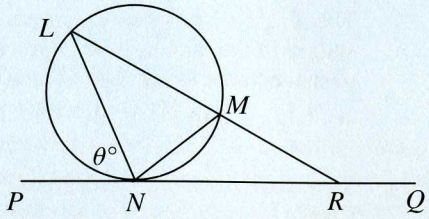
18. In the diagram, the line PQ is a tangent at N to the circle through points L, M and N. The lengths LM and LN are equal. The line LM produced meets the tangent PQ at the point R.
If $\angle PNL = \theta°$, what is the value, in degrees, of $\angle LRP$?



A $3\theta - 180$ B $180 - 2\theta$ C $180 - \theta$ D $90 - \frac{1}{2}\theta$ E θ

19. The letters S, M and C represent whole numbers. If $S \times M \times C = 240$, $S \times C + M = 46$ and $S + M \times C = 64$, what is the value of $S + M + C$?

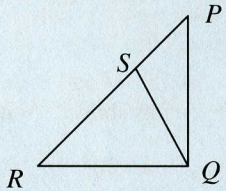
A 19 B 20 C 21 D 24 E 36

20. Which is the lowest positive integer by which 396 must be multiplied to make a perfect cube?

A 11 B 66 C 99 D 121 E 726

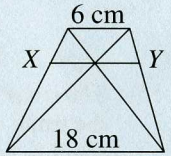
21. Triangle PQR has a right angle at Q and $PQ = QR$. The line through Q which divides the angle PQR in the ratio 1:2 meets PR at S. What is the ratio RS:SP?

A $\sqrt{2}$:1 B $\sqrt{3}$:1 C 2:1 D $\sqrt{5}$:1 E 3:1



22. In the trapezium shown (not to scale), XY is parallel to two sides and passes through the point of intersection of the diagonals. What is the length XY?

A 8 cm B 9 cm C 10 cm D 11 cm E 12 cm

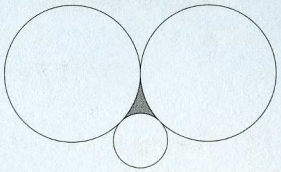


23. The number N is exactly divisible by 7. It has 4008 digits. Reading from left to right, the first 2003 digits are all 2s, the next digit is n and the last 2004 digits are all 8s. What is the value of n?

A 4 B 5 C 0 or 3 D 2 or 9 E 1 or 8

24. The diagram (not to scale) shows two circles of radius 105 which are tangent to each other and to a circle of radius 14. What is the radius of the largest circle which can be placed in the shaded region?

A 2 B 3 C 4 D 5 E 6



25. Positive integers x and y satisfy the equation: $\sqrt{x + \frac{1}{2}\sqrt{y}} - \sqrt{x - \frac{1}{2}\sqrt{y}} = 1$.
Which of the following is a possible value of y?

A 5 B 6 C 7 D 8 E 9

UKMT

UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 9 November 2004

Organised by the **United Kingdom Mathematics Trust**

and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
Once you have begun, record all your answers on the Answer Sheet.
3. Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

1. Three different positive integers add up to 12. How large could the largest of the three numbers be?

A 6 B 7 C 8 D 9 E 10

2. A dictionary defines one billion to be either one million million or one thousand million. What is the difference between these two numbers?

A 1000 B 1 000 000 C 999 000 D 999 000 000 E 999 000 000 000

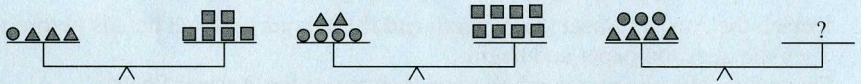
3. Milly and Molly are each given a 100g ice-lolly. They start eating their lollies at the same time, but Milly eats hers twice as fast as Molly. When Molly has three times as much of her lolly left as has Milly, what fraction of her lolly has Milly eaten?

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

4. A pieman sells pies at 4p for 5, or 1p each. If Simon simply buys 2004 pies, what is the least amount he could pay?

A £4.04 B £8.04 C £12.04 D £16.04 E £20.04

5. The first two scales shown are perfectly balanced. How many squares will be needed on the right of the third scales so that the scales balance?



A 12 B 10 C 9 D 8 E 7

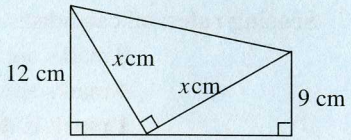
6. Pat's age is a prime number. Ten years ago, as a teenager, Pat's age was also a prime number. How old is Pat?

A 17 B 23 C 27 D 29 E more information needed

7. A ball is dropped out of a classroom window onto the playground 29 feet 3 inches below. Every time the ball hits the ground it bounces to two thirds of its previous height. What is the greatest height to which it rises following the third bounce?

A 1 foot 1 inch B 3 feet 3 inches C 8 feet 8 inches D 13 feet E 13 feet 5 inches

8. The diagram shows three right-angled triangles. What is the value of x?

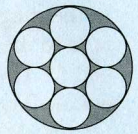


A 14 B $10\sqrt{2}$ C 15 D $12\sqrt{2}$ E $10\sqrt{3}$

9. How many numbers from 12 to 12345 inclusive have digits which are consecutive and in increasing order, reading from left to right?

A 10 B 13 C 18 D 22 E 25

10. The diagram shows seven circles of equal radius which fit snugly in the larger circle. What is the ratio of the unshaded area to the shaded area?



A 7:1 B 7:2 C $2\sqrt{3}$:1 D 9:2 E 1:1

11. Sam can mow a lawn in 3 hours. Mel takes 4 hours to mow the same lawn, and Chris takes 6 hours to do the same. If they work with a lawn mower each, and do not get in the way of each other, how long would they take to mow the lawn together?

A 1 hour 20 minutes B 1 hour 30 minutes C 3 hours D 4 hours 20 minutes E 13 hours

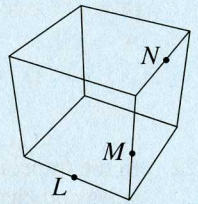
12. One face of a solid polyhedron is a regular hexagon. What is the smallest possible number of edges the polyhedron could have?

A 7 B 9 C 12 D 15 E 18

13. The value of $1^{2004} + 3^{2004} + 5^{2004} + 7^{2004} + 9^{2004}$ is calculated using a powerful computer. What is the units digit of the correct answer?

A 9 B 7 C 5 D 3 E 1

14. L, M and N are midpoints of the sides of a skeleton cube, as shown. What is the value of angle LMN?

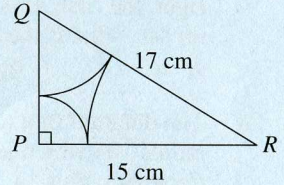


A 90° B 105° C 120° D 135° E 150°

15. The trunk of a monkey-puzzle tree has diameter 40 cm. As a protection from fire, the trunk of the tree has a bark which makes up 19% of its volume. On average, roughly how thick is the bark of the trunk?

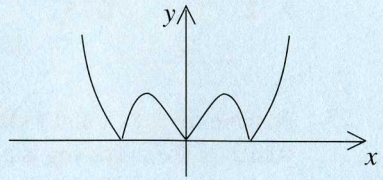
A 0.4 cm B 1.2 cm C 2 cm D 2.8 cm E 4 cm

16. In triangle PQR, angle $P = 90°$, $PR = 15$ cm and $QR = 17$ cm. Circular arcs are drawn with centres at P, Q and R, and each arc touches the other two arcs. What is the radius of the arc with centre R?



A 10 cm B 10.5 cm C 11 cm D 11.5 cm E 12 cm

17. The graph of $y = |f(x)|$ is shown. Given that the graph of $y = f(x)$ is a continuous curve, how many different possibilities are there for the graph of $y = f(x)$?



A 16 B 12 C 8 D 4 E 2