Paper collated from year	**2012**
Content	**Pure Chapters 1-13**
Marks	**100**
Time	**2 hours**

Q1.

Complete each of the following by putting the best connecting symbol ($\Leftrightarrow$, $\Leftarrow$ or $\Rightarrow$) in the box. Explain your choice, giving full reasons.

(i) $n^3 + 1$ is an odd integer ☐ n is an even integer **[2]**

(ii) $(x - 3)(x - 2) > 0$ ☐ $x > 3$ **[2]**

Q2.

Simplify

$$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$$

giving your answer in the form $b\sqrt{2}+c$, where b and c are integers.

(4)

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

(2)

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

Q4.

A rectangular garden is to have width x metres and length $(x+4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

Q5.

A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) Sketch the circle. *(2 marks)*

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$. *(2 marks)*

(ii) The equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$ *(3 marks)*

(iii) Hence find the values of k for which the line is a tangent to the circle. *(2 marks)*

Q6.

(a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leqslant x \leqslant 180°$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

Q7.

You are given that $f(x) = 2x^3 - 3x^2 - 23x + 12$.

(i) Show that $x = -3$ is a root of $f(x) = 0$ and hence factorise $f(x)$ fully. **[6]**

(ii) Sketch the curve $y = f(x)$. **[3]**

(iii) Find the x-coordinates of the points where the line $y = 4x + 12$ intersects $y = f(x)$. **[4]**

Q8.

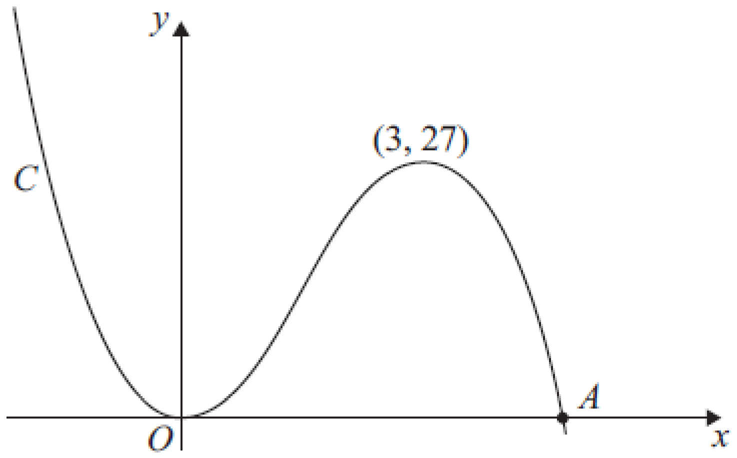


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x + 3)$

(ii) $y = f(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

Q9.

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

Q10.

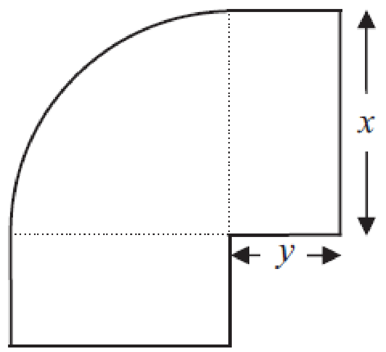


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

Q11.

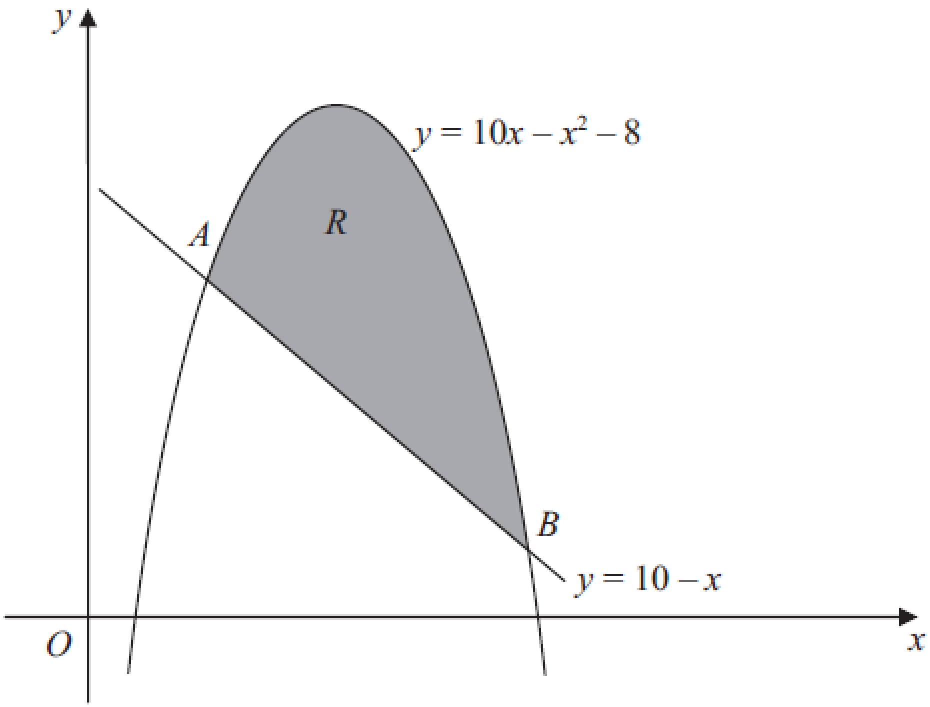


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

Q12.

The vectors **p** and **q** are given by

$$\mathbf{p} = 8\mathbf{i} + \mathbf{j} \quad \text{and} \quad \mathbf{q} = 4\mathbf{i} - 7\mathbf{j}.$$

(i) Show that **p** and **q** are equal in magnitude. **[3]**

(ii) Show that $\mathbf{p} + \mathbf{q}$ is parallel to $2\mathbf{i} - \mathbf{j}$. **[2]**

(iii) Draw $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q}$ on the grid.

Write down the angle between these two vectors. **[3]**

Q13.

Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

(3)

Mark scheme

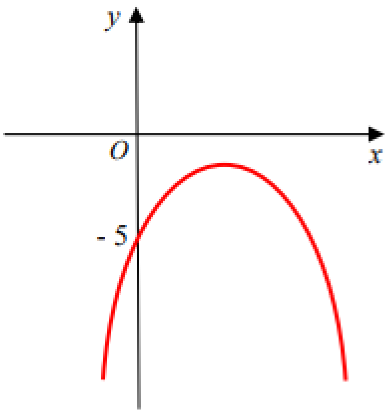
Q1. Problem Solving

(i)		'if n even then n^3 even, so $n^3 + 1$ odd' oe $\Leftarrow$ with if $n^3 + 1$ odd then n^3 even but if n^3 is even, n is not necessarily an integer or $\Leftrightarrow$ with '$n^3 + 1$ odd then n^3 even so n even', [assuming n is an integer]	B1 B1 **[2]**	must mention n^3 is even or even^3 is even or even × even = even or '$\Leftrightarrow$ with if n is odd, n^3 is odd, so $n^3 + 1$ is even' if 0 in question, allow SC1 for $\Leftrightarrow$ or $\Leftarrow$ and attempt at using general odd/even in explanation	0 for just 'if n is even, $n^3 + 1$ is odd' 0 if just examples of numbers used condone $\leftrightarrow$ instead of $\Leftrightarrow$ etc in both parts must go further than restating the info in the qn; please annotate as SC
(ii)		showing $\Leftarrow$ is true $\Leftarrow$ chosen and showing that $\Rightarrow$ [and therefore $\Leftrightarrow$] is/ are not true	B1 B1 **[2]**	eg when $x > 3$, +ve × +ve > 0 stating that true when $x < 2$ or giving a counterexample such as 1, 0 or a negative number [to show quadratic inequality also true for this number] allow B2 for $\Leftarrow$ and $x > 3$ and $x < 2$ shown/stated as soln or sketch showing two solns of $x^2 - 5x + 6 > 0$	0 for just example(s) or for simply stating it is true 0 for saying another solution $x > 2$ or B1 for this argument with another symbol

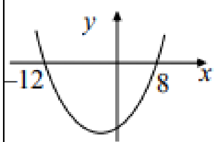
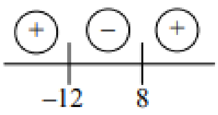
Q2. Surds Indices

$\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}$ or $\times\dfrac{-3+\sqrt{2}}{-3+\sqrt{2}}$ seen	M1
$\left[\dfrac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}=\right]\ \dfrac{a\sqrt{2}(3-\sqrt{2})}{[9-2]} \rightarrow \dfrac{3a\sqrt{2}-2a}{[9-2]}$ (or better)	dM1
$= \underline{3\sqrt{2}, -2}$	A1, A1 (4)

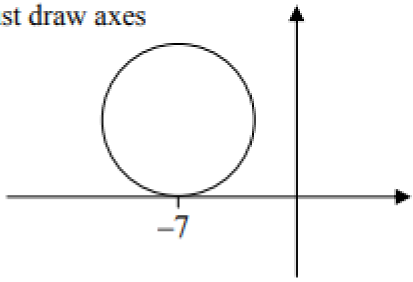
Q3. Quadratic functions

$4x-5-x^2 = q-(x-p)^2$, p, q are integers.	
$\{4x-5-x^2 =\}\ -\left[x^2-4x+5\right] = -\left[(x-2)^2-4+5\right] = -\left[(x-2)^2+1\right]$	M1
$= -1-(x-2)^2$	A1 A1
	[3]
$\{"b^2-4ac"=\}\ 4^2-4(-1)(-5) \quad \{=16-20\}$	M1
$= -4$	A1
	[2]
	
Correct ∩ shape	M1
Maximum **within** the 4th quadrant	A1
Curve cuts through -5 or $(0,-5)$ marked on the y-axis	B1
	[3]
	8

Q4. Equations and Inequalities

(a)	Sides are x and $x+4$	
	$\Rightarrow x+x+x+4+x+4>30$ or $2x+2x+8>30$ or $2(2x+4)>30$ or $4x+8>30$	
	$(\Rightarrow 4x>22)$	
	$\Rightarrow 2x>11$	B1
(b)	$x(x+4)<96$	
	$\Rightarrow x^2+4x-96<0$	B1
(c)	$(x+12)(x-8)$	M1
	Critical values $8,\ -12$	A1
	 or 	M1
	$\Rightarrow -12<x<8$	A1cso
(d)	$5\frac{1}{2}<x<8$	B1

Q5. Coordinate Geometry

(a)	$(x+7)^2+(y-5)^2$	M1 A1
	$(x+7)^2+(y-5)^2 \quad =5^2$	A1cao
)(i)	$C(-7,5)$	B1√
(ii)	$r=5$	B1√
(c)	must draw axes 	M1 A1
)(i)	$x^2+(kx+6)^2+14x-10(kx+6)+49=0$	
	$x^2+k^2x^2+12kx+36+14x$ $-10kx-60+49=0$ $(1+k^2)x^2+2kx+14x+25=0$	M1
	$\Rightarrow (k^2+1)x^2+2(k+7)x+25=0$	A1cso
(ii)	Equal roots '$b^2-4ac=0$'	B1
	$\left[2(k+7)\right]^2-4\times 25(k^2+1)$	M1
	$4\{k^2+14k+49-25k^2-25\}=0$ $-24k^2+14k+24=0$	
	$\Rightarrow 12k^2-7k-12=0$	A1
iii)	$(4k+3)(3k-4)$	M1
	$\Rightarrow k=-\frac{3}{4},\ k=\frac{4}{3}$ OE are values of k for which line is a tangent	A1

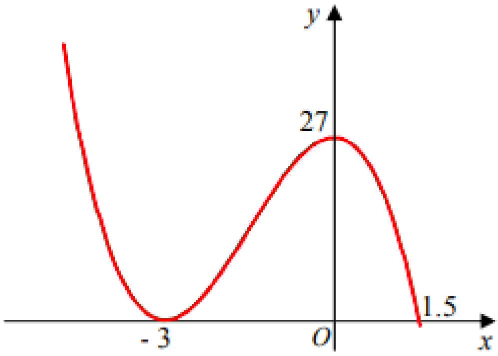
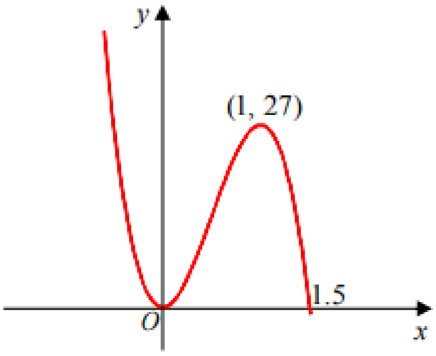
Q6. Coordinate Geometry

States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$		M1
$\frac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1-5\cos 2x) = 0$ *		A1 (2)
$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
$\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) **or** $2x = 281.54$ (or 281.6)		M1
$x = 39.2$ (or 39.3), 140.8 (or 141)		A1, A1 (5)
		7 marks

Q7. Polynomials

(i)		f(−3) used	M1
		−54 − 27 + 69 + 12 [= 0] isw	A1
		attempt at division by $(x + 3)$ as far as $2x^3 + 6x^2$ in working	M1
		correctly obtaining $2x^2 - 9x + 4$	A1
		factorising the correct quadratic factor	M1
		$(2x - 1)(x - 4)[(x + 3)]$ isw	A1
(ii)		sketch of cubic right way up, with two turning points	B1
		values of intns on x axis shown, correct (−3, 0.5 and 4) or ft from their factors or roots in (i)	B1
		12 marked on y-axis	B1
			[3]
(iii)		$2x^3 - 3x^2 - 23x + 12 = 4x + 12$ oe	M1
		$2x^3 - 3x^2 - 27x$ [= 0]	A1
		$[x](2x - 9)(x + 3)$ [= 0]	M1
		$[x =]$ 0, −3 and 9/2 oe	A1

Q8. Graphs &Transformations

(a)	{Coordinates of A are} $(4.5, 0)$ See notes below	B1	[1]
)(i)			
	Horizontal translation	M1	
	-3 and their ft 1.5 on postitive x-axis	A1 ft	
	Maximum at 27 marked on the y-axis	B1	
			[3]
i)			
	Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant.	M1	
	1.5 on x-axis	A1 ft	
	Maximum at $(1, 27)$	B1	
			[3]
:)	$\{k =\}\ -17$	B1	[1]
			8

Q9. The binomial expansion

(a).	$(1+\frac{x}{4})^8 = 1 + 2x + ..,\quad ......$	B1
	$+\frac{8\times 7}{2}(\frac{x}{4})^2 + \frac{8\times 7\times 6}{2\times 3}(\frac{x}{4})^3,$	M1 A1
	$= \quad +\frac{7}{4}x^2 + \frac{7}{8}x^3$ or $= \quad +1.75x^2 + 0.875x^3$	A1
b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. 1 + 0.2 + 0.0175 + 0.000875, = 1.2184	A1 cao (3)

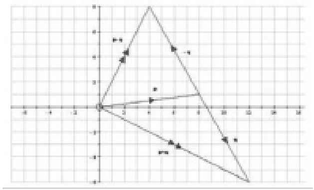
Q10. Differentiation

(a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso (3)
b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$	M1
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1 (3)
c)	$\left(\dfrac{dP}{dx} = \right) - \dfrac{8}{x^2} + 2$	M1 A1
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1
	and so $x = 2$ o.e. (ignore extra answer $x = -2$)	A1
	$P = 4 + 4 = 8$ (m)	B1 (5)
d)	$y = \dfrac{4 - \pi}{4}$, (and so width) $= 21$ (cm)	M1, A1 (2) **13**

Q11. Integration

Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic	Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic	M1
Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$	Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$	M1
Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)	Obtains $y = 8, y = 1$ (may be on diagram)	A1
Substitutes their x into a given equation to give $y =$ (may be on diagram)	Substitutes their y into a given equation to give $x =$ (may be on diagram or in part (b))	M1
$y = 8, y = 1$	$x = 2, x = 9$	A1 (5)
$\int (10x - x^2 - 8)\,dx = \dfrac{10x^2}{2} - \dfrac{x^3}{3} - 8x\ \{+c\}$		M1 A1 A1
$\left[\dfrac{10x^2}{2} - \dfrac{x^3}{3} - 8x\right]_2^9 = (......) - (......)$		dM1
$= 90 - \dfrac{4}{3} = 88\frac{2}{3}$ or $\frac{266}{3}$		
Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$		B1
So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
		12 marks

Q12. Vectors

(i)		$\lvert\mathbf{p}\rvert = \sqrt{8^2 + 1^2}$	M1	For applying Pythagoras theorem
		$\lvert\mathbf{p}\rvert = \sqrt{65}$	A1	
		$\lvert\mathbf{q}\rvert = \sqrt{4^2 + (-7)^2} = \sqrt{65}$ They are equal	A1 **[3]**	Condone no explicit statement that they are equal
(ii)		$\mathbf{p} + \mathbf{q} = 12\mathbf{i} - 6\mathbf{j}$	M1	
		$\mathbf{p} + \mathbf{q} = 6(2\mathbf{i} - \mathbf{j})$ so $\mathbf{p} + \mathbf{q}$ is parallel to $2\mathbf{i} - \mathbf{j}$	E1 **[2]**	Accept argument based on gradients being equal. "Parallel" may be implied
(iii)			B1 B1	One mark for each of $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q}$ drawn correctly SC1 if arrows missing or incorrect from otherwise correct vectors
		The angle is 90°	B1 **[3]**	Cao

Q13. Logs and Exponentials

(a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$	B1
	$\log_3 x^2 = 2\log_3 x$	B1
	Using $\log_3 3 = 1$	B1 (3)
b)	$3x^2 = 28x - 9$	M1
	Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 A1 (3) **6**