

**ADVANCED SUBSIDIARY GCE UNIT  
MATHEMATICS (MEI)**

Mechanics 1

**MONDAY 21 MAY 2007**

**4761/01**

Morning  
Time: 1 hour 30 minutes

Additional materials:  
Answer booklet (8 pages)  
Graph paper  
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

## Section A (36 marks)

- 1 Fig. 1 shows four forces in equilibrium.

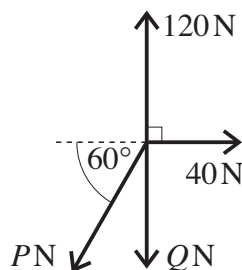


Fig. 1

(i) Find the value of  $P$ . [3]

(ii) Hence find the value of  $Q$ . [2]

- 2 A car passes a point A travelling at  $10 \text{ m s}^{-1}$ . Its motion over the next 45 seconds is modelled as follows.

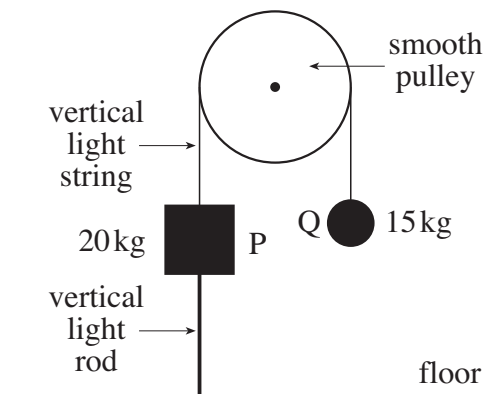
- The car's speed increases uniformly from  $10 \text{ m s}^{-1}$  to  $30 \text{ m s}^{-1}$  over the first 10 s.
- Its speed then increases uniformly to  $40 \text{ m s}^{-1}$  over the next 15 s.
- The car then maintains this speed for a further 20 s at which time it reaches the point B.

(i) Sketch a speed-time graph to represent this motion. [3]

(ii) Calculate the distance from A to B. [3]

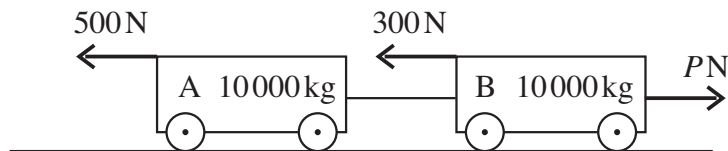
(iii) When it reaches the point B, the car is brought uniformly to rest in  $T$  seconds. The total distance from A is now 1700 m. Calculate the value of  $T$ . [2]

- 3 Fig. 3 shows a system in equilibrium. The rod is firmly attached to the floor and also to an object, P. The light string is attached to P and passes over a smooth pulley with an object Q hanging freely from its other end.



**Fig. 3**

- (i) Why is the tension the same throughout the string? [1]
- (ii) Calculate the force in the rod, stating whether it is a tension or a thrust. [3]
- 4 Two trucks, A and B, each of mass 10 000 kg, are pulled along a straight, horizontal track by a constant, horizontal force of  $P$  N. The coupling between the trucks is light and horizontal. This situation and the resistances to motion of the trucks are shown in Fig. 4.

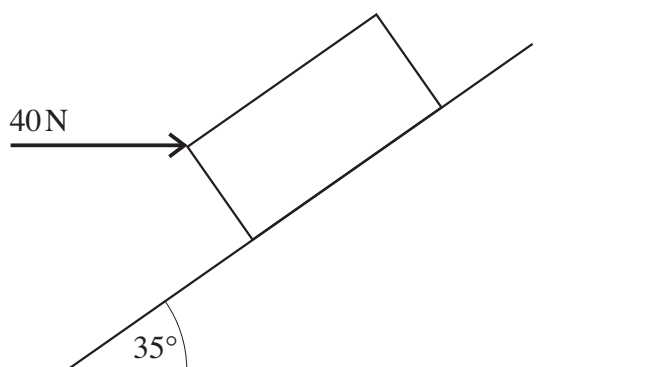


**Fig. 4**

The acceleration of the system is  $0.2 \text{ m s}^{-2}$  in the direction of the pulling force of magnitude  $P$ .

- (i) Calculate the value of  $P$ . [3]
- Truck A is now subjected to an extra resistive force of 2000 N while  $P$  does not change.
- (ii) Calculate the new acceleration of the trucks. [2]
- (iii) Calculate the force in the coupling between the trucks. [2]

- 5 A block of weight 100 N is on a rough plane that is inclined at  $35^\circ$  to the horizontal. The block is in equilibrium with a horizontal force of 40 N acting on it, as shown in Fig. 5.



**Fig. 5**

Calculate the frictional force acting on the block. [4]

- 6 A rock of mass 8 kg is acted on by just the two forces  $-80\mathbf{k}$  N and  $(-\mathbf{i} + 16\mathbf{j} + 72\mathbf{k})$  N, where  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane and  $\mathbf{k}$  is a unit vector vertically upward.

(i) Show that the acceleration of the rock is  $(-\frac{1}{8}\mathbf{i} + 2\mathbf{j} - \mathbf{k})\text{ms}^{-2}$ . [2]

The rock passes through the origin of position vectors, O, with velocity  $(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) \text{ m s}^{-1}$  and 4 seconds later passes through the point A.

(ii) Find the position vector of A. [3]

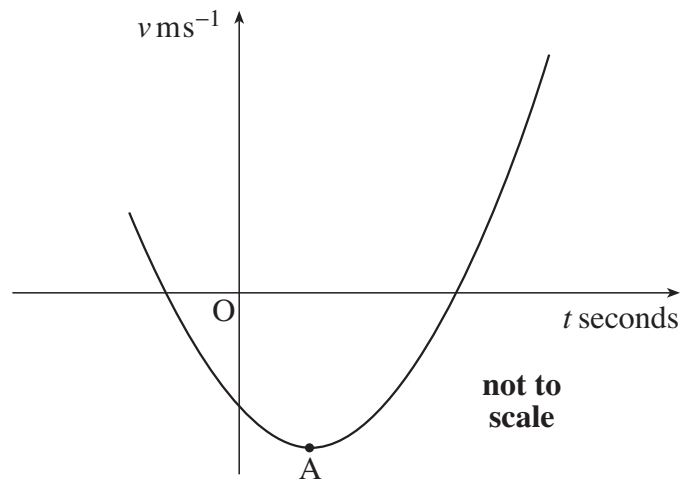
(iii) Find the distance OA. [1]

(iv) Find the angle that OA makes with the horizontal. [2]

**Section B** (36 marks)

- 7 Fig. 7 is a sketch of part of the velocity-time graph for the motion of an insect walking in a straight line. Its velocity,  $v \text{ m s}^{-1}$ , at time  $t$  seconds for the time interval  $-3 \leq t \leq 5$  is given by

$$v = t^2 - 2t - 8.$$

**Fig. 7**

- (i) Write down the velocity of the insect when  $t = 0$ . [1]
- (ii) Show that the insect is instantaneously at rest when  $t = -2$  and when  $t = 4$ . [2]
- (iii) Determine the velocity of the insect when its acceleration is zero.  
Write down the coordinates of the point A shown in Fig. 7. [5]
- (iv) Calculate the distance travelled by the insect from  $t = 1$  to  $t = 4$ . [5]
- (v) Write down the distance travelled by the insect in the time interval  $-2 \leq t \leq 4$ . [1]
- (vi) How far does the insect walk in the time interval  $1 \leq t \leq 5$ ? [3]

- 8** A ball is kicked from ground level over horizontal ground. It leaves the ground at a speed of  $25 \text{ m s}^{-1}$  and at an angle  $\theta$  to the horizontal such that  $\cos \theta = 0.96$  and  $\sin \theta = 0.28$ .

(i) Show that the height,  $y$  m, of the ball above the ground  $t$  seconds after projection is given by  $y = 7t - 4.9t^2$ . Show also that the horizontal distance,  $x$  m, travelled by this time is given by  $x = 24t$ . [3]

(ii) Calculate the maximum height reached by the ball. [2]

(iii) Calculate the times at which the ball is at half its maximum height.

Find the horizontal distance travelled by the ball between these times. [4]

(iv) Determine the following when  $t = 1.25$ .

(A) The vertical component of the velocity of the ball.

(B) Whether the ball is rising or falling. (You should give a reason for your answer.)

(C) The speed of the ball. [5]

(v) Show that the equation of the trajectory of the ball is

$$y = \frac{0.7x}{576} (240 - 7x).$$

Hence, or otherwise, find the range of the ball. [5]

<b>Q1</b>				
(i)	$\rightarrow 40 - P \cos 60 = 0$  $P = 80$	M1  A1 A1	For any resolution in an equation involving $P$ . Allow for $P = 40 \cos 60$ or $P = 40 \cos 30$ or $P = 40 \sin 60$ or $P = 40 \sin 30$ Correct equation cao	3
(ii)	$\downarrow Q + P \cos 30 = 120$  $Q = 40(3 - \sqrt{3}) = 50.7179 \dots$ so 50.7 (3 s. f.)	M1  A1	Resolve vert. All forces present. Allow $\sin \leftrightarrow \cos$ No extra forces. Allow wrong signs.  cao	2
				5

<b>Q2</b>				
(i)	Straight lines connecting (0, 10), (10, 30), (25, 40) and (45, 40)	B1  B1 B1	Axes with labels (words or letter). Scales indicated. Accept no arrows. Use of straight line segments and horiz section All correct with salient points clearly indicated	3
(ii)	$0.5(10 + 30) \times 10 + 0.5(30 + 40) \times 15 + 40 \times 20$  $= 200 + 525 + 800 = 1525$	M1 M1 A1	Attempt at area(s) or use of appropriate <i>uvast</i> Evidence of attempt to find whole area cao	3
(iii)	$0.5 \times 40 \times T = 1700 - 1525$ so $20T = 175$ and $T = 8.75$	M1 F1	Equating triangle area to $1700 - \text{their (ii)}$ $(1700 - \text{their (ii)})/20$ . Do not award for – ve answer.	2
				8

<b>Q3</b>				
(i)	String light and pulley smooth	E1	Accept pulley smooth alone	1
(ii)	5g (49) N thrust	M1 B1 A1	Three forces in equilibrium. Allow sign errors. for 15g (147) N used as a tension 5g (49) N thrust. Accept $\pm 5g$ (49). Ignore diagram. [Award SC2 for $\pm 5g$ (49) N without ‘thrust’ and SC3 if it is]	3
				4

Q4				
(i)	$P - 800 = 20000 \times 0.2$ $P = 4800$	M1  A1 A1	N2L. Allow $F = mga$ . Allow wrong or zero resistance. No extra forces. Allow sign errors. If done as 1 equn need $m = 20\,000$ . If A and B analysed separately, must have 2 equns with ' $T$ '. N2L correct.	3
(ii)	New accn $4800 - 2800 = 20000a$  $a = 0.1$	M1  A1	$F = ma$ . Finding new accn. No extra forces. Allow 500 N but not 300 N omitted. Allow sign errors. FT <b>their</b> $P$	2
(iii)	$T - 2500 = 10000 \times 0.1$  $T = 3500$ so 3500 N	M1  A1	N2L with new $a$ . Mass 10000. All forces present for A or B except allow 500 N omitted on A. No extra forces cao	2
				7

Q5				
	Take $F$ +ve up the plane $F + 40 \cos 35 = 100 \sin 35$  $F = 24.5915 \dots$ so 24.6 N (3 s. f.)  up the plane	M1  B1 A1 A1	Resolve // plane (or horiz or vert). All forces present. At least one resolved. Allow $\sin \leftrightarrow \cos$ and sign errors. Allow 100g used. Either $\pm 40 \cos 35$ or $\pm 100 \sin 35$ or equivalent seen Accept $\pm 24.5915 \dots$ or $\pm 90.1237 \dots$ even if inconsistent or wrong signs used. 24.6 N up the plane (specified or from diagram) or equiv all obtained from consistent and correct working.	4
				4

Q6				
(i)	$(-\mathbf{i} + 16\mathbf{j} + 72\mathbf{k}) + (-80\mathbf{k}) = 8\mathbf{a}$ $\mathbf{a} = \left(-\frac{1}{8}\mathbf{i} + 2\mathbf{j} - \mathbf{k}\right) \text{ m s}^{-2}$	M1 E1	Use of N2L. All forces present. Need at least the <b>k</b> term clearly derived	2
(ii)	$\mathbf{r} = 4(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) + 0.5 \times 16 \left(-\frac{1}{8}\mathbf{i} + 2\mathbf{j} - \mathbf{k}\right)$ $= 3\mathbf{i} + 4\mathbf{k}$	M1 A1 A1	Use of appropriate <b>uvas</b> t or integration (twice) Correct substitution (or limits if integrated)	3
(iii)	$\sqrt{3^2 + 4^2} = 5$ so 5 m	B1	FT <b>their</b> (ii) even if it not a displacement. Allow surd form	1
(iv)	$\arctan \frac{4}{3}$  $= 53.130\dots$ so $53.1^\circ$ (3 s. f.)	M1  A1	Accept $\arctan \frac{3}{4}$ . FT <b>their</b> (ii) even if not a displacement. Condone sign errors. (May use $\arcsin 4/5$ or equivalent. FT <b>their</b> (ii) and (iii) even if not displacement. Condone sign errors) cao	2
				8

<b>Q7</b>				
(i)	8 m s <sup>-1</sup> (in the negative direction)	B1	Allow $\pm$ and no direction indicated	1
(ii)	$(t+2)(t-4) = 0$ so $t = -2$ or 4	M1 A1	Equating $v$ to zero and solving or subst If subst used then both must be clearly shown	2
(iii)	$a = 2t - 2$  $a = 0$ when $t = 1$ $v(1) = 1 - 2 - 8 = -9$ so 9 m s <sup>-1</sup> in the negative direction  (1, -9)	M1 A1 F1  A1  B1	Differentiating Correct  Accept -9 but not 9 without comment  FT	5
(iv)	$\int_1^4 (t^2 - 2t - 8) \, dx$  $= \left[ \frac{t^3}{3} - t^2 - 8t \right]_1^4$  $= \left( \frac{64}{3} - 16 - 32 \right) - \left( \frac{1}{3} - 1 - 8 \right)$ $= -18$  distance is 18 m	M1  A1  M1 A1 A1	Attempt at integration. Ignore limits.  Correct integration. Ignore limits.  Attempt to sub correct limits and subtract Limits correctly evaluated. Award if -18 seen but no need to evaluate Award even if -18 not seen. Do not award for -18. cao	5
(v)	$2 \times 18 = 36$ m	F1	Award for $2 \times$ <b>their</b> (iv).	1
(vi)	$\int_4^5 (t^2 - 2t - 8) \, dx = \left[ \frac{t^3}{3} - t^2 - 8t \right]_4^5$  $= \left( \frac{125}{3} - 25 - 40 \right) - \left( -\frac{80}{3} \right) = 3\frac{1}{3}$  so $3\frac{1}{3} + 18 = 21\frac{1}{3}$ m	M1  A1  A1	$\int_4^5$ attempted or, otherwise, complete method seen.  Correct substitution  Award for $3\frac{1}{3} +$ <b>their</b> (positive) (iv)	3
				17

Q8				
(i)	$y = 25 \sin \theta t + 0.5 \times (-9.8)t^2$  $= 7t - 4.9t^2$  $x = 25 \cos \theta t = 25 \times 0.96t = 24t$	M1  E1  B1	Use of $s = ut + \frac{1}{2}at^2$ . Accept sin, cos, 0.96, 0.28, $\pm 9.8$ , $\pm 10$ , $u = 25$ and derivation of $-4.9$ not clear.  Shown including deriv of $-4.9$ . Accept $25 \sin \theta t = 7t$ WW  Accept $25 \times 0.96t$ or $25 \cos \theta t$ seen WW	3
(ii)	$0 = 7^2 - 19.6s$  $s = 2.5$ so 2.5 m	M1  A1	Accept sequence of $uvast$ . Accept $u=24$ but not 25. Allow $u \leftrightarrow v$ and $\pm 9.8$ and $\pm 10$ +ve answer obtained by correct manipulation.	2
(iii)	Need $7t - 4.9t^2 = 1.25$ so $4.9t^2 - 7t + 1.25 = 0$  $t = 0.209209\dots$ and $1.219361\dots$  need $24 \times (1.219\dots - 0.209209\dots)$ $= 24 \times 1.01\dots$ so 24.2 m (3 s.f.)	M1  M1  A1  B1	Equate $y$ to <b>their</b> (ii)/2 or equivalent.  Correct sub into quad formula of their 3 term quadratic being solved (i.e. allow manipulation errors before using the formula).  Both. cao. [Award M1 A1 for two correct roots WW]  FT <b>their</b> roots (only if both positive)	4
(iv) (A)  (B)  (C)	$\dot{y} = 7 - 9.8t$  $\dot{y}(1.25) = 7 - 9.8 \times 1.25 = -5.25 \text{ m s}^{-1}$  Falling as velocity is negative  Speed is $\sqrt{24^2 + (-5.25)^2}$ $= 24.5675\dots$ so 24.6 m s <sup>-1</sup> (3 s.f.)	M1  A1  E1  M1  A1	Attempt at $\dot{y}$ . Accept sign errors and $u = 24$ but not 25  Reason must be clear. FT <b>their</b> $\dot{y}$ even if not a velocity Could use an argument involving time. Use of Pythag and 24 or 7 with <b>their</b> $\dot{y}$ cao	5

(v)	$y = 7t - 4.9t^2, x = 24t$	M1	Elimination of $t$	
	so $y = \frac{7x}{24} - 4.9\left(\frac{x}{24}\right)^2$	A1	Elimination correct. Condone wrong notation with interpretation correct for the problem.	
	$y = \frac{7x}{24} - 4.9 \times \frac{x^2}{576} = \frac{0.7x}{576}(240 - 7x)$	E1	If not wrong accept as long as $24^2 = 576$ seen.	
	<b>either</b> Need $y = 0$	M1	Condone wrong notation with interpretation correct for the problem.	
	so $x = 0$ or $\frac{240}{7}$ so $\frac{240}{7}$ m	A1	Accept $x = 0$ not mentioned. Condone $0 \leq X \leq \frac{240}{7}$ .	
	<b>or</b>	B1	Time of flight $\frac{10}{7}$ s	
		B1	Range $\frac{240}{7}$ m. Condone $0 \leq X \leq \frac{240}{7}$ .	
				5
				19