19. An engineer is directed to a faulty signal, one quarter of the way into a tunnel. Whilst there, he is warned of a train heading towards the tunnel entrance. The engineer can run at 12 mph and can either run back to the tunnel entrance or forward to the exit. In either case, the engineer and the front of the train would reach the entrance or exit together. What is the speed in mph of the train?
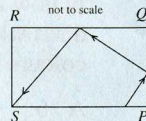
A 16   B 20   C 24   D 32   E more information needed

20. A positive number $a = [a] + \{a\}$ where $[a]$ is the integer part of a and $\{a\}$ is the fractional part of a.

Given that $x + [y] + \{z\} = 4.2$, $y + [z] + \{x\} = 3.6$, $z + [x] + \{y\} = 2.0$, and $x, y, z > 0$, what is the value of $\{y\}$?

A 0.1   B 0.3   C 0.5   D 0.7   E 0.9

21. A toy pool table is 6 feet long and 3 feet wide. It has pockets at each of the four corners P, Q, R and S. When a ball hits a side of the table, it bounces off the side at the same angle as it hit that side. A ball, initially 1 foot to the left of pocket P, is hit from the side SP towards the side PQ as shown.
How many feet from P does the ball hit side PQ if it lands in pocket S after two bounces?

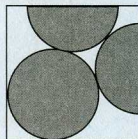A 1   B $\dfrac{6}{7}$ C $\dfrac{3}{4}$ D $\dfrac{2}{3}$ E $\dfrac{3}{5}$

22. Which positive integer n satisfies the equation

$$\frac{3}{n^3} + \frac{4}{n^3} + \frac{5}{n^3} + \cdots + \frac{n^3 - 5}{n^3} + \frac{n^3 - 4}{n^3} + \frac{n^3 - 3}{n^3} = 60 ?$$

A 5   B 11   C 31   D 60   E 2006

23. In the diagram, the circle and the two semicircles have radius 1.
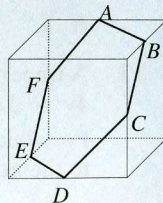What is the perimeter of the square?

A $6 + 4\sqrt{2}$ B $2 + 4\sqrt{2} + 2\sqrt{3}$ C $3\sqrt{2} + 4\sqrt{3}$
   D $4 + 2\sqrt{2} + 2\sqrt{6}$ E 12

24. A solid red plastic cube, volume $1\,\text{cm}^3$, is painted white on its outside. The cube is cut by a plane passing through the mid-points of various edges, as shown.
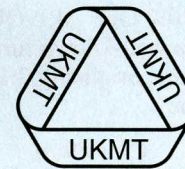What, in cm^2, is the *total* red area exposed by the cut?

A $\dfrac{3\sqrt{3}}{2}$ B 2   C $\dfrac{9\sqrt{2}}{5}$ D 3   E $\dfrac{3(\sqrt{3} + \sqrt{2})}{4}$

25. X is a positive integer in which each digit is 1; that is, X is of the form $11111\ldots$.
Given that every digit of the integer $pX^2 + qX + r$ (where p, q and r are fixed integer coefficients and $p > 0$) is also 1, irrespective of the number of digits X, which of the following is a possible value of q?

A -2 B -1 C 0   D 1   E 2

UK SENIOR MATHEMATICAL CHALLENGE

Thursday 9 November 2006

Organised by the **United Kingdom Mathematics Trust**

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The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
       0 marks are awarded for each question left unanswered;
       4 marks are awarded for each correct answer;
       1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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1. The promotion 'AMAZING! 20% OFF ALL OUR BEDFRAMES' appears on the cover of the 2006 brochure of a well-known furniture company. If 20% were to be taken off the length of a bedframe originally 2.10 m long, what would be the resulting length of the bedframe?

   A 2.00 m   B 1.90 m   C 1.89 m   D 1.78 m   E 1.68 m
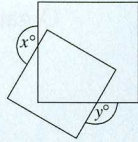
2. If $6x - y = 21$ and $6y - x = 14$, what is the value of $x - y$?

   A 1   B 2   C 3   D 4   E 5

3. The diagram shows overlapping squares.

   What is the value of $x + y$?

   A 270   B 300   C 330

   D 360   E more information needed

4. What is the value of $\sqrt{2^4} + \sqrt{3^4}$?

   A 4   B $\sqrt{20}$ C 5   D 7   E $\sqrt{97}$

5. Given that January 1st, 2006 fell on a Sunday, which day of the week will occur most frequently in 2007?

   A Monday   B Tuesday   C Wednesday D Thursday   E Friday

6. Which symbol should replace ⊕ to make the following equation true?

   $$1 \times 2 \times (3 \oplus 4 + 5) \times (6 \times 7 + 8 + 9) = 2006.$$

   A +   B −   C ÷   D ×   E none of these

7. The base of a pyramid has n edges. What is the difference between the number of edges the pyramid has and the number of faces the pyramid has?

   A $n - 2$ B $n - 1$ C n D $n + 1$ E $n + 2$

8. Matt black paint absorbs 97% of light, the remainder being reflected. Scientists have developed a new superblack coating, "10 times blacker" than matt black paint, meaning that it reflects $\frac{1}{10}$ of the light reflected by matt black paint. What percentage of light does the new coating absorb?

   A 9.7   B 90.3   C 99.7   D 99.9   E 970

9. The 80 spokes of the giant wheel *The London Eye* are made from 4 miles of cable. Roughly what is the circumference of the wheel in metres?

   A 50   B 100   C 500   D 750   E 900

10. The digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 are to be written in the squares so that every row and every column of three squares has a total of 13. Two numbers have already been entered. What is the value of n?

    A 2   B 4   C 6   D 7   E 8
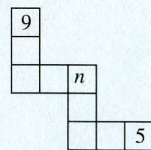
11. Three consecutive even numbers are such that the sum of four times the smallest and twice the largest exceeds three times the second by 2006. What is the sum of the digits of the smallest number?

    A 8   B 11   C 14   D 17   E 20

12. The factorial of n, written $n!$, is defined by $n! = 1 \times 2 \times 3 \times \ldots \times (n - 2) \times (n - 1) \times n$. Which of the following values of n provides a counterexample to the statement:

    "If n is a prime number, then $n! + 1$ is also a prime number"?
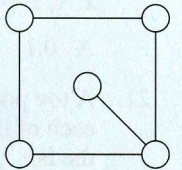
    A 1   B 2   C 3   D 4   E 5

13. The diagram shows five discs connected by five line segments. Three colours are available to colour these discs.

    In how many different ways is it possible to colour all five discs if discs which are connected by a line segment are to have different colours?

   A 6   B 12   C 30   D 36   E 48

14. Heather and Rachel each has some pennies. Heather has more than Rachel. In fact, the number of pennies that Heather has is the square of the number that Rachel has. The total number of pennies they have between them makes a whole number of pounds. What is the smallest this total could be?

    A £1   B £6   C £57   D £99   E £101

15. $PQRS$ is a square with U and V the mid-points of the sides PS and SR respectively. Line segments PV and UR meet at T.

    What fraction of the area of the square $PQRS$ is the area of the quadrilateral $PQRT$?

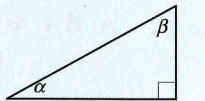   A $\frac{1}{2}$ B $\frac{5}{8}$ C $\frac{2}{3}$ D $\frac{3}{4}$ E $\frac{5}{9}$
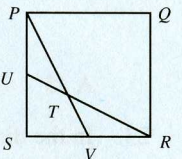
16. If $\alpha < \beta$, how many different values are there among the following expressions?

    $$\sin\alpha \sin\beta \qquad \sin\alpha \cos\beta \qquad \cos\alpha \sin\beta \qquad \cos\alpha \cos\beta$$

    A 1   B 2   C 3   D 4   E It depends on the value of α

17. A trapezium is bounded by four lines, the equations of which are $x = 0$, $x = 4$, $4y = 3x + 8$ and $y = k$, where $k < 2$.

    For which value of k is the numerical value of the perimeter of the trapezium equal to the numerical value of the area of the trapezium?

    A $\frac{3}{2}$ B 1   C $\frac{1}{2}$ D $-\frac{1}{2}$ E -1

18. What is the greatest number of the following five statements about numbers a, b which can be true at the same time?

    $$\frac{1}{a} < \frac{1}{b} \qquad a^2 > b^2 \qquad a < b \qquad a < 0 \qquad b < 0$$

    A 1   B 2   C 3   D 4   E 5