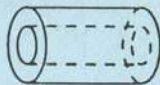18. A cylindrical hole of radius r and of length $4r$ is bored symmetrically through a solid cylinder of radius $2r$ and length $4r$. What is the total surface area of the resulting solid?

   A $22\pi r^2$ B $24\pi r^2$ C $28\pi r^2$ D $30\pi r^2$ E $36\pi r^2$

19. If $\cos\theta = 1/2$, which of these cannot equal $\sin 2\theta$?

   A $\sin\theta$ B $1/2$ C $-\sqrt{3}/2$ D $\sqrt{3}/2$ E $2\cos\theta\sin\theta$

20. A cube is inscribed in a sphere of diameter 1m. What is the surface area of the cube?

   A $2\,\text{m}^2$ B $3\,\text{m}^2$ C $4\,\text{m}^2$ D $5\,\text{m}^2$ E $6\,\text{m}^2$

21. The expression "$3 \oplus 7 \to 4$" is a short way of writing the statement "it is possible to fit a 3-sided polygon and a 7-sided polygon together (without overlap) and so make a 4-sided polygon". This statement is correct (as shown in the diagram on the right). Which of the following represents a statement which is *not* correct?
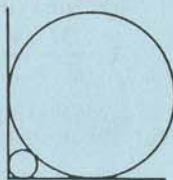
   A $3\oplus 5 \to 4$ B $3\oplus 6 \to 4$ C $3\oplus 8 \to 4$ D $4\oplus 6 \to 4$ E $4\oplus 8 \to 4$

22. Triangle ABC has $A\hat{B}C = 90°$ and $A\hat{C}B = 30°$. If a point inside the triangle is chosen at random, what is the probability that it is nearer to AB than it is to AC?

   A $\dfrac{\sqrt{3}}{2}$ B $\dfrac{1}{2}$ C $\dfrac{1}{\sqrt{3}}$ D $\dfrac{1}{3}$ E $\dfrac{1}{4}$

23. Circles with radii r and R (where $r < R$) touch each other and also touch two perpendicular lines as shown. What is the value of R/r?

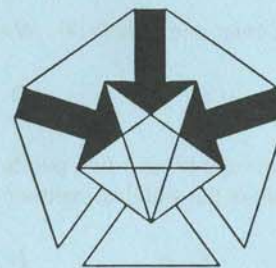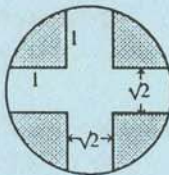   A $5 + \sqrt{2}$ B 5.75 C $\sqrt{40}$ D 6 E $3 + 2\sqrt{2}$

24. If $x^2 - 3x + 1 = 0$, what is the value of $x^2 + (1/x)^2$?

   A 7 B $(7 - 3\sqrt{5})/2$ C 9 D $(7 + 3\sqrt{5})/2$ E 10

25. A company logo has a centrally-symmetric white cross of width $\sqrt{2}$ on a dark circle. The dark corner pieces have sides of length 1 as indicated. What is the total area of the corners?

   A $\pi(2 - \sqrt{2}) + \dfrac{\sqrt{2}}{2}$ B $\pi - \dfrac{1}{\sqrt{2}}$ C $\pi(4 - \sqrt{2}) - 4\sqrt{2}$ D $\dfrac{(\pi + \sqrt{2})}{2}$ E $\dfrac{\pi(2 + \sqrt{2})}{2} - 2\sqrt{2}$

## RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. Detach the Answer Sheet (back page) and fill in your personal details before you open the question paper and begin.
   Once you have begun, record all your answers on the Answer Sheet.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules:** all candidates start out with 25 marks;

   > 0 marks are awarded for each question left unanswered;
   > 4 marks are awarded for each correct answer;
   > **1 mark is deducted** for each incorrect answer.

8. **Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

1. The sum of five consecutive even numbers is 60. What is the smallest of the five numbers?

   A 8   B 10   C 12   D 14   E 16

2. In a class of thirty pupils, there are twenty-two pupils who are right-handed and there are fourteen girls. What is the smallest number of girls who could be right-handed?

   A 0   B 2   C 4   D 6   E 8

3. The *integer part* of a positive number is the part before the decimal point; the *decimal part* is the part after the decimal point. For example, the integer part of 3.72 is '3' and the decimal part is '0.72'. Which of the following numbers has decimal part equal to exactly one eighth of the integer part?

   A 0.05   B 1.15   C 2.25   D 3.35   E 4.45

4. The smaller circle touches the larger circle, and goes through the centre of the larger circle. What fraction of the area of the larger circle is outside the smaller circle?

   A $\frac{2}{3}$ B $\frac{3}{4}$ C $\frac{4}{5}$ D $\frac{5}{6}$ E $\frac{7}{8}$

5. A square has the same perimeter as a 4cm by 2cm rectangle. What is the area of the square (in cm^2)?

   A 4   B 8   C 9   D 10   E 12

6. Observe that $18 = 4^2 + 1^2 + 1^2 + 0^2$. How many of the first fifteen positive integers can be written as the sum of the squares of four integers?

   A 11   B 12   C 13   D 14   E 15

7. Mary's height increased by 30% between her 5th birthday and her 10th birthday. It increased by 20% between her 10th birthday and her 15th birthday. By how much did her height increase between her 5th birthday and her 15th birthday?

   A 50%   B 52%   C 54%   D 56%   E 60%

8. When the following five numbers are arranged in numerical order, which one is in the middle?

   A $4\sqrt{15}$ B $5\sqrt{10}$ C $7\sqrt{5}$ D $9\sqrt{3}$ E $11\sqrt{2}$

9. Pascal, Newton, Galileo and Fermat all took the same test. The average score of all four candidates was 16; Pascal and Newton had an average of 16, Pascal and Fermat had an average of 13, while Newton and Fermat had an average of 18. What was Galileo's score?

   A 14   B 15   C 16   D 17   E 18
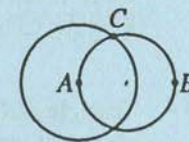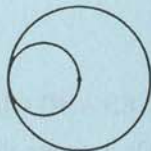
10. Roughly how many seconds are there in a day?

    A 10^3 B 10^4 C 10^5 D 10^6 E 10^7

11. Note that $1647/8235 = 1/5$. Start with $1647/8235$. First delete one digit from the numerator and one digit from the denominator leaving a fraction A/B which is equivalent to the fraction you started with. Then delete one digit from the new numerator A and one digit from the new denominator B leaving a fraction C/D which is equivalent to A/B. What is the value of the difference $D - C$?

    A 64   B 68   C 72   D 76   E 80

12. $ABCDEFGH$ is a regular octagon. P is the point inside the octagon such that triangle ABP is equilateral. What is the size of angle APC?

    A 90°   B 112.5°   C 117.5°   D 120°   E 135°

13. If $x = (1/4)^{1/2}$, what is the value of x^{-x}?

    A 1/4   B 1/2   C $1/\sqrt{2}$ D $\sqrt{2}$ E 2

14. The smaller circle has radius 10 units; AB is a diameter. The larger circle has centre A, radius 12 units and cuts the smaller circle at C. What is the length of the chord CB?

    A 8   B 10   C 12   D $10\sqrt{2}$ E 16

15. A jogger runs a certain distance at V ms^{-1}, and then walks half that distance at U ms^{-1}. If the total time for the two stages is T seconds, what is the total distance travelled (in metres)?

    A $\dfrac{3TUV}{U + 2V}$ B $\dfrac{3TUV}{2U + V}$ C $\dfrac{3T}{U + 2V}$ D $\dfrac{TUV}{2U + V}$ E $\dfrac{2TUV}{2U + V}$

16. The probability of a single ticket winning the jackpot in the National Lottery is

    $$\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44}.$$

    If I buy one ticket every week, approximately how often might I expect to win the jackpot?

    A once every hundred years   B once every twenty thousand years
    C once every hundred thousand years D once every quarter of a million years
            E once every million years

17. $(x - 1)(x^4 + 1)(x^2 + 1)(x + 1)$ equals

    A $x^8 - 1$ B $x^8 + x^6 + x^4 + x^2 + 1$ C $x^8 + 1$
    D $x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ E $x^8 - x^6 + x^4 - 1$