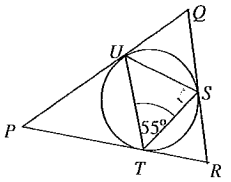
18. The year 1789 (when the French Revolution started) has three and no more than three adjacent digits (7, 8 and 9) which are consecutive integers in increasing order. How many years between 1000 and 9999 have this property?

A 130 B 142 C 151 D 169 E 180

19. The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U, as shown in the diagram.
The size of $\angle STU = 55°$. What is the size of $\angle PQR$?



A 55° B 60° C 65° D 70° E 75°

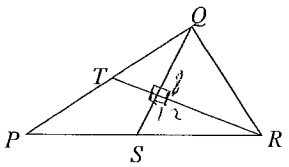
20. A triangle is cut from the corner of a rectangle. The resulting pentagon has sides of length 8, 10, 13, 15 and 20 units, though not necessarily in that order. What is the area of the pentagon?

A 252.5 B 260 C 270 D 275.5 E 282.5

21. A bracelet is to be made by threading four identical red beads and four identical yellow beads onto a hoop. How many different bracelets can be made?

A 4 B 8 C 12 D 18 E 24

22. In triangle PQR, S and T are the midpoints of PR and PQ respectively; QS is perpendicular to RT; $QS = 8$; $RT = 12$.
What is the area of triangle PQR?



A 24 B 32 C 48 D 64 E 96

23. The sum of the lengths of the 12 edges of a cuboid is x cm. The distance from one corner of the cuboid to the furthest corner is y cm. What, in cm^2, is the total surface area of the cuboid?

A $\frac{x^2 - 2y^2}{2}$ B $x^2 + y^2$ C $\frac{x^2 - 4y^2}{4}$ D $\frac{xy}{6}$ E $\frac{x^2 - 16y^2}{16}$

24. A paperweight is made from a glass cube of side 2 units by first shearing off the eight tetrahedral corners which touch at the midpoints of the edges of the cube. The remaining inner core of the cube is discarded and replaced by a sphere. The eight corner pieces are now stuck onto the sphere so that they have the same positions relative to each other as they did originally. What is the diameter of the sphere?

A $\sqrt{8} - 1$ B $\sqrt{8} + 1$ C $\frac{1}{3}(6 + \sqrt{3})$ D $\frac{4}{3}\sqrt{3}$ E $2\sqrt{3}$

25. The line with equation $y = x$ is an axis of symmetry of the curve with equation $y = \frac{px + q}{rx + s}$, where p, q, r, s are all non-zero. Which of the following is necessarily true?

A $p + q = 0$ B $r + s = 0$ C $p + r = 0$ D $p + s = 0$ E $q + r = 0$

UK SENIOR MATHEMATICAL CHALLENGE

Thursday 8 November 2007

Organised by the **United Kingdom Mathematics Trust**

and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

The United Kingdom Mathematics Trust is a Registered Charity.
http://www.ukmt.org.uk

1. What is the value of $\frac{2007}{9} + \frac{7002}{9}$?

   A 500.5 B 545 C 1001 D 1655 E 2007

2. This morning Sam told Pat "I am getting married today, aged 30." From this information, Pat may correctly deduce that Sam was born in:

   A 1976 or 1977 B 1977 C 1978 D 1979 E 1977 or 1978

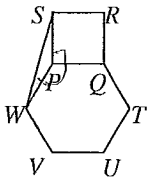
3. What is the value of $2006 \times 2008 - 2007 \times 2007$?

   A -2007 B -1 C 0 D 1 E 4 026 042

4. The diagram shows square $PQRS$ and regular hexagon $PQTUVW$.

   What is the size of $\angle PSW$?

   A 10° B 12° C 15° D 24° E 30°



5. Which of the five expressions shown has a different value from the other four?

   A 2^8 B 4^4 C $8^{8/3}$ D 16^2 E $32^{6/5}$

6. Cheryl finds a bag of coins. There are 50 coins inside and the value of the contents is £1.81. Given that it contains only two-pence and five-pence coins, how many more five-pence coins are there inside the bag than two-pence coins?

   A 4 B 6 C 8 D 10 E 12

7. How many whole numbers between 1 and 2007 are divisible by 2 but not by 7?

   A 857 B 858 C 859 D 860 E 861

8. Travelling at an average speed of 100 km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train then waited just outside the station, which reduced the average speed for the whole journey to 90 km/hr. For how many minutes was the train waiting?

   A 1 B 5 C 10 D 15 E 20

9. In a sale, a shopkeeper reduced the advertised selling price of a dress by 20%. This resulted in a profit of 4% over the cost price of the dress. What percentage profit would the shopkeeper have made if the dress had been sold at the original selling price?

   A 16% B 20% C 24% D 25% E 30%

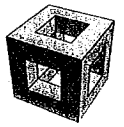
10. In 1954, a total of 6 527 mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23 450 m^2. Roughly how many million litres of water fell on Sprinkling Tarn in 1954?

    A 15 B 150 C 1 500 D 15 000 E 150 000

11. A $4 \times 4 \times 4$ cube has three $2 \times 2 \times 4$ holes drilled symmetrically all the way through, as shown.

    What is the surface area of the resulting solid?

    A 192 B 144 C 136 D 120 E 96



12. How many two-digit numbers N have the property that the sum of N and the number formed by reversing the digits of N is a square?

    A 2 B 5 C 6 D 7 E 8

13. Which of the following gives the exact number of seconds in the last six complete weeks of 2007?

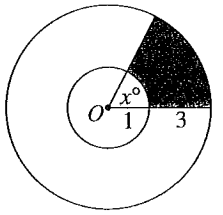
    A 9! B 10! C 11! D 12! E 13!

    {Note that $n! = 1 \times 2 \times 3 \times \ldots \times n$*.}*

14. The point O is the centre of both circles and the shaded area is one-sixth of the area of the outer circle.

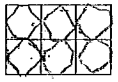
    What is the value of x?

    A 60 B 64 C 72 D 80 E 84



15. How many hexagons can be found in the diagram on the right if each side of a hexagon must consist of all or part of one of the straight lines in the diagram?

    A 4 B 8 C 12 D 16 E 20



16. Damien wishes to find out if 457 is a prime number. In order to do this he needs to check whether it is exactly divisible by some prime numbers. What is the smallest number of possible prime number divisors that Damien needs to check before he can be sure that 457 is a prime number?

    A 8 B 9 C 10 D 11 E 12

17. The two triangles have equal areas and the four marked lengths are equal.

    What is the value of x?

    A 30 B 45 C 60 D 75 E more information needed