FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3 (4757) A2

Objectives

To give students an introduction to a number of more sophisticated areas of Pure Mathematics, with a choice of options.

Assessment

Examination: (72 marks)
1 hour 30 minutes.
Candidates answer three questions out of five, each worth 24 marks.

Assumed Knowledge

Candidates are expected to know the content for *C1, C2, C3, C4, FP1* and *FP2.*

Candidates attempting Option 5 are expected to be familiar with elementary concepts of probability and with expected values.

Subject Criteria

The Units *FP1* and *FP2* are required for Advanced GCE Further Mathematics.
The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics.

Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

For Option 5, Markov Chains, a calculator with the facility to handle matrices is required.

OPTION 1:

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3		
Specification	**Ref.**	**Competence Statements**

VECTORS		
Vector (cross) product of two vectors.	FP3v1	Be able to form the vector product of two vectors in magnitude and direction, and in component form.
	2	Understand the anti-commutative and distributive properties of the vector product.
	3	Know the significance of $\mathbf{a}\times\mathbf{b}=\mathbf{0}$.
The intersection of two planes.	4	Be able to find the line of intersection of two planes.
The intersection of lines in three dimensions.	5	Be able to determine whether two lines in three dimensions are skew or intersect, and to find the point of intersection if there is one.
Distance of a point from a line or from a plane.	6	Be able to find the shortest distance from a point to a line in 2 or 3 dimensions.
	7	Be able to find the shortest distance from a point to a plane.
Scalar triple product.	8	Be able to find the scalar triple product of three vectors, and appreciate that its value is unchanged by cyclic permutation of the vectors.
Geometrical interpretation.	9	Be able to use the scalar triple product to determine the handedness of a set of vectors.
Volume of parallelepiped and tetrahedron.	10	Be able to use the scalar triple product to find the volume of a parallelepiped or tetrahedron.
Shortest distance between two skew lines.	11	Be able to use the scalar triple product to find the shortest distance between two skew lines.
Condition in three dimensions for two lines to intersect.	12	Be able to use the scalar triple product to determine whether or not two lines in 3 dimensions intersect.

OPTION 2:

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3		
Specification	**Ref.**	**Competence Statements**

MULTI-VARIABLE CALCULUS		
$z = \mathrm{f}(x, y)$ and its interpretation as a surface. Contour lines, and sections of the form $z = \mathrm{f}(a, y)$ or $z = \mathrm{f}(x, b)$.	FP3c1	Appreciate that the relation $z = \mathrm{f}(x, y)$ defines a surface in three dimensions.
Sketching of surfaces.	2	Be able to sketch contours and sections, and know how these are related to the surface.
First order partial derivatives.	3	Be able to find first order partial derivatives.
Simple applications to surfaces and stationary points.	4	Be able to use the conditions $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ to find the coordinates of stationary points on a surface.
Surfaces in three dimensions defined by $\mathrm{g}(x, y, z) = c$.	5	Appreciate that the relation $\mathrm{g}(x, y, z) = c$ defines a surface in three dimensions.
	6	Be able to find **grad** g, and to evaluate this at a point on the surface to give a normal vector.
Applications to finding the normal line and the tangent plane at a point.	7	Be able to find the equations of the normal line and tangent plane at a point on the surface.
$\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$ and its application to errors	8	Appreciate that the tangent plane gives a local approximation to the surface, and hence that $\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$, and be able to use this, or the similar result for functions of more than two variables, to estimate the consequence of errors in these variables.

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OPTION 3:

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3		
Specification	**Ref.**	**Competence Statements**

DIFFERENTIAL GEOMETRY		
Arc length.	FP3g1	Be able to calculate arc length using cartesian, parametric and polar co-ordinates.
Curved surface area and volume of a solid of revolution.	2	Be able to calculate the volume and curved surface area of a solid of revolution using cartesian or parametric co-ordinates.
Envelopes.	3	Be able to find the envelope of a family of curves by eliminating p between $\mathrm{f}(x, y, p) = 0$ and $\dfrac{\partial \mathrm{f}}{\partial p}(x, y, p) = 0$.
Intrinsic co-ordinates and intrinsic equations.	4	Understand the use of arc length and inclination of tangent as intrinsic co-ordinates.
	5	Be able to work with intrinsic equations in simple cases.
Curvature, radius of curvature.	6	Be able to use the definitions of curvature and radius of curvature.
Centre of curvature.	7	Be able to find the centre of curvature.
Evolute.	8	Be able to find the evolute as the locus of the centre of curvature and as the envelope of the normals.

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OPTION 4:

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3		
Specification	**Ref.**	**Competence Statements**
	GROUPS	
The axioms of a group.	FP3a1	Understand the group axioms and the associated language.
Illustrations of groups.	2	Be familiar with examples of groups, and of the use of group tables.
Cyclic groups.	3	Understand the meaning of the term cyclic group, and how a single element can generate such a group.
The order of a finite group; the order of an element of a group.	4	Understand the terms order of a finite group, order of an element.
Subgroups.	5	Understand the term subgroup.
Lagrange's theorem.	6	Understand and be able to use Lagrange's theorem.
Isomorphism.	7	Understand that different situations can give rise to essentially the same structure.
	8	Be able to specify an isomorphism in simple cases.

OPTION 5:

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3		
Specification	**Ref.**	**Competence Statements**

MARKOV CHAINS

The assessment of this option will be based on the assumption that candidates have a calculator with the ability to handle matrices.

Candidates who do not have such a calculator are advised not to attempt this option.

Transition matrix.	FP3m1	Understand the relationships between random processes, Markov chain models and transition matrices.
	2	Be able to represent a suitable process by means of a matrix of transition probabilities.
	3	Be able to interpret a given transition matrix in terms of an underlying process.
Limit properties of a non-periodic transition matrix.	4	Be able to use a transition matrix to calculate probabilities of future events.
	5	Know that successive powers of a non-periodic transition matrix tend towards a limit.
	6	Be able to calculate equilibrium probabilities.
Run lengths; expected values.	7	Be able to calculate the expected run length of a particular event.
Periodic states.	8	Be able to work with processes having periodic states.
Absorption and reflection.	9	Be able to work with processes having absorbing states.
	10	Be able to work with processes having reflecting barriers.