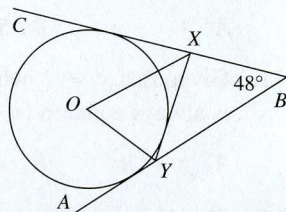


19. Note that  $2001 = 3 \times 23 \times 29$ . What is the number of the next year which can be written as the product of three distinct primes?  
 A 2002      B 2004      C 2006      D 2007      E none of these

20. In the diagram,  $AB$ ,  $CB$  and  $XY$  are tangents to the circle with centre  $O$  and  $\angle ABC = 48^\circ$ .  
 What is the size of  $\angle XOY$ ?  
 A  $42^\circ$     B  $69^\circ$     C  $66^\circ$     D  $48^\circ$     E  $84^\circ$



21. A postman's sack contains five letters, one each for the five houses in Cayley Close. Mischievously, he posts one letter through each door without looking to see if it is the correct address. In how many different ways could he do this so that exactly two of the five houses receive the correct letters?

- A 5      B 10      C 20      D 30      E 60

22. For how many values of  $n$  are both  $n$  and  $4^{\binom{n+1}{n+1}}$  integers?

- A 1      B 2      C 3      D 4      E 5

23. An equilateral triangle is inscribed in a circle, as shown. Another equilateral triangle is drawn in one of the segments so that the final diagram has a line of symmetry.  
 What is the ratio of the length of the side of the bigger triangle to the length of the side of the smaller triangle?

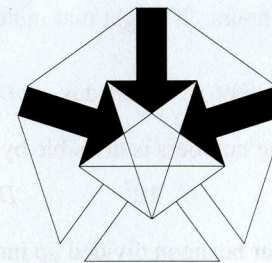
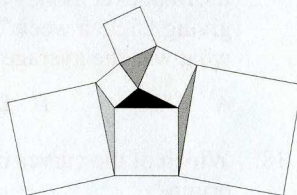


- A  $(\sqrt{5}+1):1$     B  $(2\sqrt{5}+1):1$     C  $(\sqrt{3}+\sqrt{5}):1$     D  $2\sqrt{5}:1$     E  $3:1$

24. The Queen of Hearts had some tarts, but they were eaten. Precisely one of the following statements about the tarts and the Knaves of Clubs, Diamonds and Spades is true. Which one?  
 A None of the three Knaves ate any tarts.  
 B The Knave of Clubs ate some tarts.  
 C Only one of the three Knaves ate any tarts.  
 D At least one of the Knave of Diamonds and the Knave of Spades ate no tarts.  
 E More than one of the three Knaves ate some tarts.

25. The black triangle is drawn, and a square is drawn on each of its edges. The three shaded triangles are then formed by drawing three lines which join vertices of the squares and a square is now drawn on each of these three lines. The total area of the original three squares is  $A_1$ , and the total area of the three new squares is  $A_2$ . Given that  $A_2 = kA_1$  then

- A  $k = 1$     B  $k = \frac{3}{2}$     C  $k = 2$     D  $k = 3$     E more information needed



## UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 6 November 2001

Organised by the **United Kingdom Mathematics Trust**

### RULES AND GUIDELINES (to be read before starting)

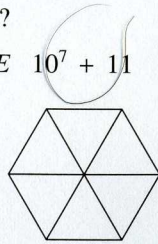
- Do not open the question paper until the invigilator tells you to do so.
- Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.  
 Once you have begun, record all your answers on the Answer Sheet.
- Time allowed: **90 minutes**.  
 No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed.  
**Calculators, measuring instruments and squared paper are forbidden.**
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked  $A, B, C, D, E$ . Only one of these is correct. Enter the letter  $A-E$  corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;  
 0 marks are awarded for each question left unanswered;  
 4 marks are awarded for each correct answer;  
**1 mark is deducted** for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

The United Kingdom Mathematics Trust is a Registered Charity.

1. A candle will burn for 100 hours. If I light it at midday on Sunday, on which day will it burn out?  
 A Tuesday    B Wednesday    C Thursday    D Friday    E Saturday

2. Exactly one of the following numbers is divisible by 11. Which is it?  
 A  $10^7 - 11$     B  $10^7 - 1$     C  $10^7$     D  $10^7 + 1$     E  $10^7 + 11$

3. The diagram shows a regular hexagon divided up into six equilateral triangles. How many quadrilaterals are there in the diagram?  
 A 6    B 8    C 10    D 12    E 14

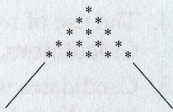


4. In the Hickey Hockey league, a team scores 5 points for a win, 2 for a draw and 0 if it loses. The Hickey Hockey Club has played 20 games in this league and has scored 21 points. What is the smallest possible number of games it could have lost?  
 A 11    B 12    C 13    D 14    E 15

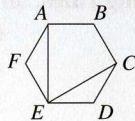
5. Goldbach's conjecture, which has not been proved, states that every even number greater than two is the sum of two primes. However, the same is not true for every odd number. Which of the following odd numbers is *not* the sum of two primes?  
 A 13    B 33    C 43    D 53    E 73

6. The mean of seven consecutive odd numbers is 21. What is the sum of the first, third, fifth and seventh of these numbers?  
 A 16    B 21    C 84    D 147    E more information needed

7. A gardener has a field in the shape of an isosceles triangle. He decided to plant it with rose bushes with the aid of his apprentice. He planted the first row with 101 bushes along the base, then his apprentice planted the next row with 100 bushes. They continued in this way, planting alternate rows, until the whole field was planted. The gardener planted the last row, which contained 1 bush. How many more bushes did the gardener plant than the apprentice?  
 A 100    B 101    C 49    D 50    E 51

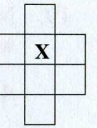


8.  $ABCDEF$  is a regular hexagon of area 60. What is the area of the kite-shaped figure  $ABCE$ ?  
 A  $20\sqrt{3}$     B 40    C 49    D 50    E 51



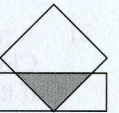
9. Which of the following numbers  $n$  gives a counter-example for the statement: 'If  $n$  is a prime number then  $n^2 + 2$  is also a prime number'?  
 A 3    B 5    C 6    D 9    E none of them

10. The digits 1 to 8 are to be inserted in the grid on the right so that any two digits which are adjacent to each other in the sequence 1 to 8 are not adjacent to each other in the grid horizontally, vertically or diagonally. Which one of the following digits could be placed in the box labelled X?



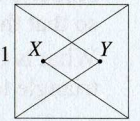
- A 4    B 5    C 6    D 7    E 8
11. Given that  $x = \frac{1}{y}$ , where  $x$  and  $y$  are unequal and non-zero, which of the following is always equal to  $(x + \frac{1}{x})(y - \frac{1}{y})$ ?  
 A  $y^2 - x^2$     B  $x^2 - y^2$     C  $2y$     D  $2x$     E 0

12. The diagram shows a  $2 \times 2$  square and a  $3 \times 1$  rectangle. One vertex of the square lies on a side of the rectangle. The sides of the rectangle are parallel to the diagonals of the square. What is the area of the shaded triangle?



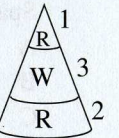
- A  $\frac{1}{2}$     B 1    C  $\frac{3}{2}$     D 2    E  $\frac{5}{2}$
13. Rosie the road-runner recently ran in two road races. The second race was 20% further in distance than the first race and Rosie's average speed was 20% slower in the second race. By what percentage was her time for the second race greater than that for the first?

- A  $33\frac{1}{3}\%$     B 40%    C 44%    D 50%    E  $66\frac{2}{3}\%$
14. The diagram shows a square and two equilateral triangles. All the sides have length 1. What is the length of  $XY$ ?  
 A  $\sqrt{3} - 1$     B  $\frac{2}{3}$     C  $\frac{3}{4}$     D  $\frac{\sqrt{3}}{2}$     E  $\frac{2 + \sqrt{3}}{4}$



15. Sam correctly calculates the value of  $5^8 \times 8^5$ . How many digits does her answer contain?  
 A 11    B 12    C 13    D 14    E 15

16. A traffic cone is painted with red (R) and white (W) bands of paint as shown. The sloping heights of the bands are in the ratio 1 : 3 : 2. What is the ratio of the area painted white to the area painted red?  
 A 5:9    B 5:7    C 1:1    D 7:5    E 9:5



17. Last year, a newspaper reported the results of a survey: 'Children in the U.K. get £3.10 pocket money a week on average. Scottish parents are the most generous, giving £5.35 a week.' Assuming that Scottish children made up 10% of the survey, what was the average pocket money per week in the rest of the U.K.?  
 A £0.85    B £1.85    C £2.85    D £3.85    E £4.85

18. Which of the curves or lines given by the following equations comes closest to the origin?  
 A  $y = x - 4$     B  $x^2 + y^2 = 4$     C  $y = \frac{4}{x}$     D  $y = x^2 + 4$     E  $y = x^4 - 4$