Paper collated from year	**2011**
Content	**Pure Chapters 1-13**
Marks	**100**
Time	**2 hours**

1. (a) Find the value of $16^{-\frac{1}{4}}$

(2)

(b) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$

(2)

2. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\, dx$$

giving each term in its simplest form.

(5)

3. Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

4.

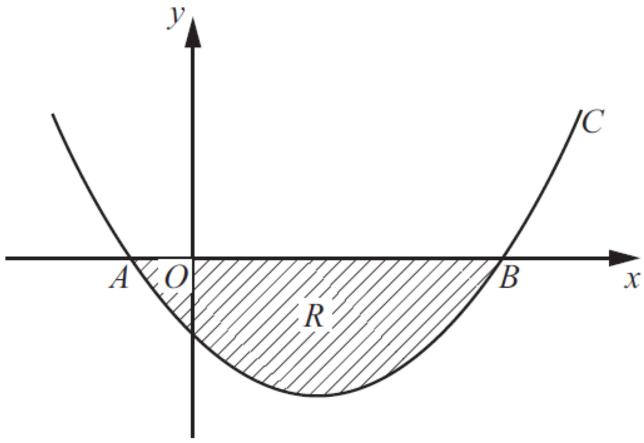


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

5.

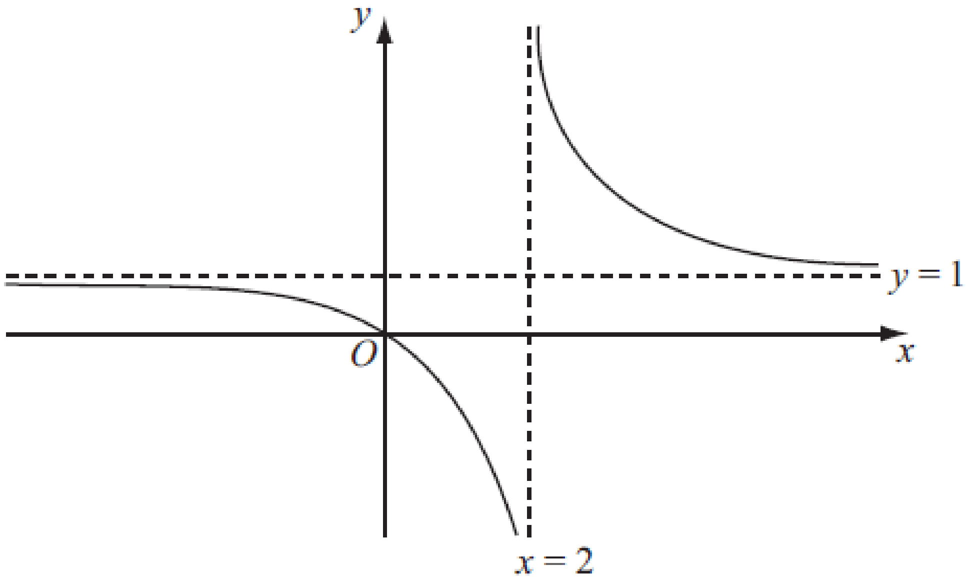


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)

6.

(a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)

7. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$.

Given that

$$\mathrm{f}'(x) = 12x^2 - 8x + 1$$

find $\mathrm{f}(x)$.

(5)

8. The equation $x^2 + (k-3)x + (3-2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

9. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A $(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

10. (a) On the axes below, sketch the graphs of

(i) $y = x(x+2)(3-x)$

(ii) $y = -\frac{2}{x}$

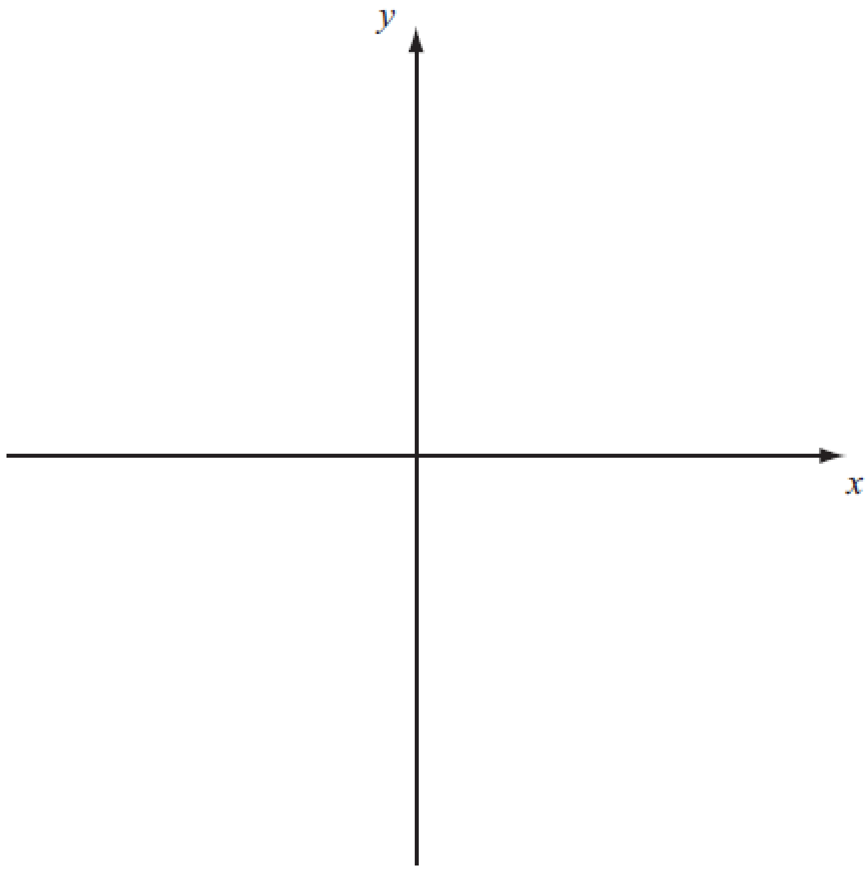
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)



11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(b) Show that the point $P(4, -8)$ lies on C.

(2)

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

12.

The points A and B have coordinates (–2, 11) and (8, 1) respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates (3, 6),

(1)

(b) find an equation for C.

(4)

(c) Verify that the point (10, 7) lies on C.

(1)

(d) Find an equation of the tangent to C at the point (10, 7), giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

13.

The volume V cm³ of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \qquad 0 < x < 5$$

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

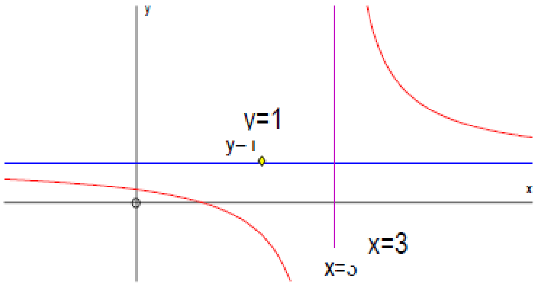
Mark scheme

1. **(a)**	$16^{\frac{1}{4}} = 2$ or $\frac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}} =\right)\ \frac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}}$ or $\frac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4$ or 16	A1 cao
		(2) **4**

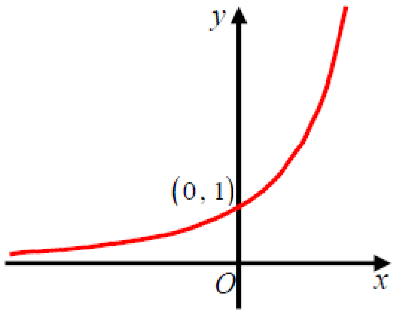
2.	$\left(\int =\right)\frac{12x^6}{6}, -\frac{3x^3}{3}, +\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}, (+c)$	M1A1,A1,A1
	$= \underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	A1
		5

3.	$\frac{5-2\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	M1
	$= \frac{\cdots}{2}$ denominator of 2	A1
	Numerator $= 5\sqrt{3} + 5 - 2\sqrt{3}\sqrt{3} - 2\sqrt{3}$	M1
	So $\frac{5-2\sqrt{3}}{\sqrt{3}-1} = -\frac{1}{2} + \frac{3}{2}\sqrt{3}$	A1
		4

Question	Scheme	Notes	Marks
4.			
(a)	Seeing -1 and 5. (See note below.)		B1
			(1)
(b)	$(x+1)(x-5) = \underline{x^2 - 4x - 5}$ or $\underline{x^2 - 5x + x - 5}$		<u>B1</u>
	$\int (x^2 - 4x - 5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (......) - (......)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) = \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36 \right\}$		
	Hence, Area $= 36$	Final answer must be 36, not -36	A1
			(6)
			[7]

Question	Scheme	Notes	Marks
5.			
(a)		Correct shape with a single crossing of each axis	B1
		$y = 1$ labelled or stated	B1
		$x = 3$ labelled or stated	B1
			(3)
(b)	Horizontal translation so crosses the x-axis at $(1, 0)$		B1
	New equation is $(y =)\dfrac{x \pm 1}{(x \pm 1) - 2}$		M1
	When $x = 0$ $y =$		M1
	$= \frac{1}{3}$		A1
			(4)
			7

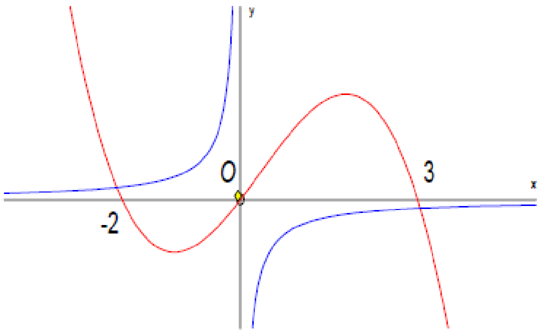
6.

(a)	Graph of $y = 7^x,\ x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$ 	At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
			(2)
(b)	$y^2 - 4y + 3 \{= 0\}$	Forming a quadratic {using $"y" = 7^x$}.	M1
		$y^2 - 4y + 3 \{= 0\}$	A1
	$\{(y-3)(y-1) = 0$ or $(7^x - 3)(7^x - 1) = 0\}$		
	$y = 3,\ y = 1$ or $7^x = 3,\ 7^x = 1$	Both $y = 3$ and $y = 1$.	A1
	$\{7^x = 3 \Rightarrow\}\ x\log 7 = \log 3$ or $x = \dfrac{\log 3}{\log 7}$ or $x = \log_7 3$	A valid method for solving $7^x = k$ where $k > 0,\ k \neq 1$	dM1
	$x = 0.5645....$	0.565 or awrt 0.56	A1
	$x = 0$	$x = 0$ stated as a solution.	B1
			(6) **[8]**

7.	$(\mathrm{f}(x) =)\dfrac{12x^3}{3} - \dfrac{8x^2}{2} + x(+c)$	M1 A1 A1
	$(\mathrm{f}(-1) = 0 \Rightarrow)\ \ 0 = 4\times(-1) - 4\times 1 - 1 + c$	M1
	$c = \underline{9}$	A1
	$\left[\mathrm{f}(x) = 4x^3 - 4x^2 + x + 9\right]$	
		5

8.			
(a)	$b^2-4ac=(k-3)^2-4(3-2k)$		M1
	$k^2-6k+9-4(3-2k)>0$ or $(k-3)^2-12+8k>0$ or better		M1
	$\underline{k^2+2k-3>0}$ *		A1cso
			(3)
(b)	$(k+3)(k-1)[=0]$		M1
	Critical values are	$k=1$ or -3	A1
	(choosing "outside" region)		M1
		$\underline{k>1 \text{ or } k<-3}$	A1 cao
			(4)
			7

9.		
(a)	$(8-3-k=0)$ so $\underline{k=5}$	B1
		(1)
(b)	$2y=3x+k$	M1
	$y=\frac{3}{2}x+...$ and so $m=\frac{3}{2}$ o.e.	A1
		(2)
(c)	Perpendicular gradient $=-\frac{2}{3}$	B1ft
	Equation of line is: $y-4=-\frac{2}{3}(x-1)$	M1A1ft
	$\underline{3y+2x-14=0}$ o.e.	A1
		(4)
(d)	$y=0,\ \Rightarrow\ B(7,0)$ or $\underline{x=7}$ $\quad x=7$ or $-\frac{c}{a}$	M1A1ft
		(2)
(e)	$AB^2=(7-1)^2+(4-0)^2$	M1
	$AB=\sqrt{52}$ or $2\sqrt{13}$	A1
		(2)
		11

10. (a)	 (i) correct shape (-ve cubic) Crossing at (-2, 0) Through the origin Crossing at (3,0) (ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch	B1 B1 B1 B1 B1 B1 (6)
(b)	"2" solutions Since only "2" intersections	B1ft dB1ft (2) **8**

11. (a)	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \Rightarrow y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$ $= 32 - 72 + 2 + 30 \qquad = \underline{-8}$ *	M1 A1cso (2)
(c)	$x=4 \Rightarrow y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$ $= 24-27-\frac{1}{2}= \qquad -\frac{7}{2}$ Gradient of the normal $= -1\div"\frac{7}{2}"$ Equation of normal: $y--8=\frac{2}{7}(x-4)$ $\underline{7y-2x+64=0}$	M1 A1 M1 M1A1ft A1 (6) **12**

12.

(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50 \left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive value.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$,} $\underline{(10-3)^2 + (7-6)^2 = 50}$, {so the point lies on C.}		B1
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x - 10)$	$y - 7 = (\text{their gradient})(x - 10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			[10]

13

(a)	$V = 4x(5-x)^2 = 4x(25 - 10x + x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}$ $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3}$, $V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3}$, $\frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$.	A1 cso
			(2)
			[10]