Paper collated from year	2015
Content	Pure Chapters 1-13
Marks	100
Time	2 hours

1. Simplify

(a) $(2\sqrt{5})^2$

(1)

(b) $\dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$ giving your answer in the form $a + \sqrt{b}$, where a and b are integers.

(4)

2. Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

3. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\dfrac{dy}{dx}$

(3)

(b) $\int y\,dx$

(3)

4.

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

5. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

6.

Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

7. (a) Factorise completely $9x - 4x^3$

(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x-axis.

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

(4)

8. A curve with equation $y = f(x)$ passes through the point (4, 9).

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

(b) Find the x coordinate of P.

(5)

9. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

10. A circle C with centre at the point (2, –1) passes through the point A at (4, –5).

(a) Find an equation for the circle C.

(3)

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

11. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

12. Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k.

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

13. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

14. Differentiate $f(x) = 8x^3 + 5$ from first principles. (4)

Mark scheme

1.(a)	20	Sight of 20. (4×5 is not sufficient)	B1
			(1)
(b)	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$	Multiplies top and bottom by a **correct** expression. This statement is sufficient. NB $2\sqrt{5}+3\sqrt{2} \equiv \sqrt{20}+\sqrt{18}$	M1
	(Allow to multiply top and bottom by $k(2\sqrt{5}+3\sqrt{2})$)		
	$= \frac{\cdots}{2}$	Obtains a denominator of 2 or sight of $\left(2\sqrt{5}-3\sqrt{2}\right)\left(2\sqrt{5}+3\sqrt{2}\right)=2$ with no errors seen in this expansion. May be implied by $\frac{\cdots}{2k}$	A1
	Note that M0A1 is not possible. The 2 must come from a correct method.		
	Note that if M1 is scored there is no need to consider the numerator. **e.g.** $\frac{2(MR?)}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{\cdots}{2}$ scores M1A1		
	Numerator = $\sqrt{2}(2\sqrt{5}\pm 3\sqrt{2}) = 2\sqrt{10}\pm 6$	An attempt to multiply the numerator by $\pm\left(2\sqrt{5}\pm 3\sqrt{2}\right)$ and obtain an expression of the form $p+q\sqrt{10}$ where p and q are integers. This may be implied by e.g. $2\sqrt{10}+3\sqrt{4}$ or by their final answer.	M1
	(Allow attempt to multiply the numerator by $k(2\sqrt{5}\pm 3\sqrt{2})$)		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{2\sqrt{10}+6}{2} = 3+\sqrt{10}$	Cso. For the answer as written or $\sqrt{10}+3$ or a statement that $a = 3$ and $b = 10$. Score when first seen and ignore any subsequent attempt to 'simplify'. Allow $1\sqrt{10}$ for $\sqrt{10}$	A1
			(4)
			(5 marks)

2.

$y=2x+4 \Rightarrow 4x^2+(2x+4)^2+20x=0$ **or** $2x=y-4$ or $x=\dfrac{y-4}{2}$ $\Rightarrow (y-4)^2+y^2+10(y-4)=0$	Attempts to rearrange the linear equation to $y=...$ or $x=...$ or $2x=...$ and attempts to **fully** substitute into the second equation.	M1
$8x^2+36x+16=0$ or $2y^2+2y-24=0$	M1: Collects terms together to produce quadratic expression = 0. The '= 0' may be implied by later work. A1: Correct three term quadratic equation in x or y. The '= 0' may be implied by later work.	M1 A1
$(4)(2x+1)(x+4)=0 \Rightarrow x=...$ or $(2)(y+4)(y-3)=0 \Rightarrow y=...$	Attempt to factorise and solve or complete the square and solve or uses a correct quadratic formula **for a 3 term quadratic.**	M1
$x=-0.5, x=-4$ or $y=-4, y=3$	Correct answers for either both values of x or both values of y (possibly un-simplified)	A1 cso
Sub into $y=2x+4$ or Sub into $x=\dfrac{y-4}{2}$	Substitutes at least one of their values of x into a **correct** equation as far as $y=...$ or substitutes at least one of their values of y into a **correct** equation as far as $y=...$	M1
$y=3,\ y=-4$ and $x=-4,\ x=-0.5$	Fully correct solutions and simplified. **Pairing not required.** If there are any extra values of x or y, score A0.	A1
		(7 marks)

	Special Case: Uses $y = -2x - 4$		
	$y=2x+4 \Rightarrow 4x^2+(-2x-4)^2+20x=0$		M1
	$8x^2+36x+16=0$		M1A1
	$(4)(2x+1)(x+4)=0 \Rightarrow x=...$		M1
	$x=-0.5, x=-4$		A0
	Sub into $y=2x+4$	Sub into $y=-2x-4$ is M0	M1
	$y=3,\ y=-4$ and $x=-4,\ x=-0.5$		A0

3.	$y = 4x^3 - \frac{5}{x^2}$		
(a)	$12x^2 + \frac{10}{x^3}$	M1: $x^n \to x^{n-1}$ e.g. Sight of x^2 or x^{-3} or $\frac{1}{x^3}$	M1A1A1
		A1: $3\times 4x^2$ or $-5\times -2x^{-3}$ (oe) (Ignore + c for this mark)	
		A1: $12x^2 + \frac{10}{x^3}$ or $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	
	Apply ISW here and award marks when first seen.		
			(3)
(b)	$x^4 + \frac{5}{x} + c$ or $x^4 + 5x^{-1} + c$	M1: $x^n \to x^{n+1}$. e.g. Sight of x^4 or x^{-1} or $\frac{1}{x^1}$ **Do <u>not</u> award for integrating their answer to part (a)**	M1A1A1
		A1: $4\frac{x^4}{4}$ or $-5\times\frac{x^{-1}}{-1}$	
		A1: For fully correct and simplified answer with + c <u>all on one line</u>. Allow $x^4 + 5\times\frac{1}{x} + c$ Allow $1x^4$ for x^4	
	Apply ISW here and award marks when first seen. Ignore spurious integral signs for all marks.		
			(3)
			(6 marks)

4.

$b^2 - 4ac < 0 \Rightarrow$ e.g. $4^2 - 4(p-1)(p-5) < 0$ or $0 > 4^2 - 4(p-1)(p-5)$ or $4^2 < 4(p-1)(p-5)$ or $4(p-1)(p-5) > 4^2$	M1: Attempts to use $b^2 - 4ac$ with at least two of *a, b* or *c* correct. May be in the quadratic formula. Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the given quadratic equation. Inequality sign not needed for this M1. There must be no *x* terms.	M1A1
	A1: For a correct un-simplified **inequality** that is not the given answer	
$4 < p^2 - 6p + 5$		
$p^2 - 6p + 1 > 0$	Correct solution with **no** errors that includes an expansion of $(p-1)(p-5)$	A1*
		(3)

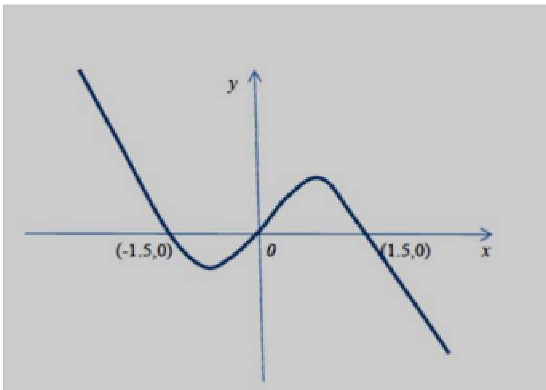
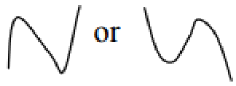
<table>
<tr><td rowspan="4">(b)</td><td colspan="2">$p^2-6p+1=0 \Rightarrow p=...$</td><td>For an attempt to solve $p^2-6p+1=0$ (not <u>their</u> quadratic) leading to 2 solutions for p (do not allow attempts to factorise – must be using the quadratic formula or completing the square)</td><td>M1</td></tr>
<tr><td>$p=3\pm\sqrt{8}$</td><td colspan="2">$p=3\pm2\sqrt{2}$ or any equivalent correct expressions e.g. $p=\dfrac{6\pm\sqrt{32}}{2}$ (May be implied by their inequalities)
Discriminant must be a single number not e.g. 36 - 4</td><td>A1</td></tr>
<tr><td colspan="3">Allow the M1A1 to score anywhere for solving the given quadratic</td><td></td></tr>
<tr><td colspan="2">$p<3-\sqrt{8}$ or $p>3+\sqrt{8}$</td><td>M1: Chooses outside region – not dependent on the previous method mark

A1: $p<3-\sqrt{8}$, $p>3+\sqrt{8}$ or equivalent e.g.
$p<\dfrac{6-\sqrt{32}}{2}$, $p>\dfrac{6+\sqrt{32}}{2}$
$\left(-\infty,\ 3-\sqrt{8}\right)\cup\left(3+\sqrt{8},\ \infty\right)$
Allow ",", "or" or a space between the answers but do not allow
$p<3-\sqrt{8}$ and $p>3+\sqrt{8}$ (this scores M1A0)
Apply ISW if necessary.</td><td>M1A1</td></tr>
<tr><td colspan="5">A correct solution to the quadratic followed by $p>3\pm\sqrt{8}$ scores M1A1M0A0</td></tr>
<tr><td colspan="5">$3+\sqrt{8}<p<3-\sqrt{8}$ scores M1A0</td></tr>
<tr><td colspan="5">Allow candidates to use x rather than p but must be in terms of p for the final A1</td></tr>
<tr><td rowspan="2"></td><td colspan="3"></td><td>(4)</td></tr>
<tr><td colspan="3"></td><td>(7 marks)</td></tr>
</table>

5(a)		
$(x^2+4)(x-3)=x^3-3x^2+4x-12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
$\frac{x^3-3x^2+4x-12}{2x}=\frac{x^2}{2}-\frac{3}{2}x+2-6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$ A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\frac{x^2}{2}$ not $\frac{x^3}{2x}$	M1A1
$\frac{dy}{dx}=x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2 \to 0$ **Dependent on both previous method marks.** A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and **isw** Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0. If they lose the previous A1 because of an **incorrect constant only** then allow recovery here and in part (b) for a correct derivative.	ddM1A1
		(5)

(b)	At $x=-1,\ y=10$	Correct value for y	B1
	$\left(\frac{dy}{dx}=\right)-1-\frac{3}{2}+\frac{6}{1}=3.5$	M1: Substitutes $x=-1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y-'10'='3.5'(x--1)$	Uses their **tangent** gradient **which must come from calculus** with $x=-1$ and their numerical y with a correct straight line method. If using $y=mx+c$, this mark is awarded for correctly establishing a value for c.	M1
	$2y-7x-27=0$	$\pm k(2y-7x-27)=0$ cso	A1
			(5)
			(10 marks)

6 (a)	$\left(4^x =\right) y^2$	Allow y^2 or $y \times y$ or "y squared" "$4^x =$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2 - 9y + 1 = (8y-1)(y-1) = 0 \Rightarrow y = ...$ or $(8(2^x)-1)((2^x)-1) = 0 \Rightarrow 2^x = ...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2 - 9x + 1 = 0$	M1
	$2^x (\text{or } y) = \frac{1}{8}, 1$	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **<u>not x</u>** unless $2^x (\text{or } y)$ is implied later	A1
	$x = -3 \quad x = 0$	M1: A correct attempt to find one **numerical value** of x from their $2^x (\text{or } y)$ **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated. A1: Both $x = -3$ and/or $x = 0$ May be implied by e.g. $2^{-3} = \frac{1}{8}$ and $2^0 = 1$ **and no extra values.**	M1A1
			(4)
			(5 marks)

7	$9x-4x^3=x(9-4x^2)$ or $-x(4x^2-9)$	Takes out a common factor of x or $-x$ **correctly.**	B1
	$9-4x^2=(3+2x)(3-2x)$ or $4x^2-9=(2x-3)(2x+3)$	$9-4x^2=(\pm3\pm2x)(\pm3\pm2x)$ or $4x^2-9=(\pm2x\pm3)(\pm2x\pm3)$	M1
	$9x-4x^3=x(3+2x)(3-2x)$	Cao but allow equivalents e.g. $x(-3-2x)(-3+2x)$ or $-x(2x+3)(2x-3)$	A1
Note: $4x^3-9x=x(4x^2-9)=x(2x-3)(2x+3)$ so $9x-4x^3=x(3-2x)(2x+3)$ **would score full marks**			
	Note: Correct work leading to $9x\left(1-\frac{2}{3}x\right)\left(1+\frac{2}{3}x\right)$ **would score full marks**		
	Allow ($x\pm0$) or ($-x\pm0$) instead of x and $-x$		
			(3)
(b)		 A cubic shape with one maximum and one minimum	M1
		Any line or curve drawn passing through (not touching) the origin	B1
		Must be the correct shape and in all four quadrants and pass through (-1.5, 0) and (1.5, 0) (Allow (0, -1.5) and (0, 1.5) or just -1.5 and 1.5 provided they are positioned correctly). **Must be on the diagram** (Allow $\sqrt{\frac{9}{4}}$ for 1.5)	A1
			(3)

(c)	$A=(-2,\ 14),\quad B=(1,\ 5)$	B1: $y=14$ **or** $y=5$ B1: $y=14$ **and** $y=5$	B1 B1
	These must be seen or used in (c)		
	$(AB=)\sqrt{(-2-1)^2+(14-5)^2}\left(=\sqrt{90}\right)$	**Correct** use of Pythagoras including the square root. Must be a correct expression for their A and B if a correct formula is not quoted	M1
	E.g. $AB=\sqrt{(-2+1)^2+(14-5)^2}$ **scores M0.** **However** $AB=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-2+1)^2+(14-5)^2}$ **scores M1**		
	$(AB=)3\sqrt{10}$	cao	A1
			(4)
			(10 marks)
Special case: Use of $4x^3-9x$ for the curve gives (-2, -14) and (1, -5) in part (c). Allow this to score a maximum of B0B0M1A1 as a special case in part (c) as the length AB comes from equivalent work.			

8(a)			
	$f(x) = x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} + 2x(+c)$	M1: $x^n \to x^{n+1}$ A1: Two terms in x correct, simplification is not required in coefficients or powers A1: All terms in x correct. Simplification not required in coefficients or powers and + c is not required	M1A1A1
	Sub $x = 4, y = 9$ into $f(x) \Rightarrow c = ...$	M1: Sub $x = 4$, $y = 9$ into $f(x)$ to obtain a value for c. If no $+c$ then M0. Use of $x = 9$, $y = 4$ is M0.	M1
	$(f(x) =)x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} + 2x + 2$	Accept equivalents **but must be simplified** e.g. $f(x) = x^{\frac{3}{2}} - 4.5\sqrt{x} + 2x + 2$ Must be all 'on one line' **and simplified**. Allow $x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			(5)
(b)	Gradient of normal is $-\frac{1}{2} \Rightarrow$ Gradient of tangent = +2	M1: Gradient of $2y + x = 0$ is $\pm\frac{1}{2}(m) \Rightarrow \frac{dy}{dx} = -\frac{1}{\pm\frac{1}{2}}$ A1: Gradient of tangent = +2 (May be implied)	M1A1
	The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2} = -\frac{1}{2}$		
	$\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2 = 2 \Rightarrow \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} = 0$	Sets the given $f'(x)$ **or their** $f'(x)$ = their **changed** m and **not** their m where m has come from $2y + x = 0$	M1
	$\times 4\sqrt{x} \Rightarrow 6x - 9 = 0 \Rightarrow x = ..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for x. If $f'(x) \neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for x. **Must be using the given** $f'(x)$ **for this mark.**	M1
	$x = 1.5$	$x = \frac{3}{2}$ (1.5) Accept equivalents e.g . $x = \frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1
			(5)
	Beware $\frac{-1}{\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2} = -\frac{1}{2} \Rightarrow \frac{-2}{3\sqrt{x}} + \frac{4\sqrt{x}}{9} - \frac{1}{2} = -\frac{1}{2}$ etc. leads to the correct answer and could score M1A1M1M0(incorrect processing)A0		
			(10 marks)

9	$\left(2-\frac{x}{4}\right)^{10}$	
	$2^{10}+\binom{10}{1}2^9\left(-\frac{1}{4}x\right)+\binom{10}{2}2^8\left(-\frac{1}{4}x\right)^2+\ldots$ For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
	First term of 1024	B1
	$=1024-1280x+720x^2$ **Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
	Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1 [4]
	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-10\times\frac{x}{8}+\frac{10\times 9}{2}\left(-\frac{x}{8}\right)^2\right)$	M1
	$1024(1\pm\ldots\ldots)$	
	$=1024-1280x+720x^2$	B1A1 A1 [4]

	Way 1	Way 2	
10	$(x \mp 2)^2+(y\pm 1)^2=k$, $k>0$	$x^2+y^2\mp 4x\pm 2y+c=0$	M1
	Attempts to use $r^2=(4-2)^2+(-5+1)^2$	$4^2+(-5)^2-4\times 4+2\times -5+c=0$	M1
	Obtains $(x-2)^2+(y+1)^2=20$	$x^2+y^2-4x+2y-15=0$	A1 (3)
	N.B. Special case: $(x-2)^2-(y+1)^2=20$ is not a circle equation but earns M0M1A0		
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)		B1
	Tangent gradient = $-\frac{1}{\text{their numerical gradient of radius}}$		M1
	Equation of tangent is $(y+5)='\tfrac{1}{2}'(x-4)$		M1
	So equation is $x-2y-14=0$ (or $2y-x+14=0$ or other integer multiples of this answer)		A1 (4)
b)Way 2	Quotes $xx'+yy'-2(x+x')+(y+y')-15=0$ and substitutes (4, -5)_		B1
	$4x-5y-2(x+4)+(y-5)-15=0$ so $2x-4y-28=0$ (or alternatives as in Way 1)		M1,M1A1 (4)
b)Way 3	Use differentiation to find expression for gradient of circle		
	Either $2(x-2)+2(y+1)\frac{dy}{dx}=0$ **or** states $y=-1-\sqrt{20-(x-2)^2}$ so $\frac{dy}{dx}=\frac{(x-2)}{\sqrt{20-(x-2)^2}}$		B1
	Substitute $x=4$, $y=-5$ after valid differentiation to give gradient =		M1
	Then as Way 1 above $(y+5)='\tfrac{1}{2}'(x-4)$ so $x-2y-14=0$		M1 A1 (4)
			[7]

Question	Scheme	Marks
11	$8^{2x+1}=24$	
	$(2x+1)\log 8 = \log 24$ or $(2x+1)=\log_8 24$ or $8^{2x}=3$ and so $(2x)\log 8 = \log 3$ or $(2x)=\log_8 3$	M1
	$x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right)$ or $x=\frac{1}{2}(\log_8 24-1)$ $x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x=\frac{1}{2}(\log_8 3)$ o.e.	dM1
	$=0.264$	A1
		(3)
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$	
	$\log_2(11y-3)-\log_2 3-\log_2 y^2=1$	M1
	$\log_2\frac{(11y-3)}{3y^2}=1$ or $\log_2\frac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$	dM1
	$\log_2\frac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\frac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
	Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example	A1
	Solves quadratic to give $y=$	ddM1
	$y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1
		(6)
		[9]

Question	Scheme	Marks
12 (a)	$4(1-\cos^2 x)+\cos x=4-k$ Applies $\sin^2 x=1-\cos^2 x$	M1
	Attempts to solve $4\cos^2 x-\cos x-k=0$, to give $\cos x=$	dM1
	$\cos x=\frac{1\pm\sqrt{1+16k}}{8}$ or $\cos x=\frac{1}{8}\pm\sqrt{\frac{1}{64}+\frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x=\frac{1\pm\sqrt{49}}{8}=1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	$x=0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1
		(3)

13	(a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
		Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
		$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
		$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
	(b)	$\left\{\dfrac{dC}{dr}=\right\}12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
		$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
		Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow $r=3$, and thus $C=$	ddM1
		Then C = awrt 483 or 484	A1cao (5)
	(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

14 $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ **(1)** States the formula for differentiation from first principles.

$f'(x)=\lim_{h\to 0}\dfrac{8(x+h)^3+5-(8x^3+5)}{h}$

$f'(x)=\lim_{h\to 0}\dfrac{8(x^3+3x^2h+3xh^2+h^3)+5-8x^3-5)}{h}$ **(1)** Correctly applies the formula to the specific function and expands and simplifies.

$f'(x)=\lim_{h\to 0}\dfrac{(24x^2h+24xh^2+8h^3)}{h}$

$f'(x)=\lim_{h\to 0}\dfrac{h(24x^2+24xh+8h^2)}{h}$ **(1)** Factorises the ‘h’ out of the numerator and divides to simplify.

$f'(x)=\lim_{h\to 0}24x^2+24xh+8h^2$

As $h\to 0, f'(x)\to 24x^2$ **(1)**