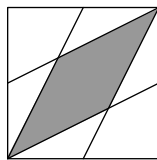
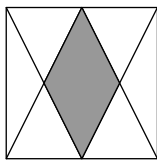


19. The diagrams show two different shaded rhombuses, each inside a square with sides of length 6.

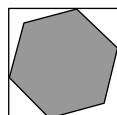


Each rhombus is formed by joining vertices of the square to midpoints of the sides of the square. What is the difference between the shaded areas?

- A 4 B 3 C 2 D 1 E 0
20. There are 10 girls in a mixed class. If two pupils from the class are selected at random to represent the class on the School Council, then the probability that both are girls is 0.15. How many boys are in the class?

- A 10 B 12 C 15 D 18 E 20

21. The diagram shows a regular hexagon, with sides of length 1, inside a square. Two vertices of the hexagon lie on a diagonal of the square and the other four lie on the edges.



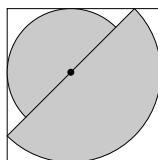
What is the area of the square?

- A $2 + \sqrt{3}$ B 4 C $3 + \sqrt{2}$ D $1 + \frac{3\sqrt{3}}{2}$ E $\frac{7}{2}$

22. If $x^2 - px - q = 0$, where p and q are positive integers, which of the following could not equal x^3 ?

- A $4x + 3$ B $8x + 5$ C $8x + 7$ D $10x + 3$ E $26x + 5$

23. The diagram shows two different semicircles inside a square with sides of length 2. The common centre of the semicircles lies on a diagonal of the square.



What is the total shaded area?

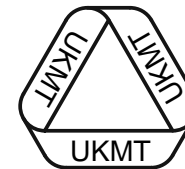
- A π B $6\pi(3 - 2\sqrt{2})$ C $\pi\sqrt{2}$ D $3\pi(2 - \sqrt{2})$ E $8\pi(2\sqrt{2} - 3)$

24. Three spheres of radius 1 are placed on a horizontal table and inside a vertical hollow cylinder of height 2 units which is just large enough to surround them. What fraction of the internal volume of the cylinder is occupied by the spheres?

- A $\frac{2}{7 + 4\sqrt{3}}$ B $\frac{2}{2 + \sqrt{3}}$ C $\frac{1}{3}$ D $\frac{3}{2 + \sqrt{3}}$ E $\frac{6}{7 + 4\sqrt{3}}$

25. All the digits of a number are different, the first digit is not zero, and the sum of the digits is 36. There are $N \times 7!$ such numbers. What is the value of N ?

- A 72 B 97 C 104 D 107 E 128



UK SENIOR MATHEMATICAL CHALLENGE

Thursday 4 November 2010

Organised by the **United Kingdom Mathematics Trust**

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The Actuarial Profession

making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

- Do not open the question paper until the invigilator tells you to do so.
- Use B or HB pencil only.** Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
- Time allowed: **90 minutes.**
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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<http://www.ukmt.org.uk>

1. What is the digit x in this cross-number?

Across

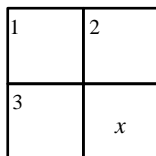
Down

1. A cube

1. One less than a cube

3. A cube

A 2 B 3 C 4 D 5 E 6



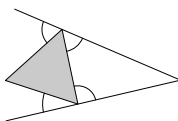
2. What is the smallest possible value of $20p + 10q + r$ when p, q and r are *different* positive integers?

A 31 B 43 C 53 D 63 E 2010

3. The diagram shows an equilateral triangle touching two straight lines.

What is the sum of the four marked angles?

A 120° B 180° C 240° D 300° E 360°



4. The year 2010 is one in which the sum of its digits is a factor of the year itself. How many more years will it be before this is next the case?

A 3 B 6 C 9 D 12 E 15

5. A notice on Morecambe promenade reads: 'It would take 20 million years to fill Morecambe Bay from a bath tap.' Assuming that the flow from the bath tap is 6 litres a minute, what does the notice imply is the approximate capacity of Morecambe Bay in litres?

A 6×10^{10} B 6×10^{11} C 6×10^{12} D 6×10^{13} E 6×10^{14}

6. Dean runs up a mountain road at 8 km per hour. It takes him one hour to get to the top. He runs down the same road at 12 km per hour. How many minutes does it take him to run down the mountain?

A 30 B 40 C 45 D 50 E 90

7. There are 120 different arrangements of the five letters in the word ANGLE. If all 120 are listed in alphabetical order starting with AEGLN and finishing with NLGEA, which position in the list does ANGLE occupy?

A 18th B 20th C 22nd D 24th E 26th

8. Which of the following is equivalent to $(x + y + z)(x - y - z)$?

A $x^2 - y^2 - z^2$ B $x^2 - y^2 + z^2$ C $x^2 - xy - xz - z^2$
 D $x^2 - (y + z)^2$ E $x^2 - (y - z)^2$

9. The symbol \diamond is defined by $x \diamond y = x^y - y^x$. What is the value of $(2 \diamond 3) \diamond 4$?

A -3 B $-\frac{3}{4}$ C 0 D $\frac{3}{4}$ E 3

10. A square is cut into 37 squares of which 36 have area 1 cm^2 . What is the length of the side of the *original* square?

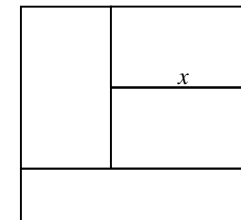
A 6 cm B 7 cm C 8 cm D 9 cm E 10 cm

11. What is the median of the following numbers?

A $9\sqrt{2}$ B $3\sqrt{19}$ C $4\sqrt{11}$ D $5\sqrt{7}$ E $6\sqrt{5}$

12. The diagram, which is not to scale, shows a square with side length 1, divided into four rectangles whose areas are equal. What is the length labelled x ?

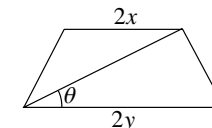
A $\frac{2}{3}$ B $\frac{17}{24}$ C $\frac{4}{5}$ D $\frac{49}{60}$ E $\frac{5}{6}$



13. How many two-digit numbers have remainder 1 when divided by 3 and remainder 2 when divided by 4?

A 8 B 7 C 6 D 5 E 4

14. The parallel sides of a trapezium have lengths $2x$ and $2y$ respectively. The diagonals are equal in length, and a diagonal makes an angle θ with the parallel sides, as shown. What is the length of each diagonal?



A $x + y$ B $\frac{x + y}{\sin \theta}$ C $(x + y) \cos \theta$ D $(x + y) \tan \theta$ E $\frac{x + y}{\cos \theta}$

15. What is the smallest prime number that is equal to the sum of two prime numbers and is also equal to the sum of three different prime numbers?

A 7 B 11 C 13 D 17 E 19

16. $PQRS$ is a quadrilateral inscribed in a circle of which PR is a diameter. The lengths of PQ, QR and RS are 60, 25 and 52 respectively. What is the length of SP ?

A $21\frac{2}{3}$ B $28\frac{11}{13}$ C 33 D 36 E 39

17. One of the following is equal to $\sqrt{9^{16x^2}}$ for all values of x . Which one?

A 3^{4x} B 3^{4x^2} C 3^{8x^2} D 9^{4x} E 9^{8x^2}

18. A solid cube of side 2 cm is cut into two triangular prisms by a plane passing through four vertices, as shown. What is the total surface area of these two prisms?

A $8(3 + \sqrt{2})$ B $2(8 + \sqrt{2})$ C $8(3 + 2\sqrt{2})$
 D $16(3 + \sqrt{2})$ E $8\sqrt{2}$

