Paper collated from year	**2014**
Content	**Pure Chapters 1-13**
Marks	**100**
Time	**2 hours**

Q1.

Factorise fully $25x - 9x^3$

(3)

Q2.

Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4)

Q3.

(a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

(1)

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2.

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(4)

Q4.

Find the set of values of x for which

(a) $3x - 7 > 3 - x$

(2)

(b) $x^2 - 9x \leq 36$

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leq 36$

(1)

Q5.

$$f(x) = 2x^3 - 7x^2 + 4x + 4$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

Q6.

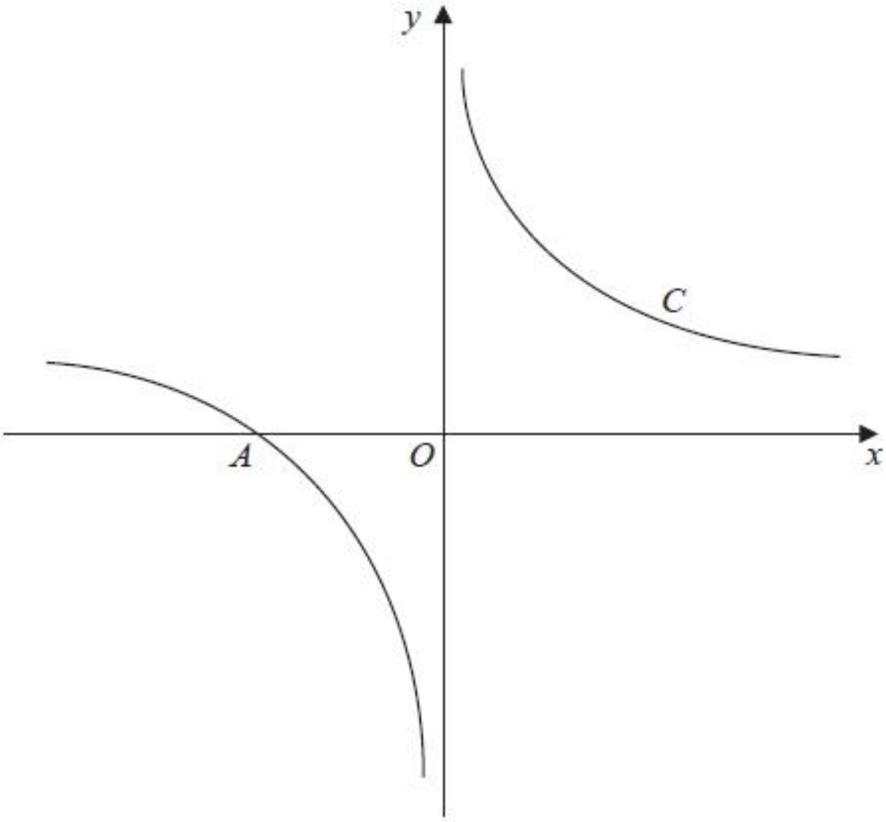


Figure 1

Figure 1 shows a sketch of the curve *C* with equation

$$y = \frac{1}{x} + 1, \qquad x \neq 0$$

The curve *C* crosses the *x*-axis at the point *A*.

(a) State the *x* coordinate of the point *A*.

(1)

The curve *D* has equation $y = x^2(x - 2)$, for all real values of *x*.

(b) Add a sketch a graph of curve *D to Figure 1*.
Show on the sketch the coordinates of each point where the curve *D* crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1.$$

(1)

Q7.

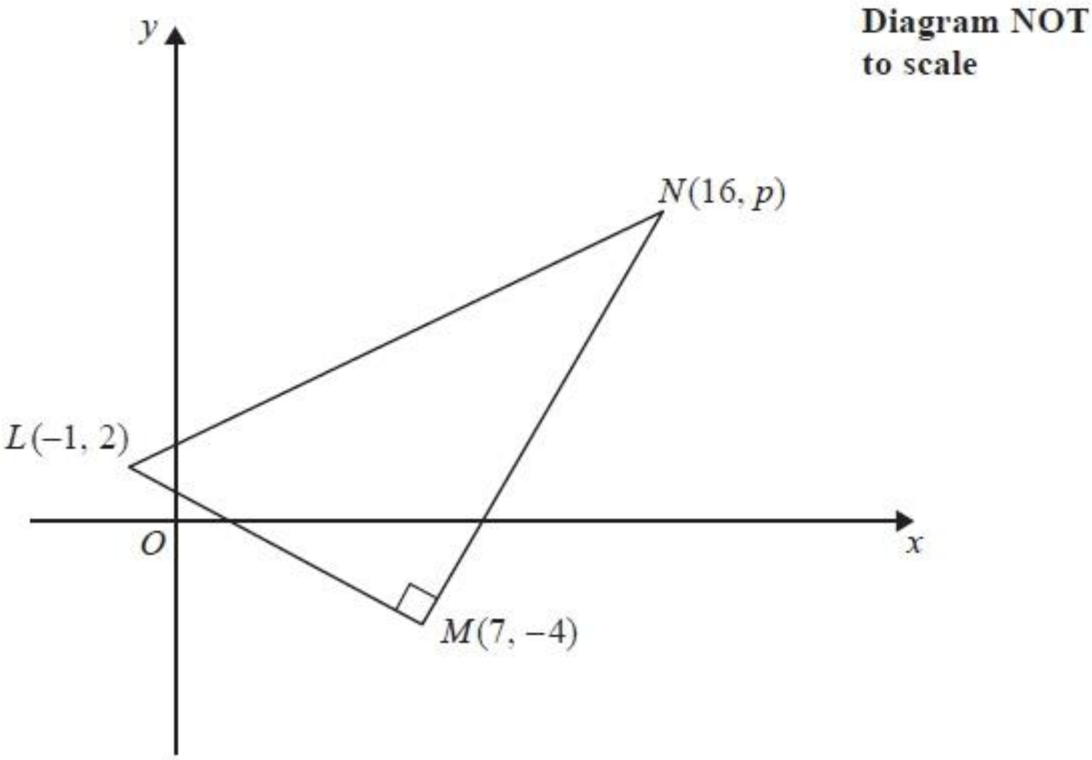


Figure 2

Figure 2 shows a right angled triangle *LMN*.

The points *L* and *M* have coordinates (−1, 2) and (7, −4) respectively.

(a) Find an equation for the straight line passing through the points *L* and *M*.

Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

Given that the coordinates of point *N* are (16, *p*), where *p* is a constant, and angle $LMN = 90°$,

(b) find the value of *p*.

(3)

Given that there is a point *K* such that the points *L, M, N*, and *K* form a rectangle,

(c) find the *y* coordinate of *K*.

(2)

Q8.

The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C,

(a) find the coordinates of A,

(2)

(b) find an equation for C.

(3)

A point R also lies on the circle C.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(d) Find the size of the angle ARQ, giving your answer to the nearest 0.1 of a degree.

(2)

Q9.

Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1 - 2x)^2$

(3)

(b) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$

(4)

Q10.

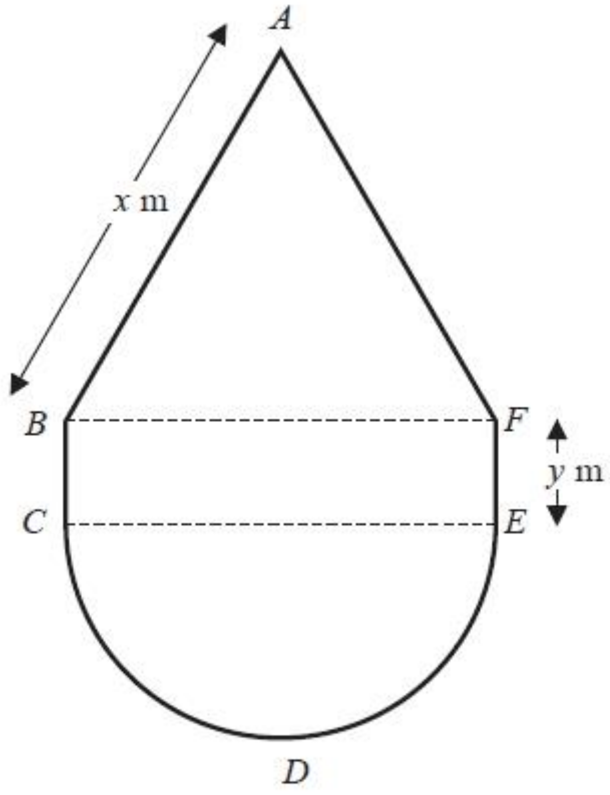


Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool *ABCDEFA* consists of a rectangle *BCEF* joined to an equilateral triangle *BFA* and a semi-circle *CDE*, as shown in Figure 4.

Given that *AB* = *x* metres, *EF* = *y* metres, and the area of the pool is 50 m^2,

(a) show that

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3})$$

(3)

(b) Hence show that the perimeter, *P* metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

(3)

(c) Use calculus to find the minimum value of *P*, giving your answer to 3 significant figures.

(5)

(d) Justify, by further differentiation, that the value of *P* that you have found is a minimum.

(2)

Q11.

Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form *a* + *b*√3, where *a* and *b* are constants to be determined.

(5)

Q12.

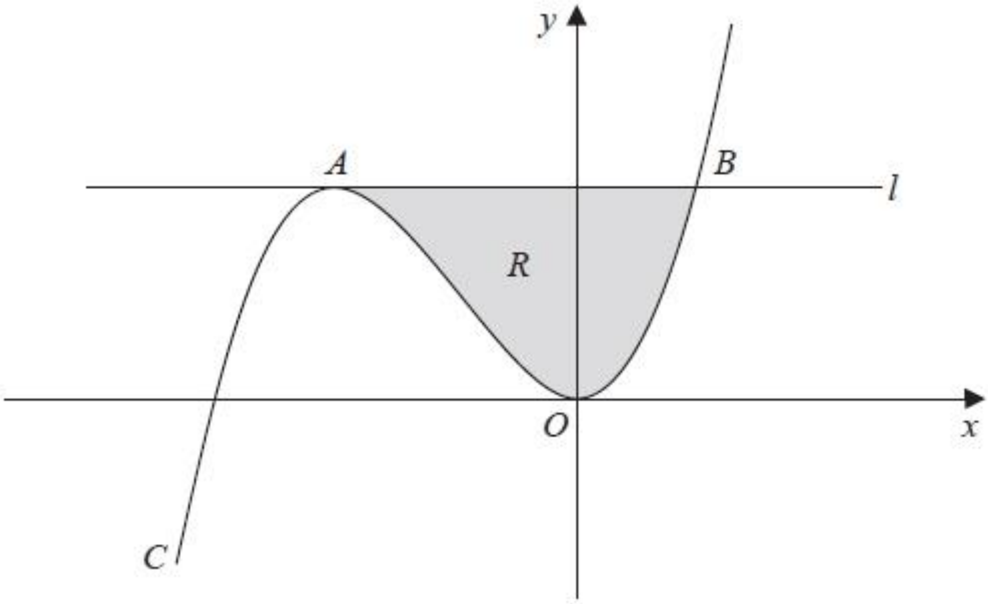


Figure 3

Figure 3 shows a sketch of part of the curve *C* with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve *C* has a maximum turning point at the point *A* and a minimum turning point at the origin *O*.

The line *l* touches the curve *C* at the point *A* and cuts the curve *C* at the point *B*.

The *x* coordinate of *A* is −4 and the *x* coordinate of *B* is 2.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve *C* and the line *l*.

Use integration to find the area of the finite region *R*.

(7)

Q13.

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

Q14.

A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geq 0, t \in \mathbb{R}.$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

Q15.

Find the exact solution, in its simplest form, to the equation

$2\ln(2x + 1) - 10 = 0$

(2)

Q16.

Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

Q17

Calculate the derivative of $g(x)=2x-3$ from first principles.

(4)

Mark scheme

Q1.

Question Number	Scheme	Marks
	$25x-9x^3 = x(25-9x^2)$	B1
	$(25-9x^2)=(5+3x)(5-3x)$	M1
	$25x-9x^3 = x(5+3x)(5-3x)$	A1
		(3)

Q2.

Question Number	Scheme	Marks
Method 1	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$	
	$\times\sqrt{2} \Rightarrow x\sqrt{16}+10\sqrt{2}=6x$	M1,A1
	$4x+10\sqrt{2}=6x \Rightarrow 2x=10\sqrt{2}$ $x=5\sqrt{2}$ or $a = 5$ and $b = 2$	M1A1
		(4)
Method 2	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$	
	$2\sqrt{2}x+10=3\sqrt{2}x$	M1A1
	$\sqrt{2}x=10 \Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$ oe	M1,A1
		(4)

Q3.

Question Number	Scheme	Marks
	(a) $80 = 5\times16$ $\sqrt{80} = 4\sqrt{5}$	B1 (1)
	Method 1 (b) $\dfrac{\sqrt{80}}{\sqrt{5}+1}$ or $\dfrac{c\sqrt{5}}{\sqrt{5}+1}$ $=\dfrac{\sqrt{80}}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}$ or $\dfrac{\sqrt{80}}{1+\sqrt{5}}\times\dfrac{1-\sqrt{5}}{1-\sqrt{5}}$ $=\dfrac{20-4\sqrt{5}}{4}$ or $\dfrac{4\sqrt{5}-20}{-4}$ $=5-\sqrt{5}$ **Method 2** $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ $p\sqrt{5}+q\sqrt{5}+p+5q = 4\sqrt{5}$ $p+5q=0$ $p+q=4$ $p=5,\ q=-1$	B1ft M1 A1 A1cao (4) **(5 marks)**

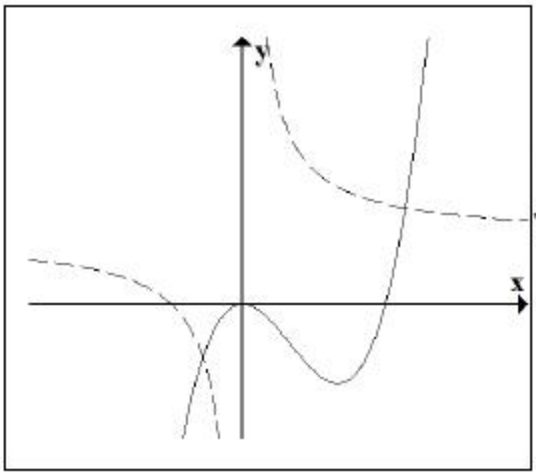
Q4.

Question Number	Scheme	Marks
	(a) $3x-7>3-x$ $4x>10$ $x>2.5,\quad x>\dfrac{5}{2},\quad \dfrac{5}{2}<x$ o.e.	M1 A1 (2)
	(b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$ e.g. $(x-12)(x+3)=0$ so $x=\ ,$ or $x=\dfrac{9\pm\sqrt{81+144}}{2}$ $12,\ -3$ $-3\le x\le12$	M1 A1 M1A1 (4)
	(c) $2.5<x\le12$	A1cso (1) **(7 marks)**

Q5.

Question Number	Scheme		Marks
	If there is no labelling, mark (a) and (b) in that order		
(a)	$f(x) = 2x^3 - 7x^2 + 4x + 4$		
	$f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$	Attempts $f(2)$ or $f(-2)$	M1
	$= 0$, and so $(x-2)$ is a factor.	$f(2) = 0$ with no sign or substitution errors ($2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient) **and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor**. Note also that a conclusion can be implied from a preamble, eg: "If $f(2) = 0$, $(x-2)$ is a factor...."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
(b)	$f(x) = \{(x-2)\}(2x^2 - 3x - 2)$	M1: Attempts long division by $(x-2)$ or other method using $(x-2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." A1: $(2x^2 - 3x - 2)$	M1 A1
	$= (x-2)(x-2)(2x+1)$ or $(x-2)^2(2x+1)$ or equivalent e.g. $= 2(x-2)(x-2)(x+\frac{1}{2})$ or $2(x-2)^2(x+\frac{1}{2})$	dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). **This is dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the factors. A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.)	dM1 A1
	Note $= (x-2)(\frac{1}{2}x - 1)(4x+2)$ would lose the last mark as it is not **fully** factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6

Q6.

Question Number	Scheme	Marks
	(a) - 1 accept $(-1, 0)$	B1 (1)
	(b) 	
	Shape	B1
	Touches at (0,0)	B1
	Crosses at (2,0) **only**	B1 (3)
	(c) 2 solutions as **curves** cross twice	B1 ft (1)
		(5 marks)

Q7.

Question Number	Scheme	Marks
(a)	Method 1: $gradient = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$ Method 2: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, so $\frac{y - y_1}{6} = \frac{x - x_1}{-8}$	M1, A1
	$y - 2 = -\frac{3}{4}(x+1)$ or $y + 4 = -\frac{3}{4}(x-7)$ or $y = their' -\frac{3}{4}' x + c$	M1
	$\Rightarrow \pm(4y + 3x - 5) = 0$	A1 (4)
	Method 3: Substitute $x = -1$, $y = 2$ and $x = 7$, $y = -4$ into $ax + by + c = 0$	M1
	$-a + 2b + c = 0$ and $7a - 4b + c = 0$	A1
	Solve to obtain $a = 3$, $b = 4$ and $c = -5$ or multiple of these numbers	M1 A1 (4)
(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ or $\frac{p+4}{16-7} = \frac{4}{3}$ Or $(y+4) = \frac{4}{3}(x-7)$ equation with $x =$ 16 substituted	M1
	$p + 4 = \frac{9 \times 4}{3} \Rightarrow p = \ldots$, $p = 8$ So $y =$, $y = 8$	M1, A1 (3)
Alternative for (b)	Attempt Pythagoras: $(p+4)^2 + 9^2 + (6^2 + 8^2) = (p-2)^2 + 17^2$	M1
	So $p^2 + 8p + 16 + 81 + 36 + 64 = p^2 - 4p + 4 + 289 \Rightarrow p = \ldots$	M1
	$p = 8$	A1 (3)
(c)	Either $(y=)\ p + 6$ or $2 + p + 4$ Or use 2 perpendicular line equations through L and N and solve for y	M1
	$(y =)\ 14$	A1 (2)
		(9 marks)

Q8.

Question Number	Scheme		Marks
(a)	$A\left(\frac{-9+15}{2}, \frac{8-10}{2}\right) = A(3, -1)$	M1: A correct attempt to find the midpoint between P and Q. Can be implied by one of x or y-coordinates correctly evaluated.	M1A1
		A1: $(3, -1)$	
			[2]
(b)	$(-9-3)^2+(8+1)^2$ or $\sqrt{(-9-3)^2+(8+1)^2}$ or $(15-3)^2+(-10+1)^2$ or $\sqrt{(15-3)^2+(-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2+(-10-8)^2$ or $\sqrt{(15+9)^2+(-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but <u>must</u> be seen in (b)**		M1
	$(x-3)^2+(y+1)^2=225\ \left(\text{or } (15)^2\right)$	$(x\pm\alpha)^2+(y\pm\beta)^2=k^2$ where $A(\alpha, \beta)$ and k is their radius.	M1
	$(x-3)^2+(y+1)^2=225$	Allow $(x-3)^2+(y+1)^2=15^2$	A1
	Accept correct answer only		
			[3]
	Alternative using $x^2+2ax+y^2+2by+c=0$		
	Uses $A(\pm\alpha, \pm\beta)$ and $x^2+2ax+y^2+2by+c=0$ e.g. $x^2+2(-3)x+y^2+2(1)y+c=0$		M1
	Uses P or Q and $x^2+2ax+y^2+2by+c=0$ e.g. $(-9)^2+2(-3)(-9)+(8)^2+2(1)(8)+c=0 \Rightarrow c=-215$		M1
	$x^2-6x+y^2+2y-215=0$		A1
(c)	Distance $=\sqrt{15^2-10^2}$	$=\sqrt{(\text{their } r)^2-10^2}$ or a correct method for the distance e.g. their $r\times\cos\left[\sin^{-1}\left(\frac{10}{\text{their } r}\right)\right]$	M1
	$\left\{=\sqrt{125}\right\}=5\sqrt{5}$	$5\sqrt{5}$	A1
			[2]

Question Number	Scheme		Marks
(d)	$\sin\left(A\hat{R}Q\right) = \frac{20}{30}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{15}\right)$	$\sin\left(A\hat{R}Q\right) = \frac{20}{(2\times \text{their } r)}$ or $\frac{10}{\text{their } r}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{\text{their } r}\right)$ or $A\hat{R}Q = \cos^{-1}\left(\frac{\text{Part}(c)}{\text{their } r}\right)$ or $A\hat{R}Q = 90 - \sin^{-1}\left(\frac{\text{Part}(c)}{\text{their } r}\right)$ or $20^2 = 15^2 + 15^2 - 2\times 15\times 15\cos(2ARQ)$ or $15^2 = 15^2 + \left(10\sqrt{5}\right)^2 - 2\times 15\times 10\sqrt{5}\cos(ARQ)$ A fully correct method to find $A\hat{R}Q$, where their $r > 10$. Must be a **correct** statement involving angle ARQ	M1
	$A\hat{R}Q = 41.8103...$	awrt 41.8	A1
			[2]
			Total 9

Q9.

Question Number	Scheme	Marks
	(a) $(1-2x)^2 = 1-4x+4x^2$ $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x$ o.e.	M1 M1A1 (3)
	Alternative method using chain rule: Answer of -4 (1 − 2 x)	M1M1A1 (3)
	(b) $\frac{x^5+6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}, = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ Attempts to differentiate $x^{-\frac{3}{2}}$ to give $k\ x^{-\frac{5}{2}}$ $= \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. Quotient Rule (May rarely appear) – See note below	M1,A1 M1 A1 (4) **(7 marks)**

Q10.

Question Number	Scheme		Marks
(a)	$\{A=\}\ xy+\frac{\pi}{2}\left(\frac{x}{2}\right)^2+\frac{1}{2}x^2\sin 60^\circ$	M1: An attempt to find 3 areas of the form: xy, $p\pi x^2$ and qx^2 A1: Correct expression for A (terms must be added)	M1A1
	$50 = xy+\frac{\pi x^2}{8}+\frac{\sqrt{3}x^2}{4} \Rightarrow y=\frac{50}{x}-\frac{\pi x}{8}-\frac{\sqrt{3}x}{4} \Rightarrow y=\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)$ * Correct proof with no errors seen		A1 *
			[3]
(b)	$\{P=\}\ \frac{\pi x}{2}+2x+2y$	Correct expression for P in terms of x and y	B1
	$P=\frac{\pi x}{2}+2x+2\left(\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)\right)$	Substitutes the given expression for y into an expression for P where P is at least of the form $\alpha x+\beta y$	M1
	$P=\frac{\pi x}{2}+2x+\frac{100}{x}-\frac{\pi x}{4}-\frac{\sqrt{3}}{2}x \Rightarrow P=\frac{100}{x}+\frac{\pi x}{4}+2x-\frac{\sqrt{3}}{2}x$		
	$\Rightarrow P=\frac{100}{x}+\frac{x}{4}\left(\pi+8-2\sqrt{3}\right)$	Correct proof with no errors seen	A1 *
			[3]
	(Note $\frac{\pi+8-2\sqrt{3}}{4}=1.919\ldots\ldots$)		
(c) and (d) can be marked together	$\frac{dP}{dx}=-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}$	M1: Either $\mu x\rightarrow\mu$ or $\frac{100}{x}\rightarrow\frac{\pm\lambda}{x^2}$ A1: Correct differentiation (need not be simplified). Allow $-100x^{-2}+(\text{awrt}\,1\cdot92)$	M1A1
	$-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}=0\Rightarrow x=\ldots$	Their $P'=0$ and attempt to solve as far as $x=\ldots$. (ignore poor manipulation)	M1
	$\Rightarrow x=\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}=7.2180574\ldots$	$\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}$ or awrt 7.2 **and no other values**	A1
	$\{x=7.218\ldots,\}\ \Rightarrow P=27.708\ldots\text{ (m)}$	awrt 27.7	A1
			[5]
	$\frac{d^2P}{dx^2}=\frac{200}{x^3}>0\Rightarrow$ Minimum	M1: Finds P'' ($x^n\rightarrow x^{n-1}$ allow for constant $\rightarrow 0$) **and** considers sign A1ft: $\frac{200}{x^3}$ (need not be simplified) and >0 and conclusion. Only follow through on a correct P'' and a single positive value of x found earlier.	M1A1ft
			[2]
			Total 13

Q11.

Question Number	Scheme		Marks
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$	M1: $x^n \to x^{n+1}$ A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$. A1: $\frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ or equivalent. e.g. $\frac{\frac{x^4}{6}}{4}+\frac{\frac{x^{-1}}{3}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)	M1A1A1
	Note that some candidates may change the function prior to integrating e.g. $\int\frac{x^3}{6}+\frac{1}{3x^2}\mathrm{d}x=\int 3x^5+6\,\mathrm{d}x$ in which case allow the M1 if $x^n \to x^{n+1}$ for their changed function and allow the M1 for limits if scored		
	$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(\sqrt{3})^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$		dM1
	2nd dM1: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. **The 2nd M1 is dependent on the 1st M1 being awarded.**		
	$=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$	$\frac{2}{3}-\frac{1}{9}\sqrt{3}$ or $a=\frac{2}{3}$ and $b=-\frac{1}{9}$. Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do **not** allow $\frac{6-\sqrt{3}}{9}$	A1cso
	This final mark is cao and cso – there must have been no previous errors		
			Total 5
	Common Errors (Usually 3 out of 5)		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\int\left(\frac{x^3}{6}+3x^{-2}\right)\mathrm{d}x=\frac{x^4}{6(4)}+\frac{3x^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\left(\frac{(\sqrt{3})^4}{24}+\frac{3(\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{3(1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{3}{\sqrt{3}}\right)-\left(\frac{1}{24}+\frac{3}{-1}\right)=\frac{10}{3}-\sqrt{3}$ A0		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\int\left(\frac{x^3}{6}+(3x)^{-2}\right)\mathrm{d}x=\frac{x^4}{6(4)}+\frac{(3x)^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\}=\left(\frac{(\sqrt{3})^4}{24}+\frac{(3\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{(3\times 1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{\sqrt{3}}{9}$ A0 **Note this is the correct answer but follows incorrect work.**		

Q12.

Question Number	Scheme		Marks
	$\int\left(\frac{1}{8}x^3+\frac{3}{4}x^2\right)\mathrm{d}x = \frac{x^4}{32}+\frac{x^3}{4}\{+c\}$	M1: $x^n \to x^{n+1}$ on either term A1: $\frac{x^4}{32}+\frac{x^3}{4}$. Any correct simplified or un-simplified form. (+ c not required)	M1A1
	$\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{2} = \left(\frac{16}{32}+\frac{8}{4}\right)-\left(\frac{256}{32}+\frac{(-64)}{4}\right)$ or $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{0} = (0)-\left(\frac{(-4)^4}{32}+\frac{(-4)^3}{4}\right)$ added to $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{0}^{2} = \left(\frac{(2)^4}{32}+\frac{(2)^3}{4}\right)-(0)$		dM1
	Substitutes limits of 2 and −4 into an "integrated function" and subtracts either way round. Or substitutes limits of 0 and −4 **and** 2 and 0 into an "integrated function" and subtracts either way round and adds the two results.		
	$=\frac{21}{2}$	$\frac{21}{2}$ or 10.5	A1
	$\{\text{At } x=-4,\ y=-8+12=4 \text{ or at } x=2,\ y=1+3=4\}$		
	Area of Rectangle $=6\times4=24$ or Area of Rectangles $=4\times4=16$ and $2\times4=8$		M1
	Evidence of $(4--2)\times$ their y_{-4} **or** $(4--2)\times$ their y_2 or Evidence of $4\times$ their y_{-4} **and** $2\times$ their y_2		
	So, area(R) $=24-\frac{21}{2}=\frac{27}{2}$	dddM1: Area rectangle – integrated answer. Dependent on all previous method marks **and requires:** **Rectangle > integration > 0** A1: $\frac{27}{2}$ or 13.5	dddM1A1
			[7]
			Total 7

	Alternative:		
	$\pm\int "\text{their}4" - \left(\frac{1}{8}x^3 + \frac{3}{4}x^2\right)\,dx$	Line – curve. **Condone missing brackets and allow either way round.**	4th M1
	$= 4x - \frac{x^4}{32} - \frac{x^3}{4}\{+c\}$	M1: $x^n \to x^{n+1}$ on either curve term A1ft: $"-\frac{x^4}{32} - \frac{x^3}{4}."$ Any correct simplified or un-simplified form of their curve terms, **follow through sign errors.** (+ c not required)	1st M1, 1st A1ft
	$[\;]_{-4}^{2} = \underline{\underline{\left(8 - \frac{16}{32} - \frac{8}{4}\right) - \left(-16 - \frac{256}{32} - \frac{(-64)}{4}\right)}}$	2nd M1 Substitutes limits of 2 and -4 into an "integrated curve" and subtracts either way round. 3rd M1 for $\pm("8" - "-16")$ **Substitutes limits into the 'line part' and subtracts either way round.** 2nd A1 for correct ± (underlined expression). **Now needs to be correct but allow ± the correct expression.**	2nd M1, 3rd M1 2nd A1
	$= \frac{27}{2}$	A1: $\frac{27}{2}$ or 13.5	3rd A1
	If the final answer is -13.5 you can withhold the final A1 **If -13.5 then "becomes" +13.5 allow the A1**		

Q13.

Question Number	Scheme		Marks
	(i) $9\sin(\theta+60^\circ)=4;\ 0\le\theta<360^\circ$ (ii) $2\tan x-3\sin x=0;\ -\pi\le x<\pi$		
(i)	$\sin(\theta+60^\circ)=\frac{4}{9}$, so $(\theta+60^\circ)=26.3877...$ $(\alpha=26.3877...)$	Sight of $\sin^{-1}\left(\frac{4}{9}\right)$ or awrt 26.4° or 0.461^c Can also be implied for $\theta=$ awrt -33.6 (i.e. $26.4-60$)	M1
	So, $\theta+60^\circ=\{153.6122...,\ 386.3877...\}$	$\theta+60^\circ=$ either "$180-$ their α" or "$360^\circ+$ their α" and not for $\theta=$ either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{93.6122...,\ 326.3877...\}$	A1: At least one of awrt 93.6° or awrt 326.4° A1: Both awrt 93.6° and awrt 326.4°	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1for any extra solutions in range		
			[4]
(ii)	$2\left(\frac{\sin x}{\cos x}\right)-3\sin x=0$	Applies $\tan x=\frac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$		
	$2\sin x-3\sin x\cos x=0$		
	$\sin x(2-3\cos x)=0$		
	$\cos x=\frac{2}{3}$	$\cos x=\frac{2}{3}$	A1
	$x=$ awrt $\{0.84,\ -0.84\}$	A1: One of either awrt 0.84 or awrt -0.84 A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x=0\Rightarrow\}\ x=0$ **and** $-\pi$	**Both** $x=0$ **and** $-\pi$ or awrt -3.14 from $\sin x=0$ **In this part of the solution, ignore extra solutions in range.**	B1
	Note solutions are: $x=\{-3.1415...,\ -0.8410...,\ 0,\ 0.8410...\}$ Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Q14.

Question Number	Scheme	Marks
(a)	$P = \frac{800e^0}{1+3e^0}, = \frac{800}{1+3} = 200$	M1,A1
		(2)
(b)	$250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t}) = 800e^{0.1t} \Rightarrow 50e^{0.1t} = 250, \Rightarrow e^{0.1t} = 5$	M1,A1
	$t = \frac{1}{0.1}\ln(5)$	M1
	$t = 10\ln(5)$	A1
		(4)

Q15.

Question Number	Scheme	Marks
(a)	$2\ln(2x+1) - 10 = 0 \Rightarrow \ln(2x+1) = 5 \quad \Rightarrow 2x+1 = e^5 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{e^5 - 1}{2}$	A1
		(2)

Q16.

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Q17.

SOLUTION

Step 1: Write down the formula for finding the derivative using first principles

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$ **M1**

Step 2: Determine $g(x+h)$

$$g(x) = 2x - 3$$

$$\begin{aligned} g(x+h) &= 2(x+h) - 3 \\ &= 2x + 2h - 3 \end{aligned}$$ **M1**

Step 3: Substitute into the formula and simplify

$$\begin{aligned} g'(x) &= \lim_{h \to 0} \frac{2x + 2h - 3 - (2x - 3)}{h} \\ &= \lim_{h \to 0} \frac{2h}{h} \\ &= \lim_{h \to 0} 2 \\ &= 2 \end{aligned}$$ **M1**

A1dep

Step 4: Write the final answer

The derivative $g'(x) = 2$.