INFORMATION

Organisation of the UK SMC

This year's United Kingdom Senior Mathematical Challenge (formerly the National Mathematics Contest) is run by the National Committee for Mathematical Contests, a Committee of The Mathematical Association.

Any comments or questions you have about the running of the UK SMC should be sent to the chair of the NCMC:

Peter Thomas, NCMC, 1 Southbrooke Close, Cambridge, CB2 2HX.

Future events will be run by The United Kingdom Mathematics Trust which is a coming together of the British Mathematical Olympiad Committee, the UK Mathematics Foundation and the NCMC.

Accessibility

With the possible exception of the last few problems on the paper, we try to ensure that UK SMC problems require very little beyond GCSE / Standard Grade mathematics. The questions are meant to be accessible in the sense that all participants should understand them and want to solve them. However, this does not mean that the problems are easy: they are designed to make students think. In particular, participants should be careful in interpreting their scores: a score of 60 or 70 may well be much more creditable than might at first appear.

Certificates

The top 6-7% of participants are eligible for Gold Certificates, the next 12-14% for Silver Certificates, and the next 18-21% for Bronze Certificates.

To receive certificates, centres must return their UK SMC results immediately after the event, in the envelope provided, to:

Heather Macklin, UK SMC, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

Past Papers

You can buy mixed packs of five assorted past NMC papers.

To obtain *N* packs, please send a cheque for £2*N*, payable to 'The Mathematical Association', to:

The Mathematical Association, 259 London Road, Leicester, LE2 3BE.

The British Mathematical Olympiad

The BMO Round 1, on Wednesday 15th January 1997, will be the first event to take place under the auspices of The United Kingdom Mathematics Trust.

BMO entry forms will be sent to all centres which return their UK SMC results promptly. A notional UK SMC qualifying score will be specified; but teachers are encouraged to use their own judgement as to which students to enter for the BMO. Entries for the BMO should be submitted well before Christmas.

Any comments or questions you have about the BMO should be sent to:

Alan West, BMO, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

Registered Charity Nº 313281

THE MATHEMATICAL ASSOCIATION

UK SENIOR MATHEMATICAL CHALLENGE 1996

(formerly the National Mathematics Contest)

INSTRUCTIONS AND INFORMATION

1. **Do not open this booklet until told to do so by the Invigilator.**
2. Detach the Answer Sheet and fill in your personal details before you open this booklet. Record all your answers on the Answer Sheet.
3. Make sure you have a pencil and a rubber. Use rough paper for rough work; don't use just the margins. **Calculators, measuring instruments and squared paper may not be used.**
4. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D, E. Only one of these is correct.
5. **Scoring Rules.** You receive 4 points for each correct answer, –1 for each incorrect answer, and 0 for each question left unanswered.
6. **Guessing.** Be careful ! Remember there is a penalty for wrong answers. All participants are advised to concentrate first on trying to solve as many as possible of the first 15 - 20 questions. Only then should you try later questions.
7. Diagrams are not necessarily drawn to scale.

FRIDAY 15th NOVEMBER 1996

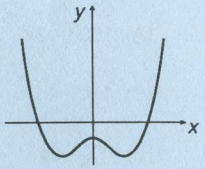
Time allowed: 90 minutes

21. A cube has its vertices removed by slicing a tetrahedron off each corner in such a way that each square face is changed into a regular octagon of side a. What is the volume of the new solid?

(A) $(6 + 4\sqrt{2})a^3$ (B) $(7 + \frac{13}{3}\sqrt{2})a^3$ (C) $7(1 + \frac{2}{3}\sqrt{2})a^3$

(D) $(7 + \frac{19}{4}\sqrt{2})a^3$ (E) $\frac{77}{3}a^3$

22. How many of the equations listed below could be the equation for the sketch graph shown?



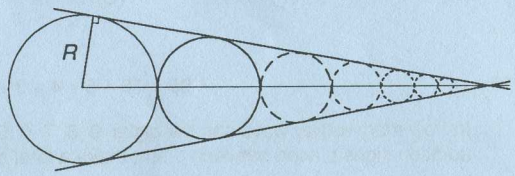
$y = x^4 - 2x^2 - 3$ $y = 3x^4 + 2x^2 - 1$

$y = x^4 + 3x^2 - 3$ $y = x^4 + 2x^2 - 3$

$y = 3x^4 - 2x^2 - 1$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

23. A pair of tangents is drawn to a circle, radius R, such that the angle between them is 60°. An infinite sequence of smaller circles is then drawn such that these tangents are tangent to all the circles. If each circle touches the next one, what is the total area of all the circles?

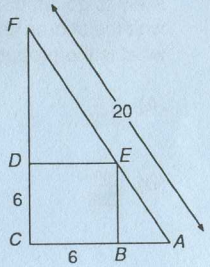


(A) $\frac{27\pi R^2}{26}$ (B) $\frac{9\pi R^2}{8}$ (C) $\frac{4\pi R^2}{3}$ (D) $\frac{3\pi R^2}{2}$ (E) none of these

24. What is the area of the largest equilateral triangle which fits inside a square of side a?

(A) $\frac{\sqrt{3}a^2}{4}$ (B) $(2\sqrt{3} - 3)a^2$ (C) $\frac{\sqrt{3}a^2}{2}$ (D) $\sqrt{3}a^2$ (E) $(3\sqrt{3} - 3)a^2$

25. A square $BCDE$ of side 6 is drawn in the triangle ACF such that E lies on AF. If AF is of length 20, what is the perimeter of the triangle ACF?



(A) $26 + 2\sqrt{109}$ (B) $26 + 2\sqrt{118}$

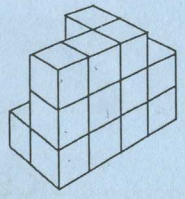
(C) 48 (D) $32 + 2\sqrt{109}$

(E) $32 + 2\sqrt{118}$

1. Which of the following is an odd number?

(A) $1^4 + 1$ (B) $3^4 + 2$ (C) $5^4 + 3$ (D) $7^4 + 5$ (E) $11^4 + 7$

2. The solid shown is made from one-centimetre cubes placed, but not glued, together. (It rests on a flat surface and some of the cubes may be hidden.) What is the minimum number of such cubes required to make the solid?



(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

3. How many square numbers between 1 and 1001 are divisible by 2?

(A) 15 (B) 16 (C) 30 (D) 32 (E) 500

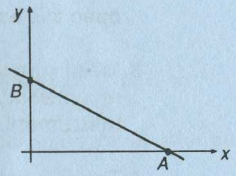
4. Which expression does **not** equal 1996?

(A) $500^2 - 498^2$ (B) $2^2(500 - 1)$ (C) $\frac{50000 - 100}{25}$

(D) $500 - 1 \times 4$ (E) $5 \times 20^2 - 2^2$

5. The line AB shown has a gradient of $\frac{-1}{k}$ $(k > 1)$.

The y co-ordinate of the point B is k. What is the x co-ordinate of the point A?



(A) $\frac{1}{k^2}$ (B) $\frac{1}{k}$ (C) 1 (D) k (E) k^2

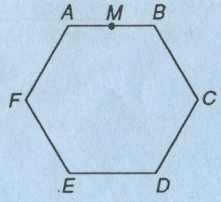
6. Your bone marrow makes approximately two hundred thousand million new red blood cells every day. (To keep the total number constant, the same number of 'worn out' red blood cells are destroyed every day by your spleen.) About how many new red blood cells will you produce while you are sitting this paper?

(A) 1×10^7 (B) 1×10^{10} (C) 1×10^{11} (D) 2×10^{11} (E) 3×10^{12}

7. The number n is a perfect square. What is the value of the next perfect square above it?

(A) $n + \sqrt{n}$ (B) $n + 2\sqrt{n} + 1$ (C) $n^2 + 1$ (D) $n^2 + n$ (E) $n^2 + 2n + 1$

8. A regular hexagon $ABCDEF$ has sides of length 2cm.
The mid-point of AB is M.
Which line segment has length $\sqrt{13}$ cm?



(A) BD (B) BE (C) EM

(D) FM (E) none of these

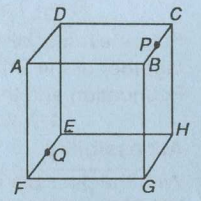
9. Arthur Weekly sold two quality used cars for £9999 each. On one he made a 10% profit and on the other a 10% loss. What was his overall profit or loss over the two transactions?

(A) Loss of £202 (B) Loss of £101 (C) Broke even (D) Profit of £101 (E) Profit of £202

10. In the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, if f and u are both halved, what is the effect on v?

(A) quadrupled (B) doubled (C) unchanged (D) halved (E) quartered

11. The solid *ABCDEFGH* is a cube. The point *P* is the mid-point of *BC* and the point *Q* is the mid-point of *EF*. Which of the following **best** describes the plane figure *APHQ*?



(A) quadrilateral (B) parallelogram (C) square

(D) rhombus (E) rectangle

12. During an experiment the radius of a circular metal disc expands by 3%. Roughly what is the expansion in its area?

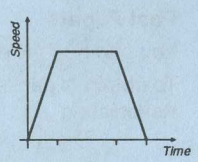
(A) 0.09% (B) 6% (C) 9% (D) 3π% (E) 9π%

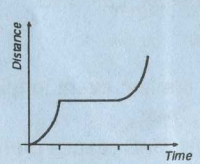
13. If two non-zero numbers, x and y, are such that their product is twice their sum, which of the following equations is correct?

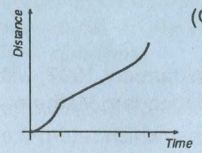
(A) $\frac{1}{x} + \frac{1}{y} = 2$ (B) $\frac{1}{x+y} = 2xy$ (C) $y = \frac{2x}{x+2}$ (D) $x = \frac{2y}{2-y}$

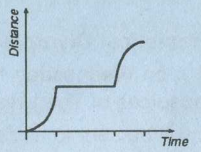
(E) $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

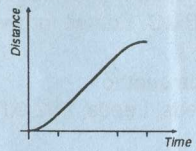
14. The diagram shows the speed-time graph of a train between two stations on its route. Which of the following graphs may describe the relationship between the distance travelled and the time during the same journey?

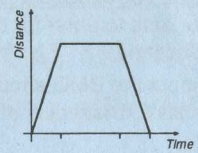


(A)


(B)


(C)


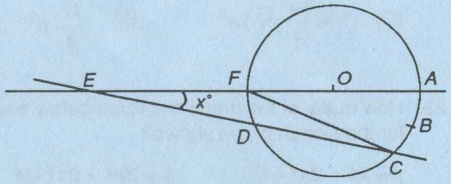
(D)


(E)


15. What is the middle digit of the product of 968880726456484032 and 875?

(A) 2 (B) 4 (C) 5 (D) 6 (E) 9

16. The centre of the circle is *O* and *EF* = *FC*. If *B* is the mid-point of the arc *AC*, what is the size of the angle *ABC*?



(A) $180^\circ - 2x^\circ$ (B) $180^\circ - x^\circ$

(C) $180^\circ - \frac{x^\circ}{2}$ (D) $90^\circ + x^\circ$

(E) $90^\circ + 2x^\circ$

17. If $x + \frac{1}{x} = 8$, what is the value of $x^4 + \frac{1}{x^4}$?

(A) 8^4 (B) $8^4 + 2$ (C) $8^4 - 2^8 + 2$ (D) $8^4 + 2^8 - 2$ (E) $8^4 + 2^8$

18. $98 + 76 + 5 + 4 + 3 + 2 + 10 = 198$

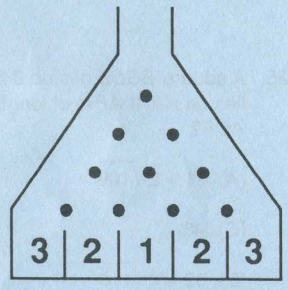
In how many **other** ways can the digits 9 8 7 6 5 4 3 2 1 0, in that order, with only addition signs placed between them, make a total of 198?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. Tamara was walking along a straight coastline at 4km/hour from east to west. Meanwhile a small fishing boat was travelling parallel to the shore. When Tamara first sighted the boat it was on a bearing of 060°. Two hours later she could still see the boat, but by then it was on a bearing of 330° from her new position. The shortest distance between the boat and the shore was always 1500m. What was the average speed of the boat in km/hour?

(A) $4 + \sqrt{3}$ (B) $8 - \sqrt{3}$ (C) $4 + 2\sqrt{3}$ (D) $8 + \sqrt{3}$ (E) $8 + 2\sqrt{3}$

20. In a fairground game, balls are rolled down a chute and bump into equally spaced pins. When they hit a pin, they are equally likely to go left or right. At the bottom the balls fall into boxes with the scores shown in the diagram. If three balls are rolled, what is the probability that the total score is 6?



(A) $\frac{11}{512}$ (B) $\frac{7}{128}$ (C) $\frac{17}{256}$

(D) $\frac{41}{256}$ (E) $\frac{17}{64}$