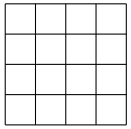
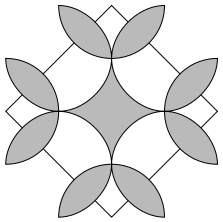
19. The 16 small squares shown in the diagram each have a side length of 1 unit. How many pairs of vertices are there in the diagram whose distance apart is an integer number of units?



A 40 B 64 C 108 D 132 E 16

20. The ratio of two positive numbers equals the ratio of their sum to their difference. What is this ratio?

A $(1+\sqrt{3}):2$ B $\sqrt{2}:1$ C $(1+\sqrt{5}):2$ D $(2+\sqrt{2}):1$ E $(1+\sqrt{2}):1$

21. 

The shaded design shown in the diagram is made by drawing eight circular arcs, all with the same radius. The centres of four arcs are the vertices of the square; the centres of the four touching arcs are the midpoints of the sides of the square. The diagonals of the square have length 1.

What is the total length of the border of the shaded design?

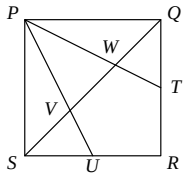
A 2π B $\frac{5\pi}{2}$ C 3π D $\frac{7\pi}{2}$ E 4π

22. Consider numbers of the form $10n + 1$, where n is a positive integer. We shall call such a number 'grime' if it cannot be expressed as the product of two smaller numbers, possibly equal, both of which are of the form $10k + 1$, where k is a positive integer.

How many 'grime numbers' are there in the sequence 11, 21, 31, 41, ..., 981, 991?

A 0 B 8 C 87 D 92 E 99

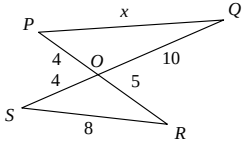
23. $PQRS$ is a square. The points T and U are the midpoints of QR and RS respectively. The line QS cuts PT and PU at W and V respectively. What fraction of the area of the square $PQRS$ is the area of the pentagon $RTWVU$?



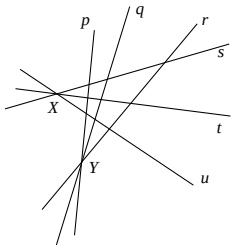
A $\frac{1}{3}$ B $\frac{2}{5}$ C $\frac{3}{7}$ D $\frac{5}{12}$ E $\frac{4}{15}$

24. The diagram shows two straight lines PR and QS crossing at O.

What is the value of x?



A $7\sqrt{2}$ B $2\sqrt{29}$ C $14\sqrt{2}$ D $7(1+\sqrt{13})$ E $9\sqrt{2}$

25. 

Challengeborough's underground train network consists of six lines, p, q, r, s, t, u, as shown. Wherever two lines meet there is a station which enables passengers to change lines. On each line, each train stops at every station.

Jessica wants to travel from station X to station Y. She does not want to use any line more than once, nor return to station X after leaving it, nor leave station Y having reached it.

How many different routes, satisfying these conditions, can she choose?

A 9 B 36 C 41 D 81 E 720

UK SENIOR MATHEMATICAL CHALLENGE

Thursday 7 November 2013

Organised by the **United Kingdom Mathematics Trust**

and supported by

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

1. Which of these is the largest number?

   A $2+0+1+3$ B $2\times0+1+3$ C $2+0\times1+3$
   D $2+0+1\times3$ E $2\times0\times1\times3$

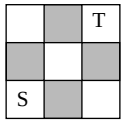
2. Little John claims he is 2m 8cm and 3mm tall. What is this height in metres?

   A 2.83m B 2.803m C 2.083m D 2.0803m E 2.0083m

3. What is the 'tens' digit of $2013^2 - 2013$?

   A 0 B 1 C 4 D 5 E 6

4. A route on the 3×3 board shown consists of a number of steps. Each step is from one square to an adjacent square of a different colour. How many different routes are there from square S to square T which pass through every other square exactly once?



   A 0 B 1 C 2 D 3 E 4

5. The numbers x and y satisfy the equations $x(y+2) = 100$ and $y(x+2) = 60$. What is the value of $x - y$?

   A 60 B 50 C 40 D 30 E 20

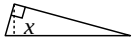
6. Rebecca went swimming yesterday. After a while she had covered one fifth of her intended distance. After swimming six more lengths of the pool, she had covered one quarter of her intended distance. How many lengths of the pool did she intend to complete?

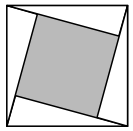
   A 40 B 72 C 80 D 100 E 120

7. In a 'ninety nine' shop, all items cost a number of pounds and 99 pence. Susanna spent £65.76. How many items did she buy?

   A 23 B 24 C 65 D 66 E 76

8. The right-angled triangle shown has a base which is 4 times its height. Four such triangles are placed so that their hypotenuses form the boundary of a large square as shown.
   What is the side-length of the shaded square in the diagram?





   A $2x$ B $2\sqrt{2}x$ C $3x$ D $2\sqrt{3}x$ E $\sqrt{15}x$

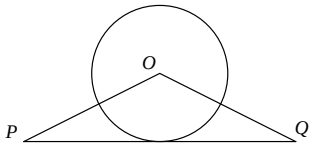
9. According to a headline, 'Glaciers in the French Alps have lost a quarter of their area in the past 40 years'. What is the approximate percentage reduction in the length of the side of a square when it loses one quarter of its area, thereby becoming a smaller square?

   A 13% B 25% C 38% D 50% E 65%

10. Frank's teacher asks him to write down five integers such that the median is one more than the mean, and the mode is one greater than the median. Frank is also told that the median is 10. What is the smallest possible integer that he could include in his list?

    A 3 B 4 C 5 D 6 E 7

11. The diagram shows a circle with centre O and a triangle OPQ. Side PQ is a tangent to the circle. The area of the circle is equal to the area of the triangle. What is the ratio of the length of PQ to the circumference of the circle?



    A 1 : 1 B 2 : 3 C $2:\pi$ D 3 : 2 E $\pi:2$

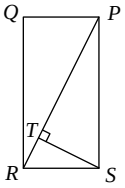
12. As a special treat, Sammy is allowed to eat five sweets from his very large jar which contains many sweets of each of three flavours − Lemon, Orange and Strawberry. He wants to eat his five sweets in such a way that no two consecutive sweets have the same flavour. In how many ways can he do this?

    A 32 B 48 C 72 D 108 E 162

13. Two entrants in a school's sponsored run adopt different tactics. Angus walks for half the time and runs for the other half, whilst Bruce walks for half the distance and runs for the other half. Both competitors walk at 3mph and run at 6mph. Angus takes 40 minutes to complete the course. How many minutes does Bruce take?

    A 30 B 35 C 40 D 45 E 50

14. The diagram shows a rectangle $PQRS$ in which $PQ : QR = 1 : 2$. The point T on PR is such that ST is perpendicular to PR. What is the ratio of the area of the triangle RST to the area of the rectangle $PQRS$?



    A $1:4\sqrt{2}$ B 1 : 6 C 1 : 8
    D 1 : 10 E 1 : 12

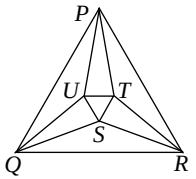
15. For how many positive integers n is $4^n - 1$ a prime number?

    A 0 B 1 C 2 D 3 E infinitely many

16. Andrew states that every composite number of the form $8n + 3$, where n is an integer, has a prime factor of the same form. Which of these numbers is an example showing that Andrew's statement is false?

    A 19 B 33 C 85 D 91 E 99

17. The equilateral triangle PQR has side-length 1. The lines PT and PU trisect the angle RPQ, the lines RS and RT trisect the angle QRP and the lines QS and QU trisect the angle PQR.
    What is the side-length of the equilateral triangle STU?



    A $\dfrac{\cos 80^\circ}{\cos 20^\circ}$ B $\frac{1}{3}\cos 20^\circ$ C $\cos^2 20^\circ$
    D $\frac{1}{6}$ E $\cos 20^\circ \cos 80^\circ$

18. The numbers 2, 3, 12, 14, 15, 20, 21 may be divided into two sets so that the product of the numbers in each set is the same. What is this product?

    A 420 B 1260 C 2520 D 6720 E 6350400