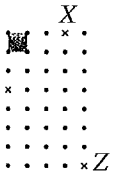
18. The shaded square of the lattice shown has area 1. What is the area of the circle through the points X, Y and Z?

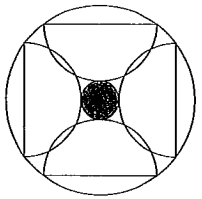


A $\frac{9\pi}{2}$ B 8π C $\frac{25\pi}{2}$ D 25π E 50π

19. How many prime numbers p are there such that $199p + 1$ is a perfect square?

A 0 B 1 C 2 D 4 E 8

20. The diagram shows four semicircles symmetrically placed between two circles. The shaded circle has area 4 and each semicircle has area 18. What is the area of the outer circle?

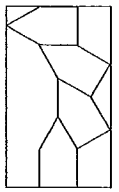


A $72\sqrt{2}$ B 100 C 98 D 96 E $32\sqrt{3}$

21. The fraction $\frac{2008}{1998}$ may be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$ where a, b, c and d are positive integers. What is the value of d?

A 2 B 4 C 5 D 199 E 1998

22. A pentagon is made by attaching an equilateral triangle to a square with the same edge length. Four such pentagons are placed inside a rectangle, as shown.
What is the ratio of the length of the rectangle to its width?



A $\sqrt{3}$:1 B 2:1 C $\sqrt{2}$:1 D 3:2 E 4:$\sqrt{3}$

23. How many pairs of real numbers (x, y) satisfy the equation $(x + y)^2 = (x + 3)(y - 3)$?

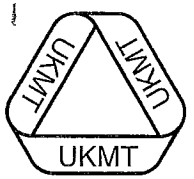
A 0 B 1 C 2 D 4 E infinitely many

24. The length of the hypotenuse of a particular right-angled triangle is given by $\sqrt{1+3+5+7+\ \ldots\ +25}$. The lengths of the other two sides are given by $\sqrt{1+3+5+\ \ldots\ +x}$ and $\sqrt{1+3+5+\ \ldots\ +y}$ where x and y are positive integers. What is the value of $x + y$?

A 12 B 17 C 24 D 28 E 32

25. What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $||x| - 2| + ||y| - 2| \leqslant 4$?

A 24 B 32 C 64 D 96 E 112



UK SENIOR MATHEMATICAL CHALLENGE

Thursday 6 November 2008

Organised by the **United Kingdom Mathematics Trust**

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The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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1. What is the value of $2 \times 2008 + 2008 \times 8$?

   A 4016 B 16064 C 20080 D 64256 E 80020

2. A giant thresher shark weighing 1250 pounds, believed to be the heaviest ever caught, was landed by fisherman Roger Nowell off the Cornish coast in November 2007. The fish was sold by auction at Newlyn Fish Market for £255. Roughly, what was the cost per pound?

   A 5p B 20p C 50p D £2 E £5

3. What is the value of $\sqrt{\frac{1}{2^6} + \frac{1}{6^2}}$?

   A $\frac{1}{10}$ B $\frac{1}{9}$ C $\frac{1}{3}$ D $\frac{5}{24}$ E $\frac{7}{24}$

4. In this subtraction, P, Q, R and S are digits. What is the value of $P+Q+R+S$?

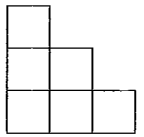
$$\begin{array}{r} 8\ Q\ 0\ S \\ -\ P\ 0\ R\ 2 \\ \hline 2\ 0\ 0\ 8 \end{array}$$

   A 12 B 14 C 16 D 18 E 20

5. 200 T-shirts have been bought for a Fun Run at a cost of £400 plus VAT at $17\frac{1}{2}\%$. The cost of entry for the run is £5 per person. What is the minimum number of entries needed in order to cover the total cost of the T-shirts?

   A 40 B 47 C 80 D 84 E 94

6. It is required to shade at least one of the six small squares in the diagram on the right so that the resulting figure has exactly one axis of symmetry. In how many different ways can this be done?

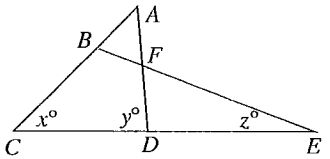


   A 6 B 9 C 10 D 12 E 15

7. A newspaper headline read 'Welsh tortoise recaptured 1.8 miles from home after 8 months on the run'. Assuming the tortoise travelled in a straight line, roughly how many minutes did the tortoise take on average to 'run' one foot?
   [1 mile = 5280 feet]

   A 3 B 9 C 16 D 36 E 60

8. In the figure shown, $AB = AF$ and ABC, AFD, BFE and CDE are all straight lines. Which of the following expressions gives z in terms of x and y?



   A $\frac{y-x}{2}$ B $y-\frac{x}{2}$ C $\frac{y-x}{3}$ D $y-\frac{x}{3}$ E $y-x$

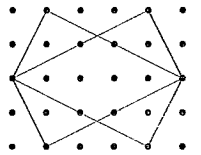
9. What is the remainder when the 2008-digit number 222 ... 22 is divided by 9?

   A 8 B 6 C 4 D 2 E 0

10. Which one of the following rational numbers *cannot* be expressed as $\frac{1}{m} + \frac{1}{n}$ where m, n are different positive integers?

    A $\frac{3}{4}$ B $\frac{3}{5}$ C $\frac{3}{6}$ D $\frac{3}{7}$ E $\frac{3}{8}$

11. The distance between two neighbouring dots in the dot lattice is 1 unit. What, in square units, is the area of the region where the two rectangles overlap?



    A 6 B $6\frac{1}{4}$ C $6\frac{1}{2}$ D 7 E $7\frac{1}{2}$

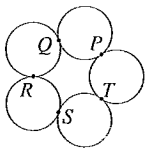
12. Mr and Mrs Stevens were married on a Saturday in July 1948. On what day of the week did their diamond wedding anniversary fall this year?

    A Monday B Tuesday C Thursday D Friday E Saturday

13. Positive integers m and n are such that $2^m + 2^n = 1280$. What is the value of $m+n$?

    A 14 B 16 C 18 D 32 E 640

14. Five touching circles each have radius 1 and their centres are at the vertices of a regular pentagon. What is the radius of the circle through the points of contact P, Q, R, S and T?



    A $\tan 18°$ B $\tan 36°$ C $\tan 45°$ D $\tan 54°$ E $\tan 72°$

15. A sequence of positive integers t_1, t_2, t_3, t_4, ... is defined by:
    $t_1 = 13$; $t_{n+1} = \frac{1}{2}t_n$ if t_n is even; $t_{n+1} = 3t_n + 1$ if t_n is odd.
    What is the value of t_{2008}?

    A 1 B 2 C 4 D 8 E None of these.

16. The numbers x, y and z satisfy the equations

$$x + y + 2z = 850, \quad x + 2y + z = 950, \quad 2x + y + z = 1200.$$

    What is their mean?

    A 250 B $\frac{1000}{3}$ C 750 D 1000 E More information is needed.

17. Andy and his younger cousin Alice both have their birthdays today. Remarkably, Andy is now the same age as the sum of the digits of the year of his birth and the same is true of Alice. How many years older than Alice is Andy?

    A 10 B 12 C 14 D 16 E 18