Paper collated from year	**2007**
Content	**Pure Chapters 1-13**
Marks	**100**
Time	**2 hours**

1.

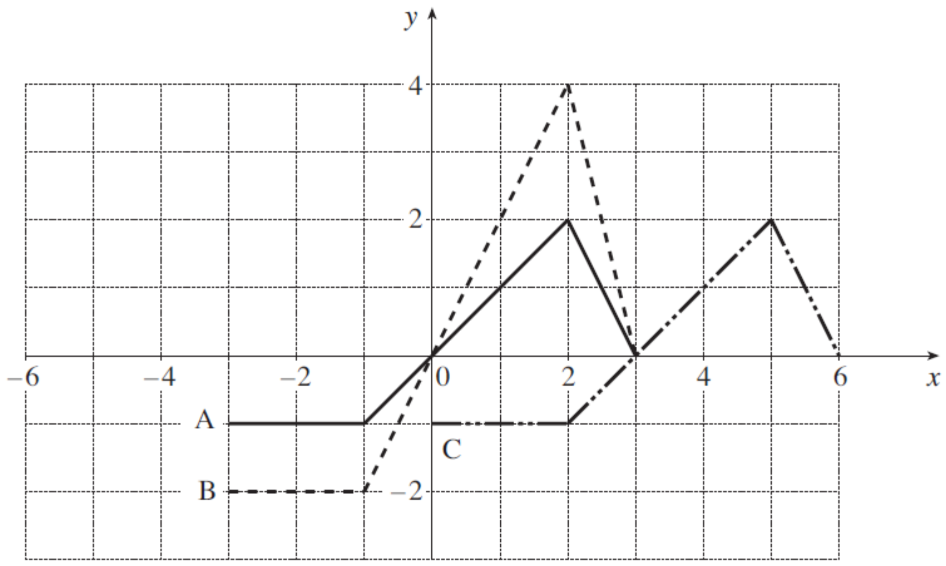


Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is $y = f(x)$.

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

2. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

3. Given that $$f(x) = \frac{1}{x}, \quad x \neq 0,$$

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

(4)

(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

(2)

4.

The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $f(x)$.

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

5.

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1 - 9x.$$

(2)

6.

A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

7.

Figure 1

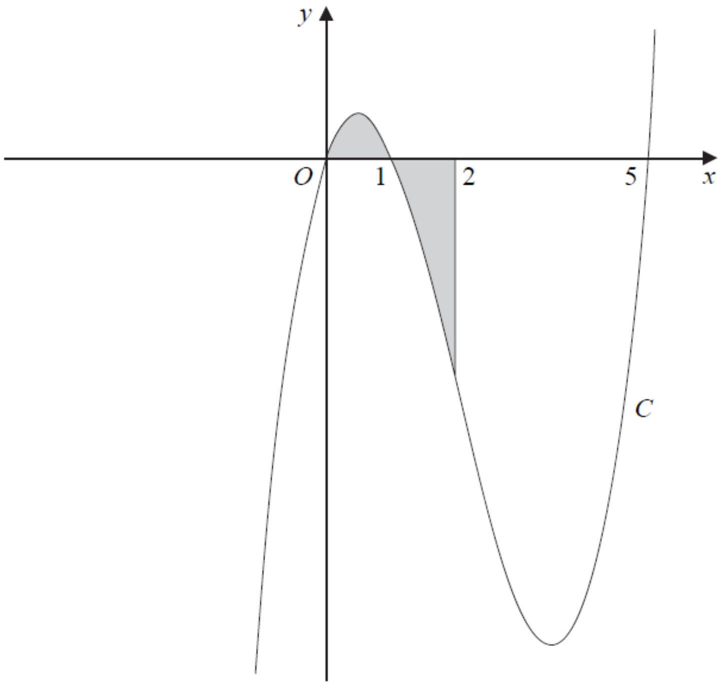


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

8.

The table gives a firm's monthly profits for the first few months after the start of its business, rounded to the nearest £100.

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800

The firm's profits, £y, for the xth month after start-up are modelled by

$$y = k \times 10^{ax}$$

where a and k are constants.

(i) Show that, according to this model, a graph of $\log_{10} y$ against x gives a straight line of gradient a and intercept $\log_{10} k$. [2]

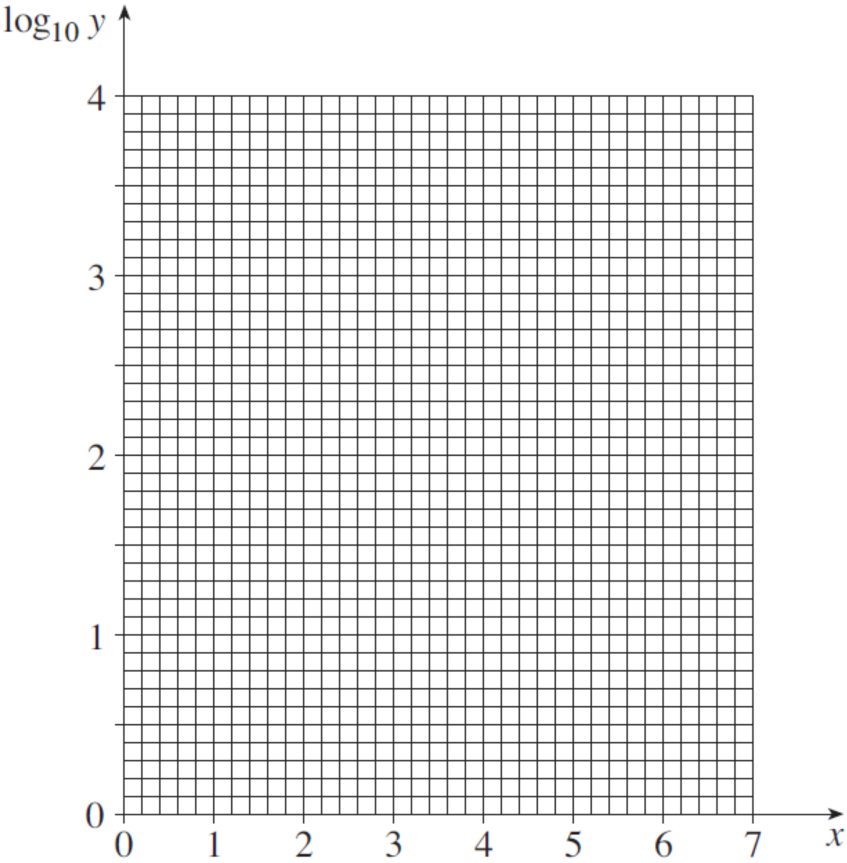
(ii) **On the insert**, complete the table and plot $\log_{10} y$ against x, drawing by eye a line of best fit. [3]

(iii) Use your graph to find an equation for y in terms of x for this model. [3]

(iv) For which month after start-up does this model predict profits of about £75 000? [3]

(v) State one way in which this model is unrealistic. [1]

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800
$\log_{10} y$	2.70					



9.

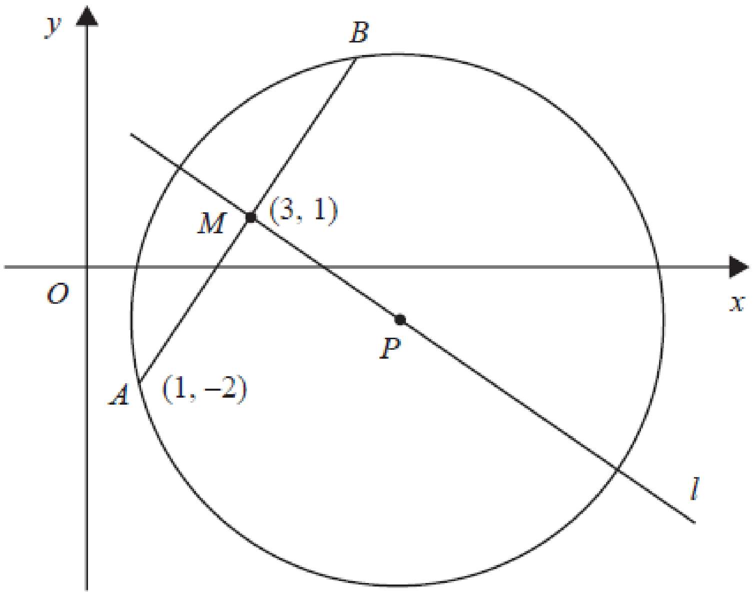


Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

10.

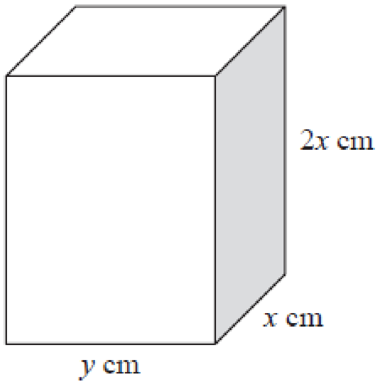


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

11.

(i) Show that the equation $2\cos^2\theta + 7\sin\theta = 5$ may be written in the form

$$2\sin^2\theta - 7\sin\theta + 3 = 0.$$ [1]

(ii) By factorising this quadratic equation, solve the equation for values of θ between $0°$ and $180°$.

[4]

12.

Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures.

[5]

13.

(a) Show that the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

has only one real root and state the exact value of this root. [2]

14.

(i) Express $\log_a x^4 + \log_a\left(\frac{1}{x}\right)$ as a multiple of $\log_a x$. [2]

(ii) Given that $\log_{10} b + \log_{10} c = 3$, find b in terms of c. [2]

Mark scheme

Question 1

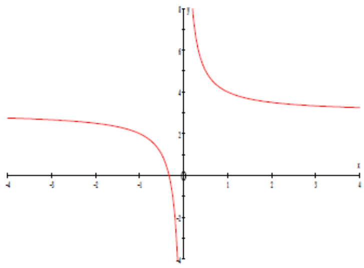
(i) $y = 2\text{f}(x)$	2	1 if 'y=' omitted [penalise only once] M1 for y = kf(x), k > 0	
(ii) $y = \text{f}(x - 3)$	2	M1 for y = f(x + 3) or y = f(x – k)	4

Question 2

(a) $6\sqrt{3}$	$(a = 6)$	B1	(1)
(b) Expanding $(2-\sqrt{3})^2$ to get 3 or 4 separate terms		M1	
$7,\ -4\sqrt{3}$	$(b = 7,\ c = -4)$	A1, A1	(3)
			4

Question 3

(a)



(a)	Shape of f(x)	B1	
	Moved up ↑	M1	
	Asymptotes: $y = 3$	B1	
	$x = 0$ (Allow "y-axis")	B1	(4)
	($y \neq 3$ is B0, $x \neq 0$ is B0).		
(b) $\frac{1}{x} + 3 = 0$	No variations accepted.	M1	
$x = -\frac{1}{3}$ (or $-0.33\ldots$)	Decimal answer requires at least 2 d.p.	A1	(2)
			6

Question 4

(a) $3x^2 \to cx^3$ or $-6 \to cx$ or $-8x^{-2} \to cx^{-1}$		M1	
$\text{f}(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1}$ $(+C)$	$\left(x^3 - 6x + \frac{8}{x}\right)$	A1 A1	
Substitute $x = 2$ and $y = 1$ into a 'changed function' to form an equation in C.		M1	
$1 = 8 - 12 + 4 + C$ $\quad C = 1$		A1cso	(5)
(b) $3 \times 2^2 - 6 - \frac{8}{2^2}$		M1	
$= 4$		A1	
Eqn. of tangent: $y - 1 = 4(x - 2)$		M1	
$y = 4x - 7$	(Must be in this form)	A1	(4)
			9

Question 5

$(1-2x)^5 = 1 + 5\times(-2x) + \frac{5\times 4}{2!}(-2x)^2 + \frac{5\times 4\times 3}{3!}(-2x)^3 + \ldots$ $= 1 - 10x + 40x^2 - 80x^3 + \ldots$	B1, M1, A1, A1 (4)
$(1+x)(1-2x)^5 = (1+x)(1-10x+\ldots)$ $= 1 + x - 10x + \ldots$ $\approx 1 - 9x$ (✱)	M1 A1 (2) **(6)**

Question 6

$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$	M1, A1
$-1400v^{-2} + \frac{2}{7} = 0$	M1
$v^2 = 4900$	**d**M1
$v = 70$	A1cso (5)
$\frac{d^2C}{dv^2} = 2800v^{-3}$	M1
$v = 70,\ \frac{d^2C}{dv^2} > 0$ $\{\Rightarrow \text{minimum}\}$ or $v = 70,\ \frac{d^2C}{dv^2} = 2800\times 70^{-3}$ $\{= \frac{2}{245} = 0.00816\ldots\}$ $\{\Rightarrow \text{minimum}\}$	A1ft (2)
$v = 70,\ C = \frac{1400}{70} + \frac{2\times 70}{7}$	M1
$C = 40$	A1 (2) **(9)**

Question 7

$y = x(x^2 - 6x + 5)$ $= x^3 - 6x^2 + 5x$	M1, A1
$\int (x^3 - 6x^2 + 5x)\,dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	M1, A1ft
$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1 = \left(\frac{1}{4} - 2 + \frac{5}{2}\right) - 0 = \frac{3}{4}$	M1
$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$	M1, A1(both)
$\therefore \text{total area} = \frac{3}{4} + \frac{11}{4}$	M1
$= \frac{7}{2}$ o.e.	A1cso **(9)**

Question 8

i	$\log_{10} y = \log_{10} k + \log_{10} 10^{ax}$ $\log_{10} y = ax + \log_{10} k$ compared to $y = mx+c$	M1 M1		 2
ii	2.9(0), 3.08, 3.28, 3.48, 3.68 plots [tol 1 mm] ruled line of best fit drawn	T1 P1f.t L1f.t.	condone one error	 3
iii	intercept = 2.5 approx gradient = 0.2 approx $y =$ their $300 \times 10^{x(\text{their } 0.2)}$ or $y = 10^{(\text{their } 2.5 + \text{their } 0.2x)}$	M1 M1 M1f.t.	or $y - 2.7 = m(x - 1)$	 3
iv	subst 75000 in any x/y eqn subst in a correct form of the relationship 11,12 or 13	M1 M1 A1	B3 with evidence of valid working	3
v	"Profits change" or any reason for this.	R1	too big, too soon	1

Question 9

(a) Gradient of AM: $\frac{1-(-2)}{3-1} = \frac{3}{2}$ or $\frac{-3}{-2}$	B1	
Gradient of l: $= -\frac{2}{3}$ M: use of $m_1 m_2 = -1$, or equiv.	M1	
$y - 1 = -\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3} = -\frac{2}{3}$ $[3y = -2x + 9]$ (Any equiv. form)	M1 A1	(4)
(b) $x = 6$: $3y = -12 + 9 = -3$ $y = -1$ (or show that for $y = -1$, $x = 6$) (*) (A conclusion is not required).	B1	(1)
(c) $(r^2 =)$ $(6-1)^2 + (-1-(-2))^2$ M: Attempt r^2 or r	M1 A1	
N.B. Simplification is not required to score M1 A1		
$(x \pm 6)^2 + (y \pm 1)^2 = k$, $k \neq 0$ (Value for k not needed, could be r^2 or r)	M1	
$(x-6)^2 + (y+1)^2 = 26$ (or equiv.) Allow $(\sqrt{26})^2$ or other exact equivalents for 26. (But… $(x-6)^2 + (y--1)^2 = 26$ scores M1 A0)	A1	(4)

Question 10

(a) $4x^2 + 6xy = 600$	M1 A1	
$V = 2x^2 y = 2x^2\left(\frac{600-4x^2}{6x}\right)$ $V = 200x - \frac{4x^3}{3}$ (*)	M1 A1cso	(4)
(b) $\frac{dV}{dx} = 200 - 4x^2$	B1	
Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x: $x^2 = 50$ or $x = \sqrt{50}$ (7.07…)	M1 A1	
Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943\ \text{cm}^3$ Allow awrt	M1 A1	(5)
(c) $\frac{d^2V}{dx^2} = -8x$ Negative, $\therefore$ Maximum	M1, A1ft	(2)
		11

Question 11

(i) $2(1 - \sin^2 \theta) + 7 \sin \theta = 5$	1	for $\cos^2 \theta + \sin^2 \theta = 1$ o.e. used	
(ii) $(2 \sin \theta - 1)(\sin \theta - 3)$	M1	1st and 3rd terms in expansion correct	
$\sin \theta = ½$	DM1	f.t. factors	
30° and 150°	A1 A1	B1,B1 for each solution obtained by any valid method, ignore extra solns outside range, 30°, 150° plus extra soln(s) scores 1	5

Question 12

$\log 3^{(2x+1)} = \log 5^{200}$	M1		Introduce logarithms throughout
$(2x+1)\log 3 = 200 \log 5$	M1		Drop power on at least one side
	A1		Obtain correct linear equation (now containing no powers)
$2x + 1 = \frac{200 \log 5}{\log 3}$	M1		Attempt solution of linear equation
$x = 146$	A1	5	Obtain $x = 146$, or better
$(2x + 1) = \log_3 5^{200}$	M1		Intoduce $\log_3$ on right-hand side
$2x + 1 = 200\log_3 5$	M1		Drop power of 200
	A1		Obtain correct equation
	M1		Attempt solution of linear equation
	A1		Obtain $x = 146$, or better

Question 13

(a) $\log_2 (x+3)^2 + \log_2 x - \log_2 (4x+2) = 1$	B1		State or imply that $2\log (x + 3) = \log (x + 3)^2$
	M1		Add or subtract two, or more, of their algebraic logs correctly
$\log_2 \left(\frac{(x+3)^2 x}{4x+2}\right) = 1$	A1		Obtain correct equation (or any equivalent, with single term on each side)
$\frac{(x+3)^2 x}{4x+2} = 2$	B1		Use $\log_2 a = 1 \Rightarrow a = 2$ at any point
$(x^2 + 6x + 9)x = 8x + 4$			
$x^3 + 6x^2 + x - 4 = 0$	A1	5	Confirm given equation correctly
(b) $x > 0$, otherwise $\log_2 x$ is undefined	B1*		State or imply that $\log x$ only defined for $x > 0$
$x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$	B1√dep*	2	State $x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$ (or x = 0.7) only, following their single positive root in (i)(b)

Question 14

(i) $3 \log_a x$	2	M1 for $4 \log_a x$ or $-\log_a x$; or $\log x^3$	
ii) $b = \frac{1000}{c}$	2	M1 for 1000 or 10^3 seen	4