2011

1. Which of the numbers below is not a whole number?

   A $\frac{2011+0}{1}$ B $\frac{2011+1}{2}$ C $\frac{2011+2}{3}$ D $\frac{2011+3}{4}$ E $\frac{2011+4}{5}$

2. Jack and Jill went up the hill to fetch a pail of water. Having filled the pail to the full, Jack fell down, spilling $\frac{2}{3}$ of the water, before Jill caught the pail. She then tumbled down the hill, spilling $\frac{2}{5}$ of the remainder.

   What fraction of the pail does the remaining water fill?

   A $\frac{11}{15}$ B $\frac{1}{3}$ C $\frac{4}{15}$ D $\frac{1}{5}$ E $\frac{1}{15}$

3. The robot *Lumber9* moves along the number line. *Lumber9* starts at 0, takes 1 step forward (to 1), then 2 steps backward (to −1), then 3 steps forward, 4 steps backward, and so on, moving alternately forwards and backwards, one more step each time. At what number is *Lumber9* after 2011 steps in total?

   A 1006 B 27 C 11 D 0 E −18

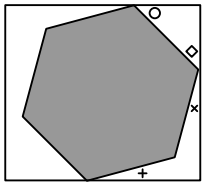
4. What is the last digit of 3^{2011} ?

   A 1 B 3 C 5 D 7 E 9

5. The diagram shows a regular hexagon inside a rectangle.

   What is the sum of the four marked angles?

   A 90° B 120° C 150° D 180° E 210°



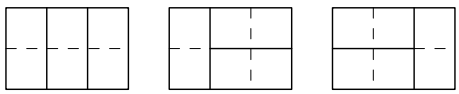
6. Granny and her granddaughter Gill both had their birthday yesterday. Today, Granny's age in years is an even number and 15 times that of Gill. In 4 years' time Granny's age in years will be the square of Gill's age in years. How many years older than Gill is Granny today?

   A 42 B 49 C 56 D 60 E 64

7. Two sides of a triangle have lengths 4 cm and 5 cm. The third side has length x cm, where x is a positive integer. How many different values can x have?

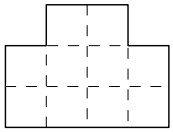
   A 4 B 5 C 6 D 7 E 8

8. A 2 × 3 grid of squares can be divided into 1 × 2 rectangles in three different ways.



   How many ways are there of dividing the bottom shape into 1 × 2 rectangles?

   A 1 B 4 C 6 D 7 E 8



9. Sam has a large collection of 1 × 1 × 1 cubes, each of which is either red or yellow. Sam makes a 3 × 3 × 3 block from twenty-seven cubes, so that no cubes of the same colour meet face-to-face.

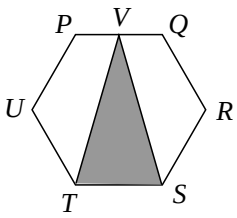
   What is the difference between the largest number of red cubes that Sam can use and the smallest number?

   A 0 B 1 C 2 D 3 E 4

10. A triangle has two edges of length 5. What length should be chosen for the third side of the triangle so as to maximise the area within the triangle?

A 5 B 6 C $5\sqrt{2}$ D 8 E $5\sqrt{3}$

11. $PQRSTU$ is a regular hexagon and V is the midpoint of PQ.
What fraction of the area of $PQRSTU$ is the area of triangle STV?

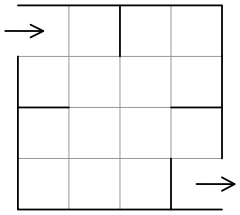


A $\frac{1}{4}$ B $\frac{2}{15}$ C $\frac{1}{3}$ D $\frac{2}{5}$ E $\frac{5}{12}$

12. The *primorial* of a number is the product of all of the prime numbers less than or equal to that number. For example, the primorial of 6 is $2 \times 3 \times 5 = 30$. How many different whole numbers have a primorial of 210?

A 1 B 2 C 3 D 4 E 5

13. The diagram represents a maze. Given that you can only move horizontally and vertically and are not allowed to revisit a square, how many different routes are there through the maze?



A 16 B 12 C 10 D 8 E 6

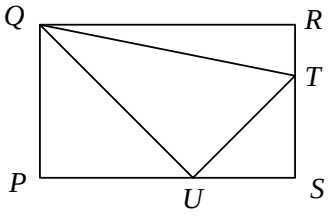
14. An equilateral triangle of side length 4 cm is divided into smaller equilateral triangles, all of which have side length equal to a whole number of centimetres. Which of the following cannot be the number of smaller triangles obtained?

A 4 B 8 C 12 D 13 E 16

15. The equation $x^2 + ax + b = 0$, where a and b are different, has solutions $x = a$ and $x = b$. How many such equations are there?

A 0 B 1 C 3 D 4 E an infinity

16. $PQRS$ is a rectangle. The area of triangle QRT is $\frac{1}{5}$ of the area of $PQRS$, and the area of triangle TSU is $\frac{1}{8}$ of the area of $PQRS$. What fraction of the area of rectangle $PQRS$ is the area of triangle QTU?



A $\frac{27}{40}$ B $\frac{21}{40}$ C $\frac{1}{2}$ D $\frac{19}{40}$ E $\frac{23}{60}$

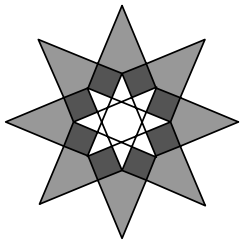
17. Jamie conducted a survey on the food preferences of pupils at a school and discovered that 70% of the pupils like pears, 75% like oranges, 80% like bananas and 85% like apples. What is the smallest possible percentage of pupils who like all four of these fruits?

A at least 10% B at least 15% C at least 20%
D at least 25% E at least 70%

18. Two numbers x and y are such that $x + y = 20$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$. What is the value of $x^2y + xy^2$?

A 80 B 200 C 400 D 640 E 800

19. The diagram shows a small regular octagram (an eight-sided star) surrounded by eight squares (dark grey) and eight kites (light grey) to make a large regular octagram. Each square has area 1.



What is the area of one of the light grey kites?

A 2 B $\sqrt{2}+1$ C $\frac{21}{8}$ D $4\sqrt{2}-3$ E $\frac{11}{4}$

20. Positive integers x and y satisfy the equation $\sqrt{x} - \sqrt{11} = \sqrt{y}$.
What is the maximum possible value of $\frac{x}{y}$?

A 2 B 4 C 8 D 11 E 44

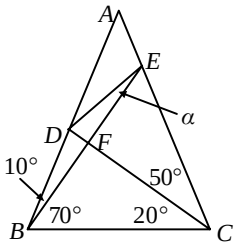
21. Each of the Four Musketeers made a statement about the four of them, as follows.

d'Artagnan: "Exactly one is lying."
Athos: "Exactly two of us are lying."
Porthos: "An odd number of us is lying."
Aramis: "An even number of us is lying."

How many of them were lying (with the others telling the truth)?

A one B one or two C two or three D three E four

22. In the diagram, $\angle ABE = 10°$; $\angle EBC = 70°$; $\angle ACD = 50°$; $\angle DCB = 20°$; $\angle DEF = \alpha$.



Which of the following is equal to $\tan\alpha$?

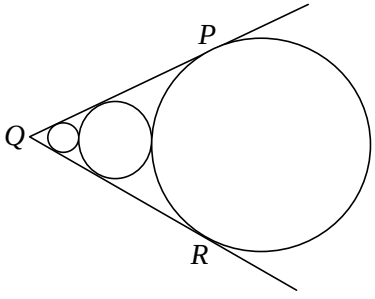
A $\frac{\tan 10° \tan 20°}{\tan 50°}$ B $\frac{\tan 10° \tan 20°}{\tan 70°}$ C $\frac{\tan 10° \tan 50°}{\tan 70°}$

D $\frac{\tan 20° \tan 50°}{\tan 70°}$ E $\frac{\tan 10° \tan 70°}{\tan 50°}$

23. What is the minimum value of $x^2 + y^2 + 2xy + 6x + 6y + 4$?

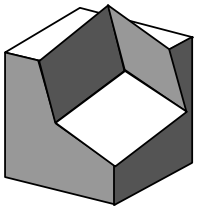
A -7 B -5 C -4 D -1 E 4

24. Three circles and the lines PQ and QR touch as shown. The distance between the centres of the smallest and the biggest circles is 16 times the radius of the smallest circle. What is the size of $\angle PQR$?



A 45° B 60° C 75° D 90° E 135°

25. A solid sculpture consists of a $4 \times 4 \times 4$ cube with a $3 \times 3 \times 3$ cube sticking out, as shown. Three vertices of the smaller cube lie on edges of the larger cube, the same distance along each.



What is the total volume of the sculpture?

A 79 B 81 C 82 D 84 E 85