Solution to the C1 June 2010 exam

Q1 We are given that the line L is parallel to $y = 3x + 1$. Therefore the gradient is 3 and the line is of the form $y = 3x + C$ to find C we use the fact that the line goes through (4,5).

Hence $5 = 3 \times 4 + C \rightarrow C = -7$. So that the eq. of line is $\boxed{y = 3x - 7}$

Q2

i) $(5a^2b)^3 \times 2b^4 = 5^3(a^2)^3b^3 \times 2b^4 = 125 \times a^6b^{3+4} \times 2 = \boxed{250 \times a^6b^7}$

ii) $\left(\frac{1}{16}\right)^{-1} = \left(\frac{16}{1}\right)^1 = \boxed{16}$

iii) $(16)^{\frac{3}{2}} = \left[(16)^{\frac{1}{2}}\right]^3 = 4^3 = \boxed{64}$

Q3 Remember you need to get the square root on its own before squaring.

$$a = \frac{\sqrt{y}-5}{c} \quad \Rightarrow ac = \sqrt{y} - 5 \quad \Rightarrow ac + 5 = \sqrt{y} \quad \Rightarrow \boxed{y = (ac+5)^2}$$

Q4 *i*) $2(1-x) > 6x + 5 \rightarrow 2 - 2x > 6x + 5$

$$\rightarrow -3 > 8x \ \ Hence \quad \boxed{x < \frac{-3}{8}}$$

ii) $(2x-1)(x+4) < 0$

<u>Graphical solution</u>: This is a quadratic with roots $x = \frac{1}{2}\ and - 4$ and as coefficient of $x^2$is positive , the curve will below *x*-axis between the roots.

Hence $\boxed{-4 < x < \frac{1}{2}}$

Or **<u>algebraic solution</u>** $(2x-1)(x+4) < 0$ means either

$(2x-1) < 0$ $and\ (x+4) > 0$ or $(2x-1) > 0$ $and\ (x+4) < 0$

$-4 < x < \frac{1}{2}$ $or \quad x < -4\ and\ \ x > 1/2$

Only first solution is possible hence $\boxed{-4 < x < \frac{1}{2}}$

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Q5*i*) $\sqrt{48}+\sqrt{27} = \sqrt{16\times 3}+\sqrt{9\times 3} = 4\sqrt{3}+3\sqrt{3} = \boxed{7\sqrt{3}}$

ii) $\frac{5\sqrt{2}}{3-\sqrt{2}} = \frac{5\sqrt{2}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3\times 5\sqrt{2}+5\sqrt{2}\sqrt{2}}{3^2-(\sqrt{2})^2} = \frac{15\sqrt{2}+5\times 2}{9-2} = \boxed{\frac{10+15\sqrt{2}}{7}}$

Q6

First we need to expand the two bracket but only need to cubic term, which will arise from the 5 times the cube and the square in 1st bracket and the *kx* in the second, so we have

$$5\times x^3+2x^2\times kx \equiv 29\,x^3 \ \ (note\ no\ m) \rightarrow 5+2k=29 \quad and \ \boxed{k=12}$$

We now use the remainder theorem which says that if f(*x*) divided by (*x*-3) has remainder 59 then $f(3)=59$.

Hence with $f(x)=x^3+kx+m$ we get with *k*=12 $f(3)=3^3+12\times 3+m=63+m$ and as $f(3)=59$ we get $\boxed{m\ =4}$

Q7 Remember the binomial expansion:

$(a+b)^4 = a^4b^0+4a^3b^1+6a^2b^2+4a^1b^3+a^0b^4$

using *a*=1 and $b=\left(\frac{1}{2}x\right)$!!!! do not forget the brackets

$$\left(1+\left(\frac{1}{2}x\right)\right)^4 = 1^4\left(\frac{1}{2}x\right)^0+4\times 1^3\left(\frac{1}{2}x\right)^1+6\times 1^2\left(\frac{1}{2}x\right)^2$$

$$+4\times 1^1\left(\frac{1}{2}x\right)^3+1^0\left(\frac{1}{2}x\right)^4$$

$$=1+4\times\frac{1}{2}x+6\times\frac{1}{4}x^2+4\times\frac{1}{8}x^3+\frac{1}{16}x^4$$

$$=\boxed{1+2x+\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{1}{16}x^4}$$

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Q8 Remember when completing the square with coefficient of x^2 not 1, factorise the coefficient out of the x^2 *and* x terms only, leave the constant.

$$5x^2 + 20x + 6 = 5[\,x^2 + 4x] + 6 = 5[\,x^2 + 4x + 4 - 4] + 6$$

$$= 5[(x+2)^2 - 4] + 6 = 5(x+2)^2 - 20 + 6$$

$$= \boxed{5(x+2)^2 - 14}$$

Q9 We are given $x - 5 = 0 \Leftrightarrow x^2 = 25$. Need to show what happens if we take one side to be true.

From LHS we get $x = 5$ which we put into RHS to give $5^2 = 25$ as required.

From RHS we get $x = \mp 5$. The +5 solution is OK but if we put $x = -5$ into LHS we get $-5 - 5 \neq 0$

Hence $x - 5 = 0 \Leftarrow x^2 = 25$ is not true and therefore

$$x - 5 = 0 \Leftrightarrow x^2 = 25 \text{ is false.}$$

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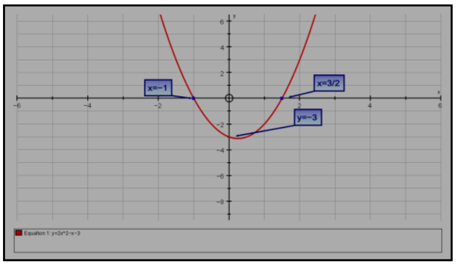
Q10 *i*) Need to factorise a quadratic. By inspection we see that we need a 2*x* and an *x* term in each bracket. The factors of -3 then need to be tried out so that they give -*x*, which gives

$$2x^2 - x - 3 = 0 \rightarrow (2x-3)(x+1) = 0 \rightarrow (2x-3) = 0 \; or \; (x+1) = 0$$

Hence the roots are $x = -\frac{3}{2} \;\; and \;\; x = -1$

ii)



Remember for sketch always need to show the roots, y-intercept, label axis with reasonable scale.

iii) $x^2 - 5x + 10 = 0$ discriminant $b^2 - 4ac = 25 - 4 \times 1 \times 10 = -15 < 0.$

Negative discriminant indicates there are no real roots.

iv) Intersection means that we need to solve simultaneous eqs.

$y = x^2 - 5x + 10 \;\; (1)$ and $y = 2x^2 - x - 3 \;\; (2)$

Putting (1)=(2) we get $x^2 - 5x + 10 = 2x^2 - x - 3$

$$x^2 + 4x - 13 = 0$$

Quadratic which we need to solve. Note the question want answer in surd form so unlikely that it is easy to factorise and better to use directly quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-13)}}{2 \times 1}$$

$$= \frac{-4 \pm 2\sqrt{4 + 13}}{2}, \text{hence} \quad \boxed{x = -2 \pm \sqrt{17}}$$

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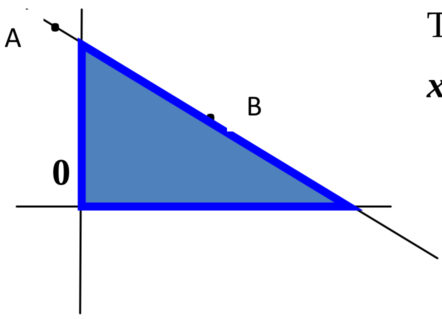
Q11 **i)** We are given two points A(-1,3) B(5,1) we need to find the gradients

$m_{AB} = \frac{3-1}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$

The line is of the form $y = -\frac{1}{3}x + C$

To find C we use the fact that the line goes through (-1,3).

Hence $3 = -\frac{1}{3} \times -1 + C \rightarrow C = \frac{8}{3}$. So that the eq. of line is $\boxed{y = -\frac{1}{3}x + \frac{8}{3}}$



The triangle we need has sides 0 to y-intercept and 0 to **x- intercept. For y-intercept we know it is C=$\frac{8}{3}$**

And for x-intercept we put $y = 0$ *giving* $x = 8$.

Hence the area is $A = \frac{1}{2} \times \frac{8}{3} \times 8 \rightarrow \boxed{A = \frac{32}{3}}$

iii) The bisector goes through then midpoint of AB and is perpendicular to AB.

Midpoint of AB is given by M: $\left(\frac{-1+15}{2}, \frac{3+1}{2}\right) = (2,2)$

As both line are perpendicular $m_{AB} \times m_{Bisector} = -1$

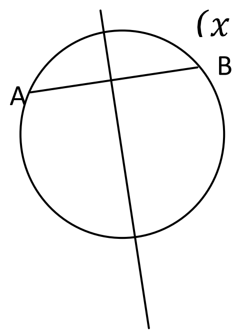
and therefore $m_{Bisector} = \frac{-1}{m_{AB}} = \frac{-1}{\frac{-1}{3}} = 3$

The line is of the form $y = 3x + C$ to find C we use the fact that the line goes through (2,2), so that $2 = 3 \times 2 + C \rightarrow C = -4$

And the equation of the bisector is $\boxed{y = 3x - 4}$ as required.

iv) <u>Often in the last part of a question you will use a result already shown before. The question you should ask yourself is why did we need to find the bisector.</u> A small sketch will help. Take another chord and you will see that the bisector goes through the centre of the circle. Hence as we know the centre lies on $x = 3$ we have $y = 3 \times 3 - 4 = 5$.

So the <u>centre is at</u> $\boxed{(3,5)}$ and equation of circle is



$(x-3)^2 + (y-5)^2 = r^2$ now using the point B(5,1) which belongs to circle we have $(5-3)^2 + (1-5)^2 = 20 = \left(2\sqrt{5}\right)^2$

eq. of circle is $\boxed{(x-3)^2 + (y-5)^2 = \left(2\sqrt{5}\right)^2}$

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Q12 *i*) The factor theorem state that if $(x-a)$ is a factor of $f(x)=0$ then $x=a$ is a root.

First we need to look at factors of constant terms 30: 1,2,3,5,10,15 and 30 and tries these out.

Can us table mode in calculator. However, we start trying by smallest factors. 1 is clearly not going to work but 2 into $f(x)=x^3+6x^2-x-30$

Gives $f(2)=2^3+6\times 2^2-2+30=8+24-2-30=0$

Hence $(x-2)$ is a factor and we can write

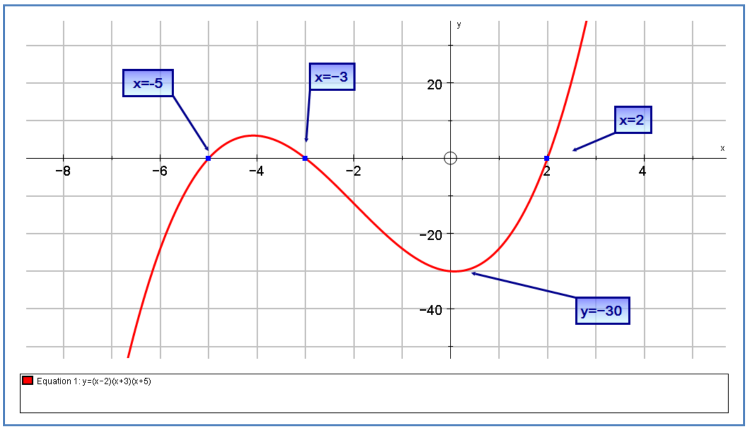
$$f(x)=x^3+6x^2-x-30=(x-2)(ax^2+bx+c)$$

By inspection we find a=1 and c=15 . To find b we can compare coefficient of x^2.

On LHS we have $+6x^2$ and on RHS $-2ax^2+bx^2$. So with a=1 we need to get

$-2+b=6$ *hence* $b=8$. And $f(x)=(x-2)(x^2+8x+15)$ Again consider factors of 15 and note that 5+3 =8! Hence $\boxed{f(x)=x^3+6x^2-x-30=(x-2)(x+3)(x+5)}$

ii) Remember diagram need to have roots and y intercept clearly indicated. Also cubic with positive coeff of cube term: $y\to+\infty$ *as* $x\to+\infty$ and $y\to-\infty$ *as* $x\to-\infty$



The transformation $\binom{1}{0}$ is a translation one unit to the left along x-axis and we have $f(x)\to f(x-1)$. Now $f(x-1)$ is $f(x)$ with x replaced by x-1, so we get

$$f(x-1)=([x-1]-2)([x-1]+3)([x-1]+5)=(x-3)(x+2)(x+4)$$

Now we expand this $(x-3)(x+2)(x+4)=(x^2-3x+2x-6)(x+4)$

$$=(x^2-x-6)(x+4)=x^3-x^2-6x$$

$$+4x^2-4x-24$$

Which gives $\boxed{=x^3+3x^2-10x-6}$