DIFFERENTIAL EQUATIONS, DE (4758) A2

Objectives

To extend the work in *Applications of Advanced Mathematics (C4)* on the formulation and solution of differential equations. Students are expected to have a reasonable degree of manipulative competence and to be able to handle more complicated problems.

Assessment

Examination (72 marks)
1 hour 30 minutes
Candidates answer three questions from four**,** each worth 24 marks.

Coursework (18 marks)
One modelling assignment involving the use of differential equations at an appropriate level of sophistication.
There are no restrictions on the context chosen.

Unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly 9.8 ms^{-2}.

Assumed Knowledge

Candidates are expected to know the content of *C1, C2, C3* and *C4*.
In addition candidates are expected to know basic kinematics and Newton's Second Law.
Relevant knowledge of complex numbers will also be required.

Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

DIFFERENTIAL EQUATIONS, DE		
Specification	**Ref.**	**Competence Statements**
MODELLING WITH DIFFERENTIAL EQUATIONS		
Constructions of models.	DEp1	Understand how to introduce and define variables to describe a given situation in mathematical terms.
	2	Be able to relate 1^{st} and 2^{nd} order derivatives to verbal descriptions and so formulate differential equations.
	3	Know the language of kinematics, and the relationships between the various terms.
	4	Know Newton's 2^{nd} law of motion.
	5	Understand how to determine the order of a differential equation.
Interpretation of solutions.	6	Be able to interpret the solution of a differential equation in terms of the original situation.
	7	Appreciate the difference between a general solution and a particular solution, i.e. one which satisfies particular prescribed conditions.
	8	Understand the significance of the number of arbitrary constants in a general solution.
	9	Be able to investigate the effect of changing a differential equation on its solution.
Tangent fields.	10	Be able to sketch the tangent field for a 1^{st} order differential equation and be able to interpret it.
	11	Be able to sketch and interpret the curve of the solution corresponding to particular conditions.
	12	Be able to identify isoclines and use them in sketching and interpreting tangent fields.
FIRST ORDER DIFFERENTIAL EQUATIONS		
Equations with separable variables.	DEc1	Be able to find both general and particular solutions of a 1^{st} order differential equation with separable variables.
First order linear differential equations.	2	Be able to solve 1^{st} order linear differential equations with constant coefficients.
	3	Be able to distinguish differential equations where the integrating factor method is appropriate, and to rearrange such equations if necessary.
	4	Be able to find an integrating factor and understand its significance in the solution of an equation.
	5	Be able to solve an equation using an integrating factor and find both general and particular solutions.

DIFFERENTIAL EQUATIONS, DE		
Specification	**Ref.**	**Competence Statements**
SECOND AND HIGHER ORDER DIFFERENTIAL EQUATIONS		
Homogeneous second order linear differential equations.	DEc6	Be able to solve homogenous 2nd order differential equations, using the auxiliary equation and complementary function.
	7	Appreciate the relationship between different cases of the solution and the nature of the roots of the auxiliary equation, and be able to interpret these different cases graphically.
	8	Be able to find the particular solution in given contexts.
The general second order linear differential equation.	9	Be able to solve the general 2nd order linear differential equation, by solving the homogeneous case and adding a particular integral.
	10	Be able to find particular integrals in simple cases. Appreciate the relationship between different cases of the solution and the nature of the roots of the auxiliary equation, and be able to interpret these different cases graphically.
	11	Be able to solve the equation for simple harmonic motion, $\ddot{x} + \omega(x + k) = 0$, and be able to relate the various forms of the solution to each other.
Damped oscillations.	12	Be able to model damped oscillations using 2nd order linear differential equations, and understand the associated terminology.
	13	Be able to interpret the solutions of equations modelling damped oscillations in words and graphically.
Higher order linear differential equations.	14	Appreciate that the same methods can be extended to higher order equations and be able to do so in simple cases.
SIMULTANEOUS DIFFERENTIAL EQUATIONS		
Simultaneous linear differential equations.	DEe1	Model situations with one independent variable and two dependent variables which lead to 1st order simultaneous differential equations, and know how to solve these by eliminating one variable to produce a single, 2nd order equation.
	2	Appreciate that the same method can be extended to more than two such equations, leading by elimination to a single higher order equation.
NUMERICAL METHODS		
Step by step methods.	DEs1	Be able to use step by step methods (e.g. Euler's method) to solve 1st order differential equations (including simultaneous equations) where appropriate.

Differential Equations (DE) Coursework: How Differential Equations are used to Solve Real-World Problems

Rationale

The aims of the coursework are that students should learn how differential equations are used to solve real-world problems and that they should appreciate how the theory they have learnt for the examination helps them to do this.

The objectives are that they should be able to undertake the various steps in the problem solving procedure shown in the flow chart in Section 5.2. The assessment criteria are closely related to these steps.

Description

There are two aspects to the work.

(i) The modelling cycle consists of pen and paper development of the consequences of the basic assumptions made, leading to a predicted outcome which must then be tested against reality.

(ii) In the experimental cycle, results are collected in order to give insight into the situation under investigation, so that a realistic model can be developed.

Level of Work

The task represents 20% of the assessment and the work involved should be consistent with that figure, both in quantity and level of sophistication. Tasks which allow only superficial or trivial treatment should be avoided.

Assessment

Each task must be assessed on one of the coursework assessment sheets, A or B. The assessor decides on the appropriate sheet according to the way the candidate has approached the particular task.

(A) In this case the modelling cycle is investigated in some depth, whilst the check against reality may use the data from published sources, from experiments which the candidate has not actually performed or from experience; there must however be a quantitative element in such data.

(B) The work presented is approximately evenly divided between developing the model, and one or more experiments conducted by the candidate to verify the quality of predictions from it and/or to inform its development.

No other mark sheet may be used, nor may these be amended in any way.

One mark is available for each criterion statement. Half marks may be awarded, but the overall total must be rounded (up or down) to an integer.

Note that in the case of Mark Scheme A, the marks for 'Manipulating the Model' may be awarded for the quality of the work either on the first or the second modelling cycles.

Task Selection

Centres are encouraged to develop their own coursework tasks. If they have any doubt about the suitability of a proposed task, they are recommended to submit details of it to the Principal Coursework Moderator, via OCR.

However, Centres which are new to the scheme are strongly recommended to use the tasks published by MEI while they are familiarising themselves with the nature of coursework. They should ensure that the material they have is that published for this specification and not that for an earlier specification.

Centres are advised that the choice of suitable tasks is crucial to the success of their candidates' coursework.