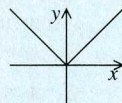


18. A cube exactly fits inside a sphere and another sphere exactly fits inside this cube. What is the ratio of the volume of the smaller sphere to the volume of the larger sphere?

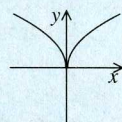
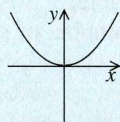
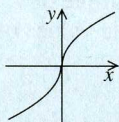
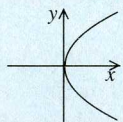
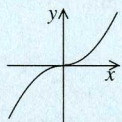
A $1:3\sqrt{3}$ B $1:4$ C $1:3$ D $2:3$ E $1:2$

19. The graph of $y = |x|$ is shown alongside.

Which of the following could be a sketch of the graph of $y = x|x|$?



A B C D E



20. It takes two weeks to clean the 3312 panes of glass in the 6000m^2 glass roof of the British Museum, a task performed once every two years. Assuming that all the panes are equilateral triangles of the same size, roughly how long is the side of each pane?

A 50 cm B 1 m C 2 m D 3 m E 4 m

21. What is the sum of the values of n for which both n and $\frac{n^2 - 9}{n - 1}$ are integers?

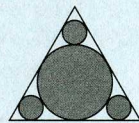
A -8 B -4 C 0 D 4 E 8

22. Given that $S = (x + 20) + (x + 21) + (x + 22) + \dots + (x + 100)$, where x is a positive integer, what is the smallest value of x such that S is a perfect square?

A 1 B 2 C 4 D 8 E 64

23. The diagram shows four touching circles, each of which also touches the sides of an equilateral triangle with sides of length 3. What is the area of the shaded region?

A $\frac{11\pi}{12}$ B π C $\frac{(4 + \sqrt{3})\pi}{6}$ D $\frac{(3 + \sqrt{3})\pi}{4}$ E $\frac{37\pi}{36}$

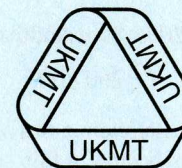


24. The factorial of n , written $n!$, is defined by $n! = 1 \times 2 \times 3 \times \dots \times (n - 2) \times (n - 1) \times n$. For how many positive integer values of k less than 50 is it impossible to find a value of n such that $n!$ ends in exactly k zeros?

A 0 B 5 C 8 D 9 E 10

25. Which of the following is equal to $\frac{1}{\sqrt{2005} + \sqrt{2005^2 - 1}}$?

A $\sqrt{1003} - \sqrt{1002}$ B $\sqrt{1005} - \sqrt{1004}$ C $\sqrt{1007} - \sqrt{1005}$
D $\sqrt{2005} - \sqrt{2003}$ E $\sqrt{2007} - \sqrt{2005}$



UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 8 November 2005

Organised by the **United Kingdom Mathematics Trust**

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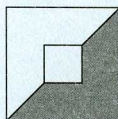
RULES AND GUIDELINES (to be read before starting)

- Do not open the question paper until the invigilator tells you to do so.
- Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
Once you have begun, record all your answers on the Answer Sheet.
- Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E . Only one of these is correct. Enter the letter $A-E$ corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

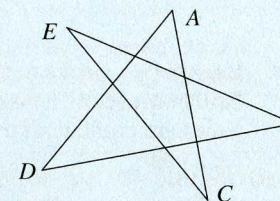
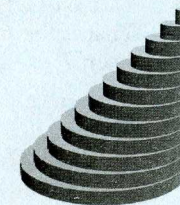
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<http://www.ukmt.org.uk>

- What is the value of 2005 plus 2005 thousandths?
A 2005.002005 B 2005.2005 C 2007.005 D 2025.05 E 2205.5
- The number 2005 is the sum of a sequence of five consecutive positive integers. Which of the following numbers occurs in this sequence?
A 395 B 400 C 405 D 410 E 415
- What is the mean of the five numbers 1^5 , 2^4 , 3^3 , 4^2 and 5^1 ?
A 6.2 B 11.4 C 12.2 D 13 E 13.8
- The diagram shows two squares, with sides of length 1 and 3, which have the same centre and corresponding sides parallel. What fraction of the larger square is shaded?
A $\frac{4}{9}$ B $\frac{4}{11}$ C $\frac{2}{5}$ D $\frac{2}{7}$ E $\frac{6}{11}$
- Last year Rachel took part in a swimathon. Every day for 9 weeks she swam the same number of lengths, either in a 25m indoor pool or a 20m outdoor pool. Later she discovered that she had swum the same total distance in each pool. On how many days did Rachel swim in the indoor pool?
A 45 B 42 C 35 D 32 E 28
- How many differently shaped triangles exist in which no two sides are the same length, each side is of integral unit length and the perimeter of the triangle is less than 13 units?
A 2 B 3 C 4 D 5 E 6
- Consider the arithmetic sequences 1998, 2005, 2012, ... and 1996, 2005, 2014, Which is the next number after 2005 that appears in both sequences?
A 2054 B 2059 C 2061 D 2063 E 2068
- An examination paper is made by taking 5 large sheets of paper, folding the pile in half and stapling it. The pages are then numbered in order from 1 to 20. What is the sum of the three page numbers that are on the same sheet of paper as page number 5?
A 13 B 21 C 33 D 37 E 41
- What is the value of the expression: $(1 + \frac{1}{2})(1 + \frac{1}{3})(1 + \frac{1}{4}) \dots (1 + \frac{1}{2004})(1 + \frac{1}{2005})$?
A 1001 B 1002 C 1003 D 1004 E 1005
- Sam and Pat were counting their money. They discovered that if Sam gave Pat £5, then Pat would have 5 times as much as Sam, but if Pat gave Sam £5, then Sam would have 5 times as much as Pat. How much did they have altogether?
A £10 B £15 C £20 D £25 E £30



- A sculpture is made up of 12 wooden cylinders, each of height 2cm. They are glued together as shown. The diameter of the top cylinder is 2cm and each of the other cylinders has a diameter 2cm more than the one immediately above it. The exhibit stands with its base on a marble table. What, in cm^2 , is the total surface area of the sculpture, excluding the base?
A 456π B 356π C 256π D 156π E 144π
- The positive integer x is a multiple of 7, and \sqrt{x} is between 15 and 16. What is the number of possible values of x ?
A 1 B 2 C 3 D 4 E 5
- In the figure shown, what is the sum of the interior angles at A, B, C, D, E?
A 90° B 135° C 150° D 180°
E more information required.
- A square number is divided by 6. Which of the following could not be the remainder?
A 0 B 1 C 2 D 3 E 4
- The four statements in the box on the right refer to a mother and her four daughters. One statement is true, three statements are false. Who is the mother?
A Alice B Beth C Carol D Diane E Ella
- A hockey team consists of 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. There are 4 substitutes: 1 goalkeeper, 1 defender, 1 midfielder and 1 forward. A substitute may only replace a player of the same category eg: midfielder for midfielder. Given that a maximum of 3 substitutes may be used and that there are still 11 players on the pitch at the end, how many different teams could finish the game?
A 110 B 118 C 121 D 125 E 132
- Eight identical regular octagons are placed edge to edge in a ring in such a way that a symmetrical star shape is formed by the interior edges. If each octagon has sides of length 1, what is the area of the star?
A $5 + 10\sqrt{2}$ B $8\sqrt{2}$ C $9 + 4\sqrt{2}$ D $16 - 4\sqrt{2}$ E $8 + 4\sqrt{2}$



Alice is the mother.
Carol and Ella are both daughters.
Beth is the mother.
One of Alice, Diane or Ella is the mother.

