Paper collated from year	**2007**
Content	**Stats chapter 14, 15, 16** **(Data Collection, Data Processing, Probability)** **Mechanics chapter 19** **(Just Kinematics)**
Marks	**62**
Time	**1 hour 15 minutes**

1 A ball is released from rest at a height h metres above ground level. The ball hits the ground 1.5 seconds after it is released. Assume that the ball is a particle that does not experience any air resistance.

(a) Show that the speed of the ball is $14.7\,\mathrm{m\,s^{-1}}$ when it hits the ground. *(2 marks)*

(b) Find h. *(2 marks)*

(c) Find the distance that the ball has fallen when its speed is $5\,\mathrm{m\,s^{-1}}$. *(3 marks)*

2 Fig. 1 is the velocity-time graph for the motion of a body. The velocity of the body is $v\,\mathrm{m\,s^{-1}}$ at time t seconds.

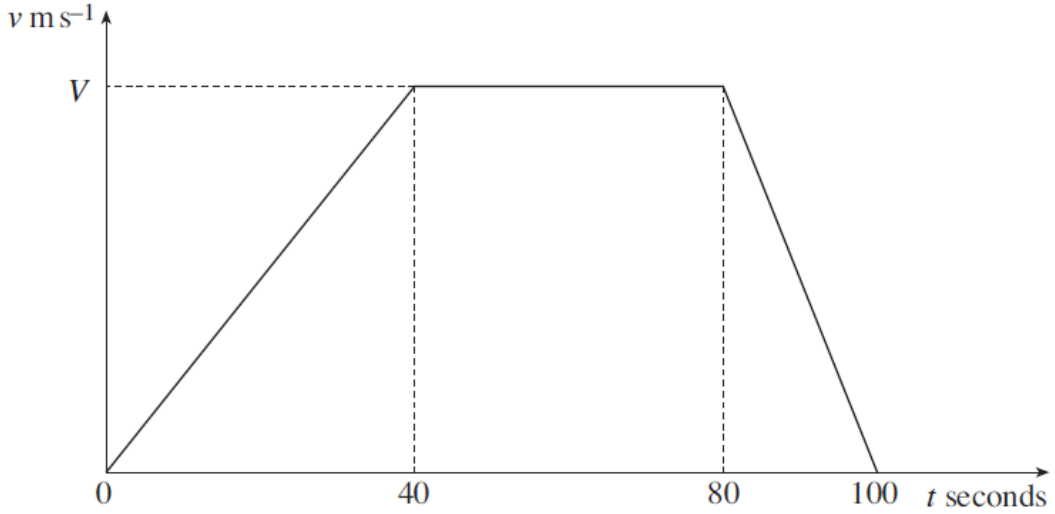


Fig. 1

The displacement of the body from $t = 0$ to $t = 100$ is 1400 m. Find the value of V. [4]

3 Fig. 7 is a sketch of part of the velocity-time graph for the motion of an insect walking in a straight line. Its velocity, v m s^{-1}, at time t seconds for the time interval $-3 \leqslant t \leqslant 5$ is given by

$$v = t^2 - 2t - 8.$$

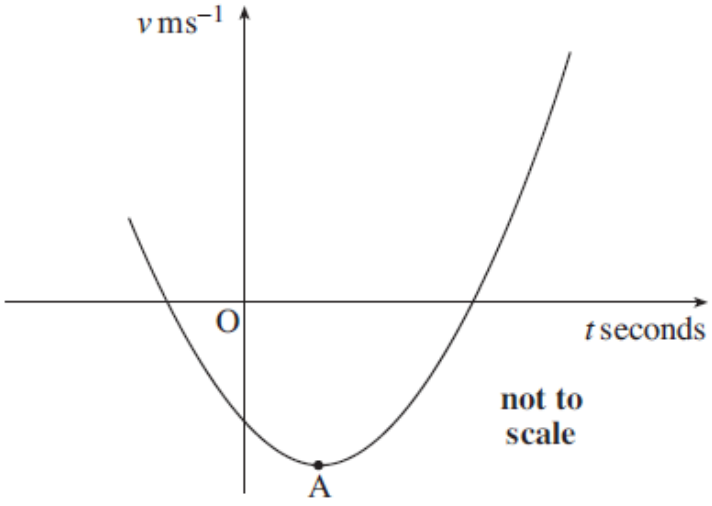


Fig. 7

(i) Write down the velocity of the insect when $t = 0$. [1]

(ii) Show that the insect is instantaneously at rest when $t = -2$ and when $t = 4$. [2]

(iii) Determine the velocity of the insect when its acceleration is zero.

Write down the coordinates of the point A shown in Fig. 7. [5]

(iv) Calculate the distance travelled by the insect from $t = 1$ to $t = 4$. [5]

(v) Write down the distance travelled by the insect in the time interval $-2 \leqslant t \leqslant 4$. [1]

(vi) How far does the insect walk in the time interval $1 \leqslant t \leqslant 5$? [3]

4 In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly selected rod is

(i) produced by machine A and is defective,

(ii) is defective.

(5)

5 The random variable X has probability function

$$P(X = x) = \frac{(2x - 1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X.

(3)

Find

(b) $P(2 < X \leqslant 5)$,

(2)

6 Summarised below are the distances, to the nearest mile, travelled to work by a random sample of 120 commuters.

Distance (to the nearest mile)	Number of commuters
0–9	10
10–19	19
20–29	43
30–39	25
40–49	8
50–59	6
60–69	5
70–79	3
80–89	1

For this distribution,

(a) describe its shape,

(1)

(b) use linear interpolation to estimate its median.

(2)

The mid-point of each class was represented by x and its corresponding frequency by f giving

$$\Sigma fx = 3550 \quad \text{and } \Sigma fx^2 = 138020$$

(c) Estimate the mean and the standard deviation of this distribution.

(3)

One coefficient of skewness is given by

$$\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}.$$

(d) Evaluate this coefficient for this distribution.

(3)

(e) State whether or not the value of your coefficient is consistent with your description in part (a). Justify your answer.

(2)

(f) State, with a reason, whether you should use the mean or the median to represent the data in this distribution.

(2)

(g) State the circumstance under which it would not matter whether you used the mean or the median to represent a set of data.

(1)

7

A teacher recorded, to the nearest hour, the time spent watching television during a particular week by each child in a random sample. The times were summarised in a grouped frequency table and represented by a histogram.

One of the classes in the grouped frequency distribution was 20–29 and its associated frequency was 9. On the histogram the height of the rectangle representing that class was 3.6 cm and the width was 2 cm.

(a) Give a reason to support the use of a histogram to represent these data.

(1)

(b) Write down the underlying feature associated with each of the bars in a histogram.

(1)

(c) Show that on this histogram each child was represented by 0.8 cm^2.

(3)

The total area under the histogram was 24 cm^2.

(d) Find the total number of children in the group.

(2)

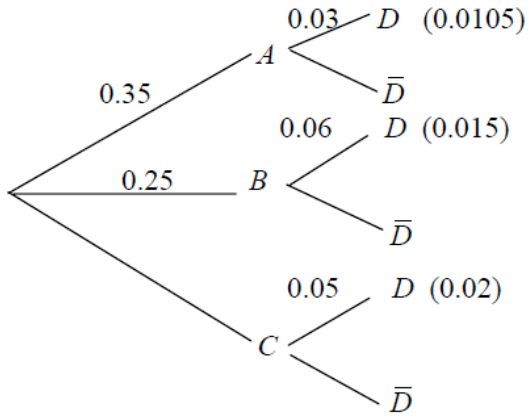
Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$v = 0 + 1.5 \times 9.8$	M1		Use of constant acceleration equation to find v
	$= 14.7 \text{ ms}^{-1}$	A1	2	AG Correct v from correct working $1.5 \times 9.8 = 14.7$ is not enough on its own
(b)	$h = \frac{1}{2} \times 9.8 \times 1.5^2$	M1		Use of constant acceleration equation with $a = 9.8$ to find h
	$= 11.0$ m (to 3 sf)	A1	2	Correct h Allow 11 m; ignore negative signs
(c)	$5^2 = 0^2 + 2 \times 9.8s$	M1		Use of constant acceleration equation with $u = 0$ to find s
		A1		Correct equation
	$s = \frac{25}{19.6} = 1.28$ m (to 3 sf)	A1	3	Correct s Accept 1.27
	OR			
	$t = \frac{5}{9.8} = 0.510$			
	$s = \frac{1}{2}(0+5)\frac{5}{9.8} = 1.28$ m			
	OR			
	$s = 0 + \frac{1}{2} \times 9.8 \times \left(\frac{5}{9.8}\right)^2 = 1.28$ m			

2	**either**			
		M1		Attempt at area. If not trapezium method at least one part area correct. Accept equivalent.
	70V obtained	A1		Or equivalent – need not be evaluated.
	So 70V = 1400	M1		Equate **their** 70V to 1400. Must have attempt at complete areas or equations.
	and V = 20	A1		cao
	or	M1		Attempt to find areas in terms of ratios (at least one correct)
		A1		Correct total ratio – need not be evaluated. (Evidence may be 800 or 400 or 200 seen).
		M1		Complete method. (Evidence may be 800/40 or 400/20 or 200/10 seen).
	V = 20	A1		cao [Award 3/4 for 20 seen WWW]
				4

Q7				
(i)	8 m s^{-1} (in the negative direction)	B1	Allow $\pm$ and no direction indicated	1
(ii)	$(t+2)(t-4)=0$ so $t=-2$ or 4	M1 A1	Equating v to zero and solving or subst If subst used then both must be clearly shown	2
(iii)	$a=2t-2$ $a=0$ when $t=1$ $v(1)=1-2-8=-9$ so 9 m s^{-1} in the negative direction $(1,-9)$	M1 A1 F1 A1 B1	Differentiating Correct Accept -9 but not 9 without comment FT	5
(iv)	$\int_1^4 \left(t^2-2t-8\right)\,dx$ $=\left[\frac{t^3}{3}-t^2-8t\right]_1^4$ $=\left(\frac{64}{3}-16-32\right)-\left(\frac{1}{3}-1-8\right)$ $=-18$ distance is 18 m	M1 A1 M1 A1 A1	Attempt at integration. Ignore limits. Correct integration. Ignore limits. Attempt to sub correct limits and subtract Limits correctly evaluated. Award if -18 seen but no need to evaluate Award even if -18 not seen. Do not award for -18. cao	5
(v)	$2\times 18=36\text{ m}$	F1	Award for $2\times$ **their** (iv).	1
(vi)	$\int_4^5 \left(t^2-2t-8\right)\,dx=\left[\frac{t^3}{3}-t^2-8t\right]_4^5$ $=\left(\frac{125}{3}-25-40\right)-\left(-\frac{80}{3}\right)=3\frac{1}{3}$ so $3\frac{1}{3}+18=21\frac{1}{3}$ m	M1 A1 A1	$\int_4^5$ attempted or, otherwise, complete method seen. Correct substitution Award for $3\frac{1}{3}$ + **their** (positive) (iv)	3
				17

4

. (a)		Correct tree shape	M1
		A, *B* and *C* and 0.35 and 0.25	A1
		D (x3) and 0.03, 0.06, 0.05	A1 (3)
		(May be implied by seeing	
		$P(A \cap D)$ etc at the ends)	
)(i)	$P(A \cap D) = 0.35 \times 0.03$, $= \underline{\mathbf{0.0105}}$ or $\frac{21}{2000}$		M1, A1
		$P(C) = 0.4$ (anywhere)	B1
(ii)	$P(D) =$ (i) + 0.25x 0.06 + (0.4x0.05)		M1
	$= \underline{\mathbf{0.0455}}$ or $\frac{91}{2000}$		A1 (5)
(c)	$P(C \mid D) = \frac{P(C \cap D)}{P(D)}$, $= \frac{0.4 \times 0.05}{\text{(ii)}}$		M1, A1ft
	$= 0.43956\ldots$ or $\frac{40}{91}$	**0.44** or awrt **0.440**	A1 (3)
	[Correct answers only score full marks in each part]		**11 marks**

5

(a)	N.B. Part (a) doesn't have to be in a table, could be a list $P(X = 1) = \ldots$etc		B1, B1, B1

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306 (Accept awrt 3 s.f) (3)

(b)	$P(3) + P(4) + P(5) =$, $\frac{21}{36}$ or $\frac{7}{12}$ or awrt 0.583		M1, A1 (2)

6

(a)	Positive skew (both bits)	B1	(1)
(b)	$19.5 + \frac{(60-29)}{43} \times 10$, $= 26.7093\ldots$ awrt **26.7**	M1, A1	(2)
	(N.B. Use of 60.5 gives 26.825… so allow awrt 26.8)		
(c)	$\mu = \frac{3550}{120} = 29.5833...$ or $29\frac{7}{12}$ awrt **29.6**	B1	
	$\sigma^2 = \frac{138020}{120} - \mu^2$ or $\sigma = \sqrt{\frac{138020}{120} - \mu^2}$	M1	
	$\sigma = 16.5829...$ or $(s = 16.652...)$ awrt **16.6** (or $s = 16.7$)	A1	(3)
(d)	$\frac{3(29.6-26.7)}{16.6}$	M1A1ft	
	$= 0.52\ldots$ awrt **0.520** (or with s awrt 0.518)	A1	(3)
	(N.B. 60.5 in (b) ...awrt 0.499[or with s awrt 0.497])		
(e)	$0.520 > 0$ correct statement about their (d) being >0 or < 0	B1ft	
	So it is consistent with (a) ft their (d)	dB1ft	(2)
(f)	Use Median	B1	
	Since the data is skewed or less affected by outliers/extreme values	dB1	(2)
(g)	If the data are symmetrical or skewness is zero or normal/uniform distribution	B1	(1)
	("mean =median" or "no outliers" or "evenly distributed" all score B0)		**14 marks**

7

(a)	Time is a continuous variable or data is in a grouped frequency table	B1	(1)
(b)	Area is proportional to frequency or $A \propto f$ or $A = kf$	B1	(1)
(c)	$3.6 \times 2 = 0.8 \times 9$	M1 dM1	
	1 child represented by 0.8	A1 cso	(3)
(d)	(Total) $= \frac{24}{0.8}$, $= \mathbf{30}$	M1, A1	(2)
			7 marks