# STATISTICS 4, S4 (4769) A2

## Objectives

To provide students with the opportunity to explore ideas in more advanced statistics to a greater depth.

### Assessment

# Examination(72 marks)<br/>1 hour 30 minutesThere are four options in this unit and there will be one question on each option.<br/>Candidates are expected to answer three questions, each worth 24 marks.

#### Assumed Knowledge

Candidates are expected to know the content for *C1*, *C2*, *C3* and *C4*, for *FP1* and for *S1*, *S2* and *S3*. Candidates who wish to study the Generating Functions option must ensure they have appropriate pure mathematics facility in respect of calculus and the summation of series.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

## Option 1:

		STATISTICS 4, S4
Specification	Ref.	Competence Statements
		ESTIMATION
Expectation and variance of a function of a random variable.	S4E1	Be able to find the expectation and variance of a function of a discrete or continuous random variable.
Estimators as random variables.	2	Understand the idea of an estimator as a random variable and its sampling distribution.
Biased and unbiased estimators.	3	Understand the meaning of biased and unbiased estimators.
	4	Be able to determine whether a given estimator is biased or unbiased.
	5	Be able to construct estimators in simple cases.
Unbiased estimators for population mean and variance, from single and pooled samples.	6	Be able to find unbiased estimators for the mean and variance of a population from single and pooled samples.
Maximum likelihood.	7	Understand the meaning of the likelihood of a set of outcomes for a discrete or continuous random variable.
	8	Be able to find the maximum likelihood estimator of a population parameter or parameters, for a discrete or continuous random variable, in simple cases.
Standard error of an estimator.	9	Understand and be able to obtain and use the standard error of an estimator.
Mean square error.	10	Understand the use of mean square error (MSE) for biased estimators.
	11	Be able to obtain a mean square error.
Relative efficiency of estimators.	12	Understand that estimators can be compared by considering their standard errors or mean square errors.

# Option 2:

		STATISTICS 4, S4
Specification	Ref.	Competence Statements
		GENERATING FUNCTIONS
Probability generating function (pgf).	S4f1	Know that the coefficients of a generating function represent probabilities for a discrete random variable.
	2	Understand the uniqueness of the relationship between a distribution and its probability generating function.
Derivation of the pgf of a discrete random variable.	3	Be able to derive a probability generating function for a discrete random variable from its probability distribution.
Use of the pgf to find the distribution of the sum of random variables.	4	Know that the probability generating function of a sum of two or more independent discrete random variables is the product of their probability generating functions and be able to use this.
Use of the pgf to	5	Be able to derive the mean and variance of a discrete random variable from its pgf
find the mean and variance of a random variable.	6	Be able to derive the probability generating function of a linear transformation of a random variable.
Moment generating function (mgf).	7	Know that the coefficients of a generating function may be used to derive moments for a discrete or continuous random variable.
	8	Understand the uniqueness of the relationship between a distribution and its moment generating function.
Derivation of the mgf of a random variable.	9	Be able to derive a moment generating function for a discrete or continuous random variable given its probability function or probability density function.
Use of the mgf to find the distribution of the sum of random variables.	10	Know that the moment generating function of a sum of two or more independent random variables is the product of their moment generating functions and be able to use this.
Use of the mgf to find the mean and variance of a random variable.	11	Be able to derive the mean and variance of a random variable by inspection of coefficients or differentiation.
	12	Be able to derive the moment generating function of a linear transformation of a random variable.

		STATISTICS 4, S4
Specification	Ref.	Competence Statements
		INFERENCE
Two-sample and paired tests.	S4I1	Understand the difference between a two-sample test and a paired sample test.
Test for the difference of means of unpaired samples.	2	Know when to use the Normal distribution and when to use the t distribution in testing for differences of population means.
	3	Know when to use pooled unbiased estimators for common mean and variance for samples from two populations and be able to do so.
Test for the difference of medians of unpaired samples using the Wilcoxon (Mann- Whitney) rank sum test.	4	Know when to use the Wilcoxon rank sum (Mann-Whitney) test for differences of population medians.
Confidence intervals for the difference of means of unpaired populations.	5	Be able to construct and interpret confidence intervals for the difference in mean of unpaired populations using the <i>t</i> distribution.
Type I and Type II errors.	6	Know what is meant by a Type I and a Type II error in a hypothesis test.
	7	Understand that the probability of a Type I error is the significance level of the test.
	8	Understand that the probability of a Type II error depends on the true population distribution, which is unknown, as well as the sample size and significance level.
	9	Be able to calculate the probability of a Type II error in a specific case.
Operating characteristic, power function.	10	Be able to calculate, plot and interpret the operating characteristic and the power function of a hypothesis test.

## **Option 4:**

STATISTICS 4, S4				
Specification	Ref.	Competence Statements		
	D	ESIGN AND ANALYSIS OF EXPERIMENTS		
Principles of experimental design.	S4x1	Be able to explain the need for randomisation and replication in a particular experimental design.		
	2	Be able to explain the purpose of particular designs in given simple situations.		
	3	Be able to suggest a suitable experimental design for a given simple situation.		
One-way analysis of variance.	4	Understand the usual one-way analysis of variance model, for the case of commor population variance.		
	5	Know that a one-way analysis of variance is used to test for equality of population means.		
	6	Be able to carry out a one-way analysis of variance using an F test.		
	7	Be able to interpret the conclusions of such a test.		