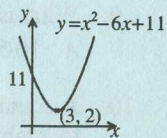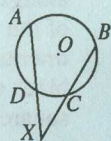18. The curve $y = x^2 - 6x + 11$ is rotated through 180° about the origin. What is the equation of the new curve?

    A $y = x^2 + 6x + 11$ B $y = x^2 - 6x - 11$ C $y = -x^2 + 6x - 11$

    D $y = -x^2 - 6x + 11$ E $y = -x^2 - 6x - 11$

19. In the diagram, O is the centre of the circle, $\angle AOB = \alpha$ and $\angle COD = \beta$. What is the size of $\angle AXB$ in terms of α and β?

    A $\frac{1}{2}\alpha - \frac{1}{2}\beta$ B $90° - \frac{1}{2}\alpha - \frac{1}{2}\beta$ C $\alpha - \beta$

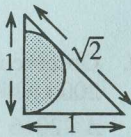   D $180° - \alpha - \beta$ E more information needed

20. What is the maximum possible value of the product of a list of positive integers, which are not necessarily all different, given that the sum of these numbers is 100?

    A 10^{10} B 2^{50} C $2^{20} \times 3^{20}$ D 5^{20} E $2^2 \times 3^{32}$

21. What is the radius of the shaded semicircle?

    A $\sqrt{2} - 1$ B $\frac{1}{\sqrt{2}}$ C $3 - 2\sqrt{2}$ D $\frac{1}{2}$ E $2 - \sqrt{2}$

22. Given that $0 < b < a$ and $a^2 + b^2 = 6ab$, what is the value of $\frac{a+b}{a-b}$?

    A $\frac{1}{\sqrt{2}}$ B $\sqrt{2}$ C $\frac{1}{\sqrt{2} - 1}$ D $2\sqrt{2}$ E $\sqrt{6}$

23. A tennis club has n left-handed players and $2n$ right-handed players, but in total there are fewer than 20 players. At last summer's tournament, in which every player in the club played every other player exactly once, no matches were drawn and the ratio of the number of matches won by left-handed players to the number of matches won by right-handed players was 3 : 4.
What is the value of n?

    A 3   B 4   C 5   D 6   E more information needed

24. How many pairs of positive integers (x, y) satisfy the equation $\sqrt{x} - \sqrt{17} = \sqrt{y}$?

    A 0   B 1   C 2   D 17   E infinitely many

25. The figure on the right shows two parallel lines, ℓ_1, and ℓ_2. Line ℓ_1 is a tangent to circles C_1 and C_3, line ℓ_2 is a tangent to circles C_2 and C_3 and the three circles touch as shown. Circles C_1 and C_2 have radius s and t respectively.
What is the radius of circle C_3?

    A $2\sqrt{s^2 + t^2}$ B $s + t$ C $2\sqrt{st}$ D $\frac{4st}{s+t}$

    E more information needed
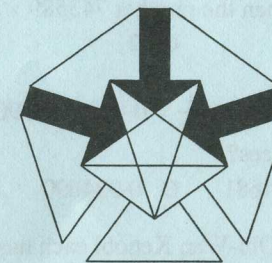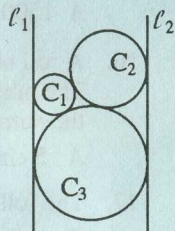

UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 14 November 2000

Organised by the **United Kingdom Mathematics Trust**

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.

2. Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
Once you have begun, record all your answers on the Answer Sheet.

3. Time allowed: **90 minutes**.
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.

4. The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.

5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).

6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.

7. **Scoring rules**: all candidates start out with 25 marks;
        0 marks are awarded for each question left unanswered;
        4 marks are awarded for each correct answer;
        1 mark is deducted for each incorrect answer.

8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

1. What is the remainder when the number 743589×301647 is divided by 5?

    A 0   B 1   C 2   D 3   E 4

2. When written in decimal form, $\frac{1}{81} = 0.0123456790123\ldots$. What is the value of $\frac{2}{81}$ correct to six decimal places?

    A 0.02468   B 0.024681   C 0.024690   D 0.024691   E 0.0246914

3. Annakin Skywalker and Obi-Wan Kenobi each has some coins in his pocket. If Annakin gave Obi-Wan Kenobi one coin then Obi-Wan Kenobi would have twice as many coins as Annakin, but if Obi-Wan Kenobi gave Annakin one coin, they would each have the same number of coins. Altogether, how many coins do they have?

    A 8   B 10   C 12   D 14   E 16

4. An attempt by two men to sit in every one of the 72 000 seats in Cardiff's Millennium Stadium to raise money for charity ended in painful failure, despite their wearing man-made-fibre tracksuits to help them slide from seat to seat. Before the pain stopped them, they had managed 64 000 seats between them in 27 hours. On average, roughly how long did each man take per seat?

    A 0.5 seconds B 1 second   C 1.5 seconds D 2 seconds   E 3 seconds

5. $ABCD$ is a rectangle. P is the midpoint of AD; the length of BQ is one third of the length of BC. What fraction of the area of the rectangle is the area of the shaded quadrilateral $ABQP$?

    A $\frac{5}{12}$ B $\frac{2}{5}$ C $\frac{3}{8}$ D $\frac{1}{3}$ E $\frac{7}{16}$

6. Last year, a newspaper reported that the mean forecourt price of petrol was 73.3 pence per litre for unleaded petrol. The price before tax, however, was 15.2 pence per litre. What was the approximate percentage increase from the price before tax to the mean forecourt price?

    A 20%   B 40%   C 80%   D 400%   E 500%

7. The three regular hexagons are all the same size. X, Y and Z denote the values of the shaded areas in the hexagons, as shown.
    Which of the following statements is true?

    A X is equal to Y but not to Z B X is equal to Z, but not to Y
    C Y is equal to Z, but not to X D X is equal to Y and to Z
    E X, Y and Z are all different

8. A van travels from Newcastle to South Shields at an average speed of 30 mph and returns by the same route at an average speed of 40 mph. Which of the following is closest to the van's average speed for the whole journey?

    A 33 mph   B 34 mph   C 35.5 mph   D 36 mph   E more information needed
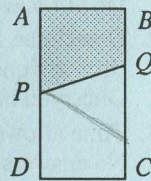
9. What is the value of $(61^2 - 39^2) \div (51^2 - 49^2)$?

    A 10.5   B 11   C 12   D 21   E 22

10. A square piece of wood, of side 8 cm, is painted black and fixed to a table. An equal square, painted white, is placed on the table alongside the black square and has a point P marked one quarter of the way along a diagonal, as shown. Whilst keeping the same orientation on the table and always remaining in contact with the black square, the white square now slides once around the black square. Through what distance does P move?

    A 32 cm   B 48 cm   C 64 cm   D 72 cm   E 80 cm

11. Given that a and b are integers greater than zero, which of the following equations could be true?

    A $a - b = a \div b$ B $a + b = a \div b$ C $a - b = a \times b$
    D $a + b = a - b$ E $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$

12. Observe that $2000 = 2^4 \times 5^3$. What is the number of the next year after the year 2000 which can be written $a^b \times c^d$ where a, b, c, d are 2, 3, 4, 5 in some order?

    A 2016   B 2025   C 2040   D 2048   E 2050

13. A trapezium has parallel sides of length a and b, and height h. Sides a and b are both decreased by 10% and the height h is increased by 10%.
    What is the percentage change in the area of the trapezium?

    A 10% decrease   B 1% decrease   C no change
             D 10% increase   E 30% increase

14. Which shape cannot be obtained as the cross-section (in any direction) of this solid, which is a triangular prism with three rectangular faces?

    A triangle   B rectangle   C trapezium   D pentagon   E hexagon

15. The diagram represents the addition of three 3-digit numbers, which between them use all the digits from 1 to 9. Which of the following cannot be obtained as the answer to the addition?

    A 1500   B 1503   C 1512   D 1521   E 1539

16. A roll of adhesive tape is wound round a central cylindrical core of radius 3 cm. The outer radius of a roll containing 20 m of tape is 4 cm. Approximately, what is the outer radius of a roll containing 80 m of tape?

    A 5 cm   B 5.5 cm   C 6 cm   D 7 cm   E 12 cm

17. The following equation is true for all a, b and c:
    $a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b + c)(ab + bc + ca) + kabc$
    What is the value of k?

    A −6   B −3   C 0   D 3   E 6