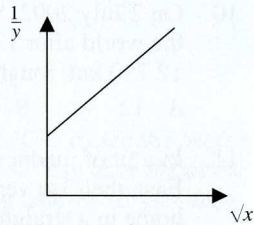


19. The straight line shows the graph of $\frac{1}{y}$ plotted against \sqrt{x} . Which of the following could be a possibility for the equation linking y and x ?

A $y^2 = \frac{1}{x-1}$ B $y^2 = \frac{1}{x^2+1}$ C $y^2 = x-1$
 D $y^2 = \frac{1}{x-2\sqrt{x+1}}$ E $y^2 = \frac{1}{x+2\sqrt{x+1}}$

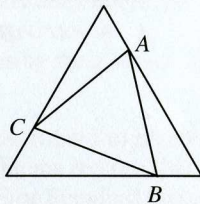


20. All six vertices of hexagon $UVWXYZ$ lie on the circumference of a circle; $\angle ZUV = 88^\circ$ and $\angle XYZ = 158^\circ$. What is the size of $\angle VWX$?

A 92° B 114° C 120° D 132° E it is impossible to determine

21. The outer equilateral triangle has area 1. The points A , B , C are a quarter of the way along the sides as shown. What is the area of the equilateral triangle ABC ?

A $\frac{3}{8}$ B $\frac{7}{16}$ C $\frac{1}{2}$ D $\frac{9}{16}$ E $\frac{5}{8}$

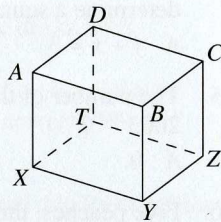


22. Given an unlimited supply of 50p, £1 and £2 coins, in how many different ways is it possible to make a sum of £100?

A 1326 B 2500 C 2601 D 5050 E 10 000

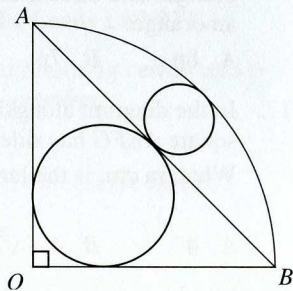
23. The cube, $XYZTABCD$, is cut into four pieces by cutting along two planes, $BCTX$ and $BDTY$. What fraction of the volume of the cube is occupied by the piece containing corner A ?

A $\frac{3}{8}$ B $\frac{1}{3}$ C $\frac{3}{10}$ D $\frac{5}{18}$ E $\frac{1}{4}$



24. AOB is an isosceles right-angled triangle drawn in a quadrant of a circle of radius 1 unit. The largest possible circle is drawn in the minor segment cut off by the line AB . This circle has radius r . The radius of the inscribed circle of the triangle AOB is R . What is the value of $\frac{R}{r}$?

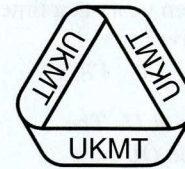
A 2 B $2\sqrt{2} - 1$ C $\sqrt{2} + 1$
 D $\frac{9}{5}$ E $\sqrt{3}$



25. How many pairs of positive integers (x, y) are solutions of the equation

$$\frac{1}{x} + \frac{2}{y} = \frac{3}{19}?$$

A 0 B 1 C 2 D 3 E more than 3



UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 11 November 2003

Organised by the **United Kingdom Mathematics Trust**

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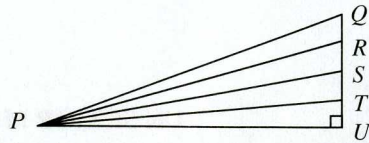
RULES AND GUIDELINES (to be read before starting)

- Do not open the question paper until the invigilator tells you to do so.
- Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin. Once you have begun, record all your answers on the Answer Sheet.
- Time allowed: **90 minutes**. No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed. **Calculators, measuring instruments and squared paper are forbidden.**
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E . Only one of these is correct. Enter the letter $A-E$ corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;
 - 0 marks are awarded for each question left unanswered;
 - 4 marks are awarded for each correct answer;
 - 1 mark is deducted** for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

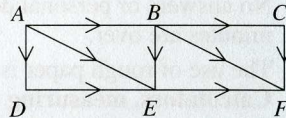
The United Kingdom Mathematics Trust is a Registered Charity.

1. If you kept counting back eleven years at a time from 2003, at which of the following years would you arrive?
 A 1505 B 1605 C 1705 D 1805 E 1905

2. Triangle PQU has a right angle at U . The points R, S and T divide the side QU into quarters. Which of the following statements about the areas of the triangles PQR, PRS, PST, PTU is true?



- A All have the same area B ΔPQR is biggest C ΔPRS is biggest
 D ΔPST is biggest E ΔPTU is biggest
3. If $a \oplus b = \sqrt{ab + 4}$, then what is the value of $(2 \oplus 6) \oplus 8$?
 A 6 B 8 C 10 D 12 E 18
4. Susan is taller than Sophie, but shorter than Sandra. Stephanie is taller than Sarah, but shorter than Susan. Who is the tallest of these five girls?
 A Susan B Sophie C Sandra D Stephanie E Sarah
5. One of the oldest sporting events in the world is the Kiplingcotes Derby, a horse race which has been held in the East Yorkshire Wolds almost every year since 1519. Each rider pays a fee of £4.25 to enter the race. The first prize in the race is the sum of £50, but the second prize is the total of the entry fees minus an administration cost of 25p per rider. In 2000, 18 riders competed in the Kiplingcotes Derby. How much greater than the first prize was the second prize?
 A £22 B £26.50 C £46 D £72 E £76.50
6. The engineering company, Sparks and Tensor, has a complicated system of conveyor belts in its factory. Components must travel along these belts in the directions shown by the arrows.



How many different routes are there from A to F along the conveyor belts?

- A 4 B 5 C 6 D 7 E 8
7. Climbers use ropes of different diameters. A 50m rope which is 9mm in diameter weighs about 2.7kg. Roughly what would a 50m rope of the same material, but of diameter 11mm, weigh?
 A 2.7kg B 3.3kg C 4kg D 4.9kg E 6kg
8. The difference between two numbers is one quarter of their sum. What is the ratio of the smaller number to the larger number?
 A 3 : 8 B 1 : 2 C 5 : 7 D 1 : 4 E 3 : 5
9. Mary received a 10% pay rise, and Margaret received a 5% pay rise. This gave them both salaries of £23 100 per year. How much more, per year, did Margaret earn than Mary before they received these pay rises?
 A £1155 B £1000 C £850 D £760 E £550

10. On 2 July 2002, Steve Fossett completed the first solo balloon circumnavigation of the world after $13\frac{1}{2}$ days. Assuming the balloon travelled along a circle of diameter 12 750 km, roughly what was the average speed of the balloon in km/h?
 A 12 B 40 C 75 D 120 E 300

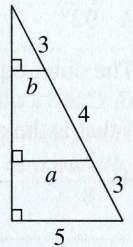
11. In a fit of madness, the bee Zerk left the hive and flew 1m due North, then 1m due East, then 1m vertically up. She then made a beeline for the hive, flying directly home in a straight line.

How far, in m, did she fly altogether?

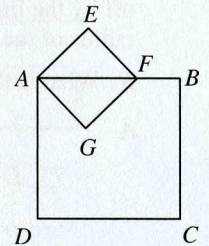
- A 4 B $3 + \sqrt{2}$ C $3 + \sqrt{3}$ D $5\frac{1}{4}$ E 6

12. From the information given in the diagram shown alongside, what is the value of $a + b$?

- A 3 B 4 C 5 D 6 E 7



13. Which of the following is divisible by 3 for every whole number x ?
 A $x^3 - x$ B $x^3 - 1$ C x^3 D $x^3 + 1$ E $x^3 + x$
14. Which of the following straight lines should be omitted to leave four lines which determine a square?
 A $y + x = 3$ B $y = x - 1$ C $y + x = 1$ D $y = x + 1$ E $y + x = 2$
15. The number of this year, 2003, is prime. How many square numbers are factors of 2003^{2003} ?
 A 0 B 1 C 44 D 1002 E 2003
16. Five peaches, three oranges and two melons cost £3.18. Four peaches, eight oranges and three melons cost £4.49. How much more expensive is a peach than an orange?
 A 8p B 7p C 6p D 5p E more information needed
17. In the diagram alongside, square $ABCD$ has side 4cm, and square $AEFG$ has side 2cm. What, in cm, is the length of CE ?



- A 4 B $4 + \sqrt{2}$ C 6 D 7 E $4 + 2\sqrt{2}$

18. What is the value of $2^{2003} - 2^{2002} - 2^{2001} - 2^{2000}$?

- A -2^{2002} B 0 C 2^{-4000} D 64 E 2^{2000}