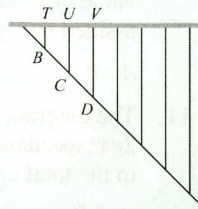


18. When $n = 81$, what is the value of $\frac{n^{20}}{3^n}$?

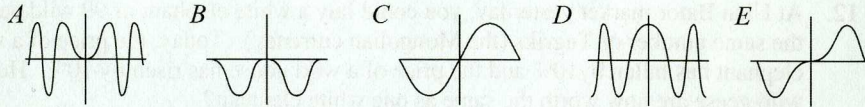
- A less than $\frac{1}{100}$ B $\frac{1}{3}$ C 1 D 3 E more than 100

19. A sculpture consists of a row of 2 metre rods each placed with one end resting on horizontal ground and the other end resting against a vertical wall. The diagram shows how the rods BT, CU, DV, \dots look from above. The bases B, C, D, \dots lie on a straight line on the ground at 45° to the wall. The top ends of the rods T, U, V, \dots lie on part of a curve on the wall. What curve is it?



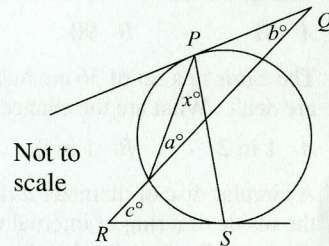
- A a straight line B a parabola C a circle D a sine curve E a quartic curve

20. Which of the following could be the graph of $y = \sin(x^2)$?



21. In the figure, PQ and RS are tangents to the circle. Given that $a = 20$, $b = 30$ and $c = 40$, what is the value of x ?

- A 20 B 25 C 30
D 35 E 40



Not to scale

22. Given that $y = \frac{x}{x + \frac{x}{x+y}}$, for which of the following values of x is y not a real number?

- A -6 B -3 C 1 D 3 E 6

23. A square $XABD$ of side length 1 is drawn inside a circle with diameter XY of length 2. The point A lies on the circumference of the circle. Another square $YCBE$ is drawn. What is the ratio of the area of square $XABD$ to the area of square $YCBE$?

- A 1:2 B 1:3 C $1:\sqrt{2}(2-\sqrt{3})$ D $1:(\sqrt{2}-1)$ E $1:(2-\sqrt{3})$

24. A function f has the property that $f(n+3) = \frac{f(n)-1}{f(n)+1}$ for all positive integers n .

Given that $f(2002)$ is non-zero, what is the value of $f(2002) \times f(2008)$?

- A 1 B -1 C 2 D -2 E more information needed

25. Let N be a positive integer less than 10^{2002} . When the digit 1 is placed after the last digit of N , the number formed is three times the number formed when the digit 1 is placed in front of the first digit of N . How many different values of N are there?

- A 1 B 42 C 333 D 667 E 2002



UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 12 November 2002

Organised by the **United Kingdom Mathematics Trust**

and supported by

The Actuarial Profession

making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

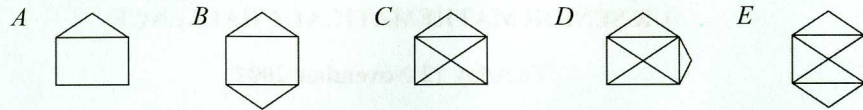
- Do not open the question paper until the invigilator tells you to do so.
- Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
Once you have begun, record all your answers on the Answer Sheet.
- Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E . Only one of these is correct. Enter the letter $A-E$ corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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1. How many squares of area 1 cm^2 have a total perimeter length which is equal to the perimeter of a single square of area 4 cm^2 ?

A 2 B 4 C 8 D 16 E 32

2. Which of the following networks is not traversable? (A traversable network is one which can be drawn without taking the pen off the paper and without going over any line more than once.)



3. Peri the winkle starts at the point $(1, 1)$. Each day Peri crawls from point (x, y) to point $(y, x + y)$, so that at the end of the first day Peri has reached $(1, 2)$.

Where is Peri at the end of the sixth day?

A $(6, 7)$ B $(6, 12)$ C $(13, 21)$ D $(21, 34)$ E $(144, 233)$

4. What is the remainder when $123456789 \times 987654321$ is divided by 6?

A 1 B 2 C 3 D 4 E 5

5. The value of the product $wxyz$ is 2002 and w, x, y and z are prime numbers. What is the value of $w^2 + x^2 + y^2 + z^2$?

A 66 B 203 C 260 D 285 E 343

6. In the village of Much-Pedling-in-the-Marsh, one third of the children can swim, two thirds can ride a bicycle and one seventh can both swim and ride a bicycle (though not necessarily at the same time). Given that there are fewer than 40 children in Much-Pedling-in-the-Marsh, how many of them can neither swim nor ride a bicycle?

A 1 B 2 C 3 D 4 E 5

7. This crossnumber must be completed so that there is a different non-zero digit in each square. Which digit goes in the square marked X?

1.	2.
3.	X

Clues

Across

Down

1. Cube

1. Square

3. Sum of two squares

2. Prime

A 1 B 3 C 5 D 7 E 9

8. A furlong is 220 yards long and a yard is 36 inches. A chain is 44 cubits long and a cubit is 54 barleycorns. There are 10 chains in a furlong. How many barleycorns are there in one inch?

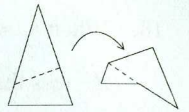
A 6 B 5 C 4 D 3 E 2

9. When forest trees are planted 1 metre apart in a particular repeating pattern, covering a large area of ground, the density of trees is about 10 000 per hectare.

If, instead, the trees were planted 2 metres apart in the same pattern, approximately how many trees per hectare would there be?

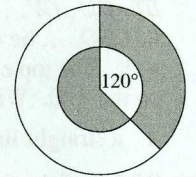
A 2 500 B 5 000 C 10 000 D 20 000 E 40 000

10. The diagram shows a triangular piece of paper that has been folded once to produce a shape with the outline of a pentagon. If a *rectangular* piece of paper is folded once, what is the smallest value of n (greater than 4) for which it is not possible to create an n -sided polygon in the same way?



A 6 B 7 C 8 D 9 E 10

11. The diagram shows two concentric circles of radii r and $2r$ respectively. What is the ratio of the total shaded area to the total unshaded area?



A 5:7 B 7:5 C 1:1 D 2:3 E 3:2

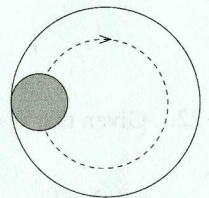
12. At Ulan Bator market yesterday, you could buy a white elephant or 99 wild geese for the same number of Tugriks (the Mongolian currency). Today, the price of a white elephant has fallen by 10% and the price of a wild goose has risen by 10%. How many wild geese are now worth the same as one white elephant?

A 81 B 90 C 98.01 D 99 E 121

13. The cards in a set of 36 are numbered 1 to 36. The cards are shuffled and four cards are dealt. What are the chances of them being dealt in descending order?

A 1 in 2 B 1 in 8 C 1 in 16 D 1 in 24 E 1 in 36

14. A circular disc of diameter d rolls without slipping around the inside of a ring of internal diameter $3d$, as shown in the diagram. By the time that the centre of the inner disc returns to its original position for the first time, how many times will the inner disc have turned about its centre?

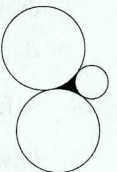


A 1 B π C 3 D 2π E 2

15. For how many integer values of n does the equation $x^2 + nx - 16 = 0$ have integer solutions?

A 2 B 3 C 4 D 5 E 6

16. Three circles touch, as shown in the diagram. The two larger circles both have radius 1 and the smaller circle has radius $\sqrt{2} - 1$. What is the perimeter of the shaded region?



A $\frac{\pi}{4}(\sqrt{2} - 1)$ B $\frac{\pi}{2}(\sqrt{2} - 1)$ C $\frac{\pi}{2}$ D $\frac{\pi}{4}(\sqrt{2} + 1)$ E $\frac{\pi}{\sqrt{2}}$

17. I have a ten volume collection of mathematical puzzles. Each volume contains the same number of puzzles and the puzzles are numbered consecutively throughout (so, for example, the number of the first puzzle in volume 2 is one more than that of the last puzzle in volume 1). Whilst browsing one night, I was particularly intrigued by puzzle number 351 in volume 5 and puzzle number 689 in volume 8. How many puzzles are there in each volume?

A 70 B 71 C 85 D 87 E more information needed