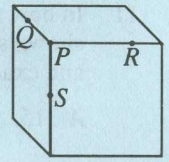
19. P is a vertex of a cuboid and Q, R and S are three points on the edges as shown. $PQ = 2$ cm, $PR = 2$ cm and $PS = 1$ cm. What is the area, in cm^2, of triangle QRS?



$A\ \sqrt{15}/4$ $B\ 5/2$ $C\ \sqrt{6}$ $D\ 2\sqrt{2}$ $E\ \sqrt{10}$

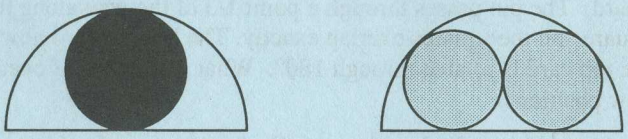
20. What is the 1999th term of the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ... ?

$A\ 59$ $B\ 60$ $C\ 61$ $D\ 62$ $E\ 63$

21. Just one of the following is a prime number. Which one is it?

$A\ 1000^2 + 111^2$ $B\ 555^2 + 666^2$ $C\ 2000^2 - 999^2$
$D\ 1001^2 + 1002^2$ $E\ 1001^2 + 1003^2$

22.



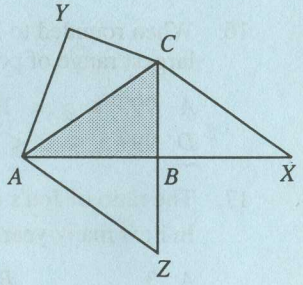
The area of each large semicircle is 2. What is the difference between the black and grey shaded areas?

$A\ 0$ $B\ \frac{1}{2}$ $C\ 1 + 2\sqrt{2}$ $D\ \frac{5}{9}$ $E\ 23 - 16\sqrt{2}$

23. The statement "There are exactly four integer values of n for which $(2n + y)/(n - 2)$ is itself an integer" is true for certain values of y only. For how many values of y in the range $1 \leqslant y \leqslant 20$ is the statement true?

$A\ 0$ $B\ 7$ $C\ 8$ $D\ 10$ $E\ 20$

24. The figure shows a hexagon $AZBXCY$ made from four congruent tiles. The shape and position of the tiles are given by triangle ABC and the three reflections of triangle ABC in the lines determined by its sides. For example, ABZ is the image of ABC when reflected in the line determined by AB. If a polygon is made from five tiles whose shape and position are determined by a quadrilateral and the four reflections of that quadrilateral in the lines determined by its sides, what is the smallest possible number of sides of the resulting polygon?

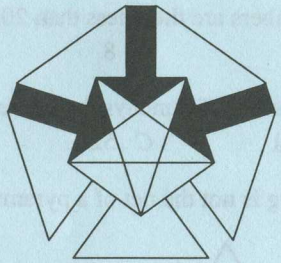


$A\ 4$ $B\ 5$ $C\ 6$ $D\ 7$ $E\ 8$

25. What is the sum to infinity of the convergent series

$$\frac{1}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{5}{32} + \frac{8}{64} + \frac{13}{128} + \frac{21}{256} + \frac{34}{512} + \ldots ?$$

$A\ \frac{7}{4}$ $B\ 2$ $C\ \sqrt{5}$ $D\ \frac{9}{4}$ $E\ \frac{7}{3}$



UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 9 November 1999

Organised by the **United Kingdom Mathematics Trust**

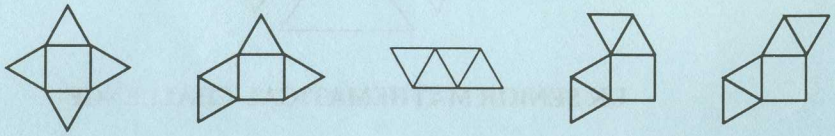
RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
   Once you have begun, record all your answers on the Answer Sheet.
3. Time allowed: **90 minutes.**
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

1. How many prime numbers are there less than 20?
   A 6 *B* 7 *C* 8 *D* 9 *E* 10

2. What is the largest number of Sundays that there can be in any one year?
   A 50 *B* 51 *C* 52 *D* 53 *E* 54

3. Which of the following is not the net of a pyramid?



   A *B* *C* *D* *E*

4. I need to buy 12 films for my camera before my holiday. They normally cost £4.50 each, but a number of shops have "special offers". Which of these is the best deal?
   A One fifth off all prices! *B* Two for the price of four! *C* Buy two – get one free!
   D 30% price cut! *E* Pay only three quarters of the normal price!

5. In 1998 a newspaper reported that "The world record for remembering the value of π to the greatest number of decimal places is 40 000 places, which took the record holder 17 hours and 21 minutes to recite."
   What was the average number of decimal places recited per minute, approximately?
   A 20 *B* 40 *C* 200 *D* 400 *E* 2000

6. Our ancient Ancient History teacher's copy of Homer's *Odyssey* cost 40p in 1974. A similar edition today costs £5. What percentage increase is this?
   A 12.5% *B* 1150% *C* 1250% *D* 12400% *E* 12500%

7. The size of each exterior angle of a regular polygon is one quarter of the size of an interior angle. How many sides does the polygon have?
   A 6 *B* 8 *C* 9 *D* 10 *E* 12

8. Two numbers differ by 9 and have sum 99. What is the ratio of the larger number to the smaller?
   A 5:4 *B* 6:5 *C* 7:6 *D* 8:7 *E* 9:8

9. The factorial of n, written $n!$, is defined by $n! = 1 \times 2 \times 3 \times \ldots \times (n-2) \times (n-1) \times n$
   e.g. $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$.
   What is the smallest positive integer which is *not* a factor of 50! ?
   A 51 *B* 52 *C* 53 *D* 54 *E* 55

10. Which is the largest of the following?
    A $\sqrt{1999}$ *B* $1\sqrt{999}$ *C* $19\sqrt{99}$ *D* $199\sqrt{9}$ *E* $1999\sqrt{0}$

11. In how many different ways can I circle letters in the grid shown so that there is exactly one circled letter in each row and exactly one circled letter in each column?

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

    A 15 *B* 24 *C* 60 *D* 100 *E* 120

12. Earlier this year, the White Rabbit said to me, "Two days ago, Alice was still thirteen, but her sixteenth birthday will be next year." When is Alice's birthday?
    A Jan 1st *B* Feb 28th *C* Feb 29th *D* Dec 30th *E* Dec 31st

13. Two square pieces of card, each 3 cm × 3 cm, are attached by a single pin to a board. The pin passes through a point 1/3 of the way along the diagonal of each square and the squares overlap exactly. The bottom card now remains fixed, while the top card is rotated through 180°. What is the area of overlap of the cards in this new position?
    A 1 cm^2 *B* 2 cm^2 *C* 4 cm^2 *D* 6 cm^2 *E* 9 cm^2

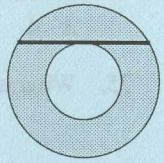
14. The line whose equation is $y = 3x + 4$ is reflected in the line whose equation is $y = -x$. What is the equation of the image line?
    A $3y = x + 4$ *B* $3y = x - 4$ *C* $y = 3x - 4$ *D* $y = -3x - 4$ *E* $y = 4x + 3$

15. Three people each think of a number which is the product of two different primes. Which of the following could be the product of the three numbers which are thought of?
    A 120 *B* 144 *C* 240 *D* 3000 *E* 12100

16. When rounded to 3 significant figures, the number x is written as 1000. What is the largest range of possible values of x?
    A $999 \leqslant x < 1001$ *B* $995 \leqslant x < 1005$ *C* $990 \leqslant x < 1010$
    D $999.5 \leqslant x < 1005$ *E* $999.5 \leqslant x < 1000.5$

17. The ratio of Jon's age to Jan's age is 3 : 1. Three years ago the ratio was 4 : 1.
    In how many years time will the ratio be 2 : 1?
    A 3 *B* 6 *C* 9 *D* 12 *E* 15

18. The diagram shows two concentric circles. The chord of the large circle is a tangent to the small circle and has length $2p$. What is the area of the shaded region?



    A πp^2 *B* $2\pi p^2$ *C* $3\pi p^2$ *D* $4\pi p^2$
    E more information needed