives the remainder when

$$f(2) = 42 = 4(2)^3 + 2k + 6 \quad f(5)$$

$$42 = 32 + 2k + 6 \quad \text{is divided by } (x-2), \text{ by}$$

$$4 = 2k$$

$$\underline{k = 2}$$

$$f(x) = 0 \Rightarrow 4x^3 + 2x + 6 = 0$$

$$2x^3 + x + 3 = 0$$

$$f(1) = 2 + 1 + 3 \neq 0$$

$$f(-1) = 2(-1) - 1 + 3 = 0$$

∴ $\underline{x = -1}$ is a root of $f(x) = 0$

9(i) $n^2 + (n+1)^2 + (n+2)^2 = n^2 + n^2 + 2n + 1 + n^2 + 4n + 4$

$$= \underline{\underline{3n^2 + 6n + 5}} \quad (\text{does not factorise})$$

(ii) using odd \times odd gives even AND even \times anything gives even

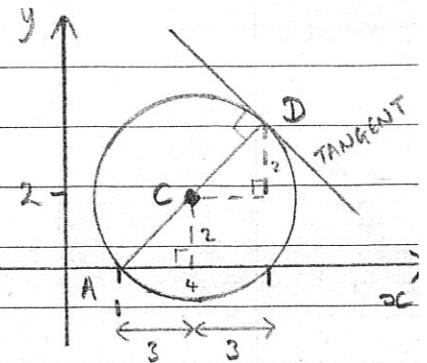
∴ $6n + 5$ is even + odd, so odd for any n .

$3n^2$ is odd for odd n , or even for even n .

so $3n^2 + 6n + 5$ is even for odd n , or odd for even n .

∴ the sum of the three squares is EVEN ONLY for

ODD INTEGERS n



10(i) by symmetry, B is pt. (7, 0)

(ii) radius is $\sqrt{(4-1)^2 + (2-0)^2} = \sqrt{13}$

circle has eqn $(x-4)^2 + (y-2)^2 = 13$

(iii) $\vec{AC} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \therefore \vec{CD} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow D$ is pt. (7, 4)

(iv) grad AD is $\frac{4-0}{7-1} = \frac{2}{3} \therefore$ grad of tangent is $-\frac{3}{2}$

D lies on tangent so we (7, 4) as (x_1, y_1) in

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{3}{2}(x - 7)$$

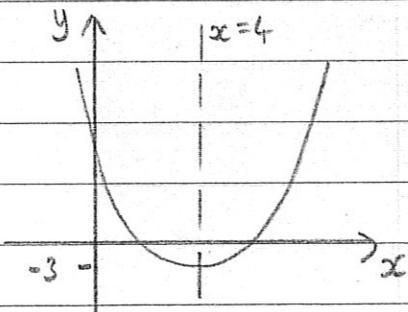
$$y - 4 = -\frac{3}{2}x + \frac{21}{2}$$

$$\underline{\underline{y = -\frac{3}{2}x + \frac{21}{2}}}$$

11 (i) $y = (x-4)^2 - 3$

line of sym. $x=4$

min. pt. (4, -3)



(ii) y-int. where $x=0 \quad y = (0-4)^2 - 3$
 $= 16 - 3$

\therefore y-intercept at (0, 13)

x-int. where $y=0 \quad 0 = (x-4)^2 - 3$

$$3 = (x-4)^2$$

$$\pm\sqrt{3} = x - 4$$

$$4 \pm \sqrt{3} = x$$

$$\downarrow +3$$

$$\downarrow \sqrt{3}$$

$$\downarrow +4$$

\therefore x-int at $(4-\sqrt{3}, 0)$ and $(4+\sqrt{3}, 0)$

(iii) translation (2) has eqn $f(x-2)$

$$f(x) = (x-4)^2 - 3$$

$$f(x-2) = (x-2-4)^2 - 3$$

$$= (x-6)^2 - 3$$

$$= x^2 - 12x + 36 - 3$$

$$= \underline{\underline{x^2 - 12x + 33}} \quad \text{Q.E.D.}$$

(iv) subst. $y = 8-2x$ into $y = x^2 - 12x + 33$

$$8-2x = x^2 - 12x + 33$$

$$0 = x^2 - 10x + 25$$

$$0 = (x-5)^2$$

$x = 5$ repeated root \therefore meet at ONLY ONE point

y-coord is $8-2(5) = -2$

\therefore coords of meeting pt. (5, -2)

12(i) roots at $-5, -2, \frac{3}{2}$, y-int. at -30

$$5 \times 2x - 3 = -30$$

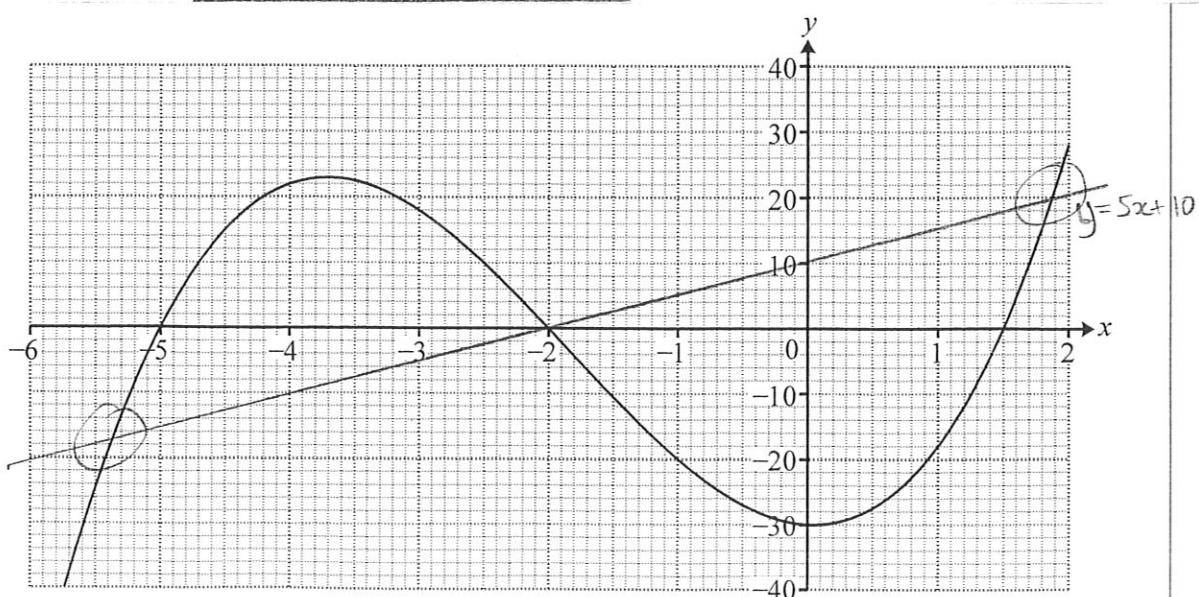
$$\therefore \underline{\underline{y = (x+5)(x+2)(2x-3)}}$$

(ii) $y = (x^2 + 7x + 10)(2x - 3)$

$$= 2x^3 + 14x^2 + 20x - 3x^2 - 21x - 30$$

$$= \underline{\underline{2x^3 + 11x^2 - x - 30}}$$

(iii)



intersections at x-coords $-2, \underline{-5.35}, \underline{1.88}$

(iv) intersections where $5x+10 = 2x^3 + 11x^2 - x - 30$

$$0 = 2x^3 + 11x^2 - 6x - 40$$

one root is $x = -2$, $\therefore (x+2)$ is a factor by the FACTOR THM.

Divide the cubic by $(x+2)$

	x	$2x^2 + 7x - 20$	
	x	$2x^3 + 7x^2 - 20x$	
	$+2$	$4x^2 \quad 14x \quad -40$	

\therefore the cubic is $(x+2)(2x^2 + 7x - 20)$

\therefore intersections where $2x^2 + 7x - 20 = 0$

$$a=2, b=7, c=-20$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{49 - 4(2)(-20)}}{4}$$

$$= \frac{-7 \pm \sqrt{209}}{4}$$