Paper collated from year	**2010**
Content	**Pure Chapters 1-13**
Marks	**103**
Time	**2 hours**

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(4)

2. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(3)

(b) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

3. Find the set of values of x for which

(a) $3(x-2) < 8-2x$

(2)

(b) $(2x-7)(1+x) < 0$

(3)

(c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$

(1)

4. In this question the unit vectors **i** and **j** are pointing east and north respectively.

(i) Calculate the bearing of the vector $-4\mathbf{i} - 6\mathbf{j}$. **[2]**

The vector $-4\mathbf{i} - 6\mathbf{j} + k(3\mathbf{i} - 2\mathbf{j})$ is in the direction $7\mathbf{i} - 9\mathbf{j}$.

(ii) Find k. **[4]**

5. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Solve, for $0 \leqslant x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)

6. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx$$

giving each term in its simplest form.

(4)

7. (a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1$.

(2)

8.

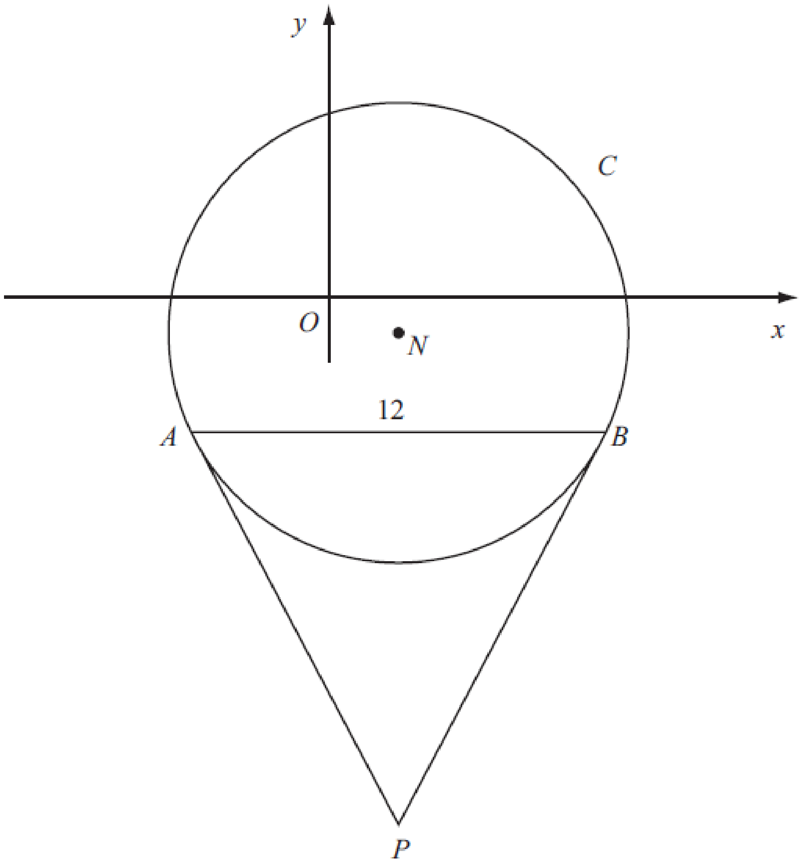


Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x-2)^2+(y+1)^2=\frac{169}{4}$$

(a) Write down the coordinates of N.

(2)

(b) Find the radius of C.

(1)

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in Figure 3.

(c) Find the coordinates of A and the coordinates of B.

(5)

(d) Show that angle $ANB = 134.8°$, to the nearest 0.1 of a degree.

(2)

The tangents to C at the points A and B meet at the point P.

(e) Find the length AP, giving your answer to 3 significant figures.

(2)

9. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

10. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

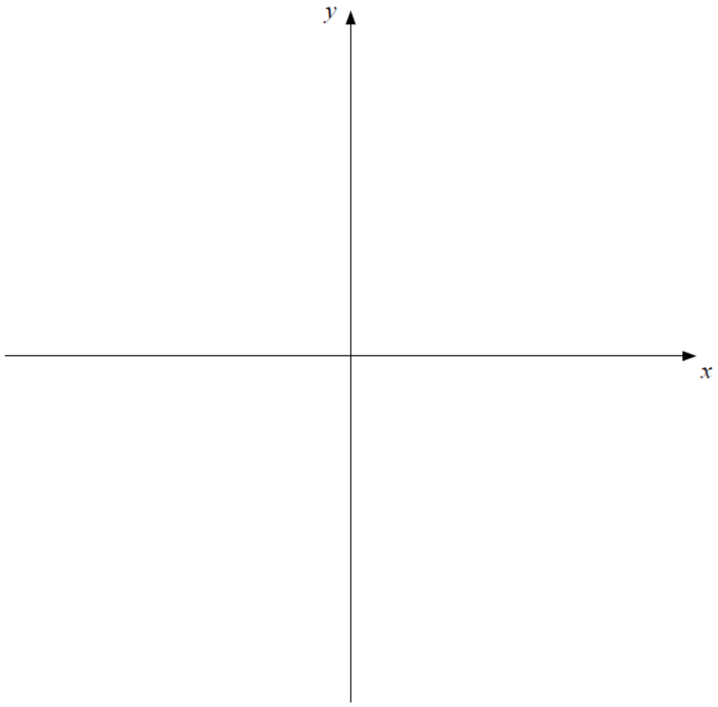
are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3},\ r + s\sqrt{3})$, where p, q, r and s are integers.

(7)



11 Show that the following statement is false.

$$x - 5 = 0 \iff x^2 = 25$$ **[2]**

12 You are given that $f(x) = x^3 + 6x^2 - x - 30$.

(i) Use the factor theorem to find a root of $f(x) = 0$ and hence factorise $f(x)$ completely. **[6]**

(ii) Sketch the graph of $y = f(x)$. **[3]**

(iii) The graph of $y = f(x)$ is translated by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Show that the equation of the translated graph may be written as

$$y = x^3 + 3x^2 - 10x - 24.$$ **[3]**

13

(i) Find the equation of the tangent to the curve $y = x^4$ at the point where $x = 2$. Give your answer in the form $y = mx + c$. **[4]**

(ii) Calculate the gradient of the chord joining the points on the curve $y = x^4$ where $x = 2$ and $x = 2.1$. **[2]**

(iii) (A) Expand $(2 + h)^4$. **[3]**

(B) Simplify $\dfrac{(2+h)^4 - 2^4}{h}$. **[2]**

(C) Show how your result in part **(iii)** (B) can be used to find the gradient of $y = x^4$ at the point where $x = 2$. **[2]**

Mark scheme

Q1	$\left[(3-x)^6 =\right] 3^6 + 3^5 \times 6 \times (-x) + 3^4 \times \binom{6}{2} \times (-x)^2$	M1
	$= 729, \quad -1458x, \quad +1215x^2$	B1,A1, A1 **[4]**

Q2	(a) $(7+\sqrt{5})(3-\sqrt{5}) = 21 - 5 + 3\sqrt{5} - 7\sqrt{5}$ Expand to get 3 or 4 terms	M1
	$= 16, \ -4\sqrt{5}$ (1st A for 16, 2nd A for $-4\sqrt{5}$) (i.s.w. if necessary, e.g. $16 - 4\sqrt{5} \rightarrow 4 - \sqrt{5}$)	A1, A1 (3)
	(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ (This is sufficient for the M mark)	M1
	Correct denominator without surds, i.e. $9-5$ or 4	A1
	$4-\sqrt{5}$ or $4-1\sqrt{5}$	A1 (3) **[6]**

3. **(a)**	$3x - 6 < 8 - 2x \rightarrow 5x < 14$ (Accept $5x - 14 < 0$ (o.e.))	M1	
	$x < 2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\leq$)	A1	(2)
(b)	Critical values are $x = \frac{7}{2}$ and -1	B1	
	Choosing "inside" $-1 < x < \frac{7}{2}$	M1 A1	(3)
(c)	$-1 < x < 2.8$	B1ft	(1)
	Accept any exact equivalents to -1, 2.8, 3.5		**6**

4

	mark	notes
$270 - \arctan\left(\frac{6}{4}\right)$	M1	Award for arctan p seen where $p = \pm\frac{6}{4}$ or $\frac{4}{6}$, or equivalent
$= 213.69\ldots$ so 214°	A1	cao
	2	
Need $(-4 + 3k)\mathbf{i} + (-6 - 2k)\mathbf{j} = \lambda(7\mathbf{i} - 9\mathbf{j})$ *	M1	Attempt to get LHS in the direction of $(7\mathbf{i} - 9\mathbf{j})$. Could be done by finding (tangents of) angles. Accept the use of $\lambda = 1$.
either		
so $\frac{-4+3k}{-6-2k} = \frac{7}{-9}$. or equivalent	M1	Attempt to solve **their** *. Allow = $\frac{7}{9}, \frac{9}{7}, -\frac{9}{7}$
	A1	Expression correct
$k = 6$	A1	Award full marks for $k = 6$ found WWW
or		
	M1	Attempt to solve **their** *. Must have both equations.
$-4 + 3k = 7\lambda$	A1	Correct equations
$-6 - 2k = -9\lambda$		
$k = 6$	A1	Award full marks for $k = 6$ found WWW
trial and error method		M1 any attempt to find the value of k and 'test' M1 Systematic attempt in (the equivalent of) **their** * Award full marks for $k = 6$ found WWW
	4	

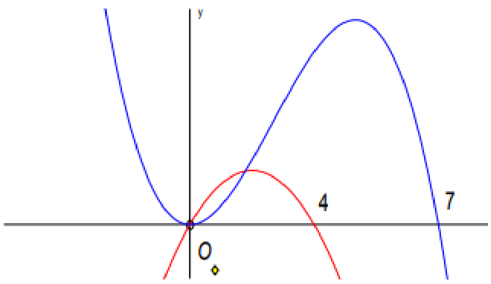
5	(a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) Requires the correct value with no incorrect working seen.	**B1**	**(1)**
	(b) awrt 21.8 (α) (Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft) (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	**B1**	
	$180 + \alpha$ $(= 201.8)$, or $90 + (\alpha/2)$ (if division by 2 has already occurred) (α found from $\tan 2x = \ldots$ or $\tan x = \ldots$ or $\sin 2x = \pm\ldots$ or $\cos 2x = \pm\ldots$)	**M1**	
	$360 + \alpha$ $(= 381.8)$, or $180 + (\alpha/2)$ (α found from $\tan 2x = \ldots$ or $\sin 2x = \ldots$ or $\cos 2x = \ldots$) OR $540 + \alpha$ $(= 561.8)$, or $270 + (\alpha/2)$ (α found from $\tan 2x = \ldots$)	**M1**	
	Dividing at least one of the angles by 2 (α found from $\tan 2x = \ldots$ or $\sin 2x = \ldots$ or $\cos 2x = \ldots$)	**M1**	
	$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt)	**A1**	**(5)** **6**

6	$\frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$	M1 A1
	$= 2x^4 + 4x^{\frac{3}{2}}, -5x + c$	A1 A1
		4

7	(a) $2\log_3(x-5) = \log_3(x-5)^2$	B1
	$\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$	M1
	$\log_3 3 = 1$ seen or used correctly	B1
	$\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$ $\left\{\frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)\right\}$	M1
	$x^2 - 16x + 64 = 0$ (*)	A1 cso (5)
	(b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ <u>Must</u> be seen in part (b). Or: Substitute $x = 8$ into original equation and verify. Having additional solution(s) such as $x = -8$ loses the A mark. $x = 8$ with no working scores both marks.	M1 A1 (2) **7**

Q8	(a)	$N(2, -1)$	B1, B1 (2)
	(b)	$r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$	B1 (1)
	(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1 + x_2}{2} = 2$ then solve	M1
		To obtain $x_1 = -4$, $x_2 = 8$	A1ft A1ft
		Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$	M1
		So $y_2 = y_1 = -3.5$	A1 (5)
	(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \frac{6}{"6.5"} \Rightarrow \theta = (67.38)...$	M1
		So angle ANB is 134.8 *	A1 (2)
	(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta = \frac{AP}{"6.5"}$	M1
		Therefore $AP = 15.6$	A1cao (2)
			[12]

Q9	(a)	$\left[y=12x^{\frac{1}{2}}-x^{\frac{3}{2}}-10\right]$	
		$[y'=]\quad 6x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}$	M1 A1
		Puts their $\frac{6}{x^{\frac{1}{2}}}-\frac{3}{2}x^{\frac{1}{2}}=0$	M1
		So $x=\ ,\frac{12}{3}=4$ (If $x=0$ appears also as solution then lose A1)	M1, A1
		$x=4,\ \Rightarrow\ y=12\times 2-4^{\frac{3}{2}}-10$, so $y=6$	dM1,A1 (7)
	(b)	$y''=-3x^{-\frac{3}{2}}-\frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
	(c)	[Since $x>0$] It is a maximum	B1 (1) [10]

Question Number	Scheme	Marks
10. (a)		
	(i) ∩ shape (anywhere on diagram)	B1
	Passing through or stopping at (0, 0) and (4,0) only(Needn't be ∩ shape)	B1
	(ii) correct shape (-ve cubic) with a max and min drawn anywhere	B1
	Minimum or maximum at (0,0)	B1
	Passes through or stops at (7,0) but <u>NOT</u> touching. (7, 0) should be to right of (4,0) or B0	B1 (5)
	Condone (0,4) or (0, 7) marked correctly on x-axis. Don't penalise poor overlap near origin. **Points must be marked on the sketch...not in the text**	
(b)	$x(4-x)=x^2(7-x)$ $(0=)x[7x-x^2-(4-x)]$	M1
	$(0=)x[7x-x^2-(4-x)]$ (o.e.)	B1ft
	$0=x\left(x^2-8x+4\right)$ *	A1 cso (3)
(c)	$\left(0=x^2-8x+4\Rightarrow\right)x=\frac{8\pm\sqrt{64-16}}{2}$ or $(x\pm4)^2-4^2+4(=0)$	M1
	$(x-4)^2=12$	A1
	$=\frac{8\pm4\sqrt{3}}{2}$ or $(x-4)=\pm2\sqrt{3}$	B1
	$x=4\pm2\sqrt{3}$	A1
	From sketch A is $x=4-2\sqrt{3}$	M1
	So $y=\left(4-2\sqrt{3}\right)\left(4-[4-2\sqrt{3}]\right)$ (dependent on 1st M1)	M1
	$\underline{=-12+8\sqrt{3}}$	A1 (7)
		15

Notes

11		mention of -5 as a square root of 25 or $(-5)^2 = 25$	M1	condone $-5^2 = 25$
		$-5 - 5 \neq 0$ o.e. or $x + 5 = 0$	M1	or, dep on first M1 being obtained, allow **M1** for showing that 5 is the only soln of $x - 5 = 0$ allow M2 for $x^2 - 25 = 0$ $(x + 5)(x - 5)$ [= 0] so $x - 5 = 0$ or $x + 5 = 0$

12	**(i)**	trials of at calculating f(x) for at least one factor of 30	**M1**	**M0** for division or inspection used
		details of calculation for f(2) or f(−3) or f(−5)	**A1**	
		attempt at division by $(x - 2)$ as far as $x^3 - 2x^2$ in working	**M1**	or equiv for $(x + 3)$ or $(x + 5)$; or inspection with at least two terms of quadratic factor correct
		correctly obtaining $x^2 + 8x + 15$	**A1**	or B2 for another factor found by factor theorem
		factorising a correct quadratic factor	**M1**	for factors giving two terms of quadratic correct; M0 for formula without factors found
		$(x - 2)(x + 3)(x + 5)$	**A1**	condone omission of first factor found; ignore '= 0' seen allow last four marks for $(x - 2)(x + 3)(x + 5)$ obtained; for all 6 marks must see factor theorem use first
12	**(ii)**	sketch of cubic right way up, with two turning points	**B1**	0 if stops at x-axis
		values of intns on x axis shown, correct (−5, −3, and 2) or ft from their factors/ roots in (i)	**B1**	on graph or nearby in this part mark intent for intersections with both axes
		y-axis intersection at −30	**B1**	or $x = 0$, $y = -30$ seen in this part if consistent with graph drawn

12	(iii)	$(x - 1)$ substituted for x in either form of eqn for $y = f(x)$	M1	correct or ft their (i) or (ii) for factorised form; condone one error; allow for new roots stated as −4,−2 and 3 or ft
		$(x - 1)^3$ expanded correctly (need not be simplified) or two of their factors multiplied correctly	M1 dep	or **M1** for correct or correct ft multiplying out of all 3 brackets at once, condoning one error $[x^3 - 3x^2 + 4x^2 + 2x^2 + 8x - 6x - 12x - 24]$
		correct completion to given answer [condone omission of '$y =$']	M1	unless all 3 brackets already expanded, must show at least one further interim step allow **SC1** for $(x + 1)$ subst <u>and</u> correct exp of $(x + 1)^3$ or two of their factors ft <u>or</u>, for those using given answer: **M1** for roots stated or used as −4,−2 and 3 or ft **A1** for showing all 3 roots satisfy given eqn **B1** for comment re coefft of x^3 or product of roots to show that eqn of translated graph is not a multiple of RHS of given eqn

13	(i)	$\frac{dy}{dx} = 4x^3$	M1	
		when $x = 2$, $\frac{dy}{dx} = 32$ s.o.i.	A1	i.s.w.
		when $x = 2$, $y = 16$ s.o.i.	B1	
		$y = 32x - 48$ c.a.o.	A1	
	(ii)	34.481	2	**M1** for $\frac{2.1^4 - 2^4}{0.1}$
	(iii) (*A*)	$16 + 32h + 24h^2 + 8h^3 + h^4$ c.a.o.	3	**B2** for 4 terms correct **B1** for 3 terms correct
	(iii) (*B*)	$32 + 24h + 8h^2 + h^3$ or ft	2	**B1** if one error
	(iii) (*C*)	as $h \to 0$, result $\to$ their 32 from (iii) (B)	1	
		gradient of tangent is limit of gradient of chord	1	