OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Registered Charity 1066969
[Turn over

Section A (36 marks)

1 A particle is thrown vertically upwards and returns to its point of projection after 6 seconds. Air resistance is negligible.

Calculate the speed of projection of the particle and also the maximum height it reaches. [4]

2 Force $\mathbf{F}_1$ is $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$N and force $\mathbf{F}_2$ is $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$N, where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are vectors east and north respectively.

(i) Calculate the magnitude of $\mathbf{F}_1$, correct to three significant figures. [2]

(ii) Calculate the direction of the force $\mathbf{F}_1 - \mathbf{F}_2$ as a bearing. [3]

Force $\mathbf{F}_2$ is the resultant of all the forces acting on an object of mass 5 kg.

(iii) Calculate the acceleration of the object and the change in its velocity after 10 seconds. [3]

3 A train consists of an engine of mass 10 000 kg pulling one truck of mass 4000 kg. The coupling between the engine and the truck is light and parallel to the track.

The train is accelerating at 0.25 m s^{-2} along a straight, level track.

(i) What is the resultant force on the train in the direction of its motion? [2]

The driving force of the engine is 4000 N.

(ii) What is the resistance to the motion of the train? [1]

(iii) If the tension in the coupling is 1150 N, what is the resistance to the motion of the truck? [2]

With the same overall resistance to motion, the train now climbs a uniform slope inclined at 3° to the horizontal with the same acceleration of 0.25 m s^{-2}.

(iv) What extra driving force is being applied? [3]

[Turn over

4 Fig. 4 shows the unit vectors **i** and **j** in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

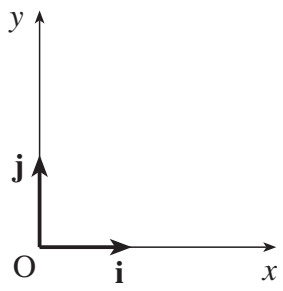


Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the **j** direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

5 You should neglect air resistance in this question.

A small stone is projected from ground level. The maximum height of the stone above horizontal ground is 22.5 m.

(i) Show that the vertical component of the initial velocity of the stone is 21 m s^{-1}. [2]

The speed of projection is 28 m s^{-1}.

(ii) Find the angle of projection of the stone. [2]

(iii) Find the horizontal range of the stone. [4]

Section B (36 marks)

6 A toy car is travelling in a straight horizontal line.

One model of the motion for $0 \leqslant t \leqslant 8$, where t is the time in seconds, is shown in the velocity–time graph Fig. 6.

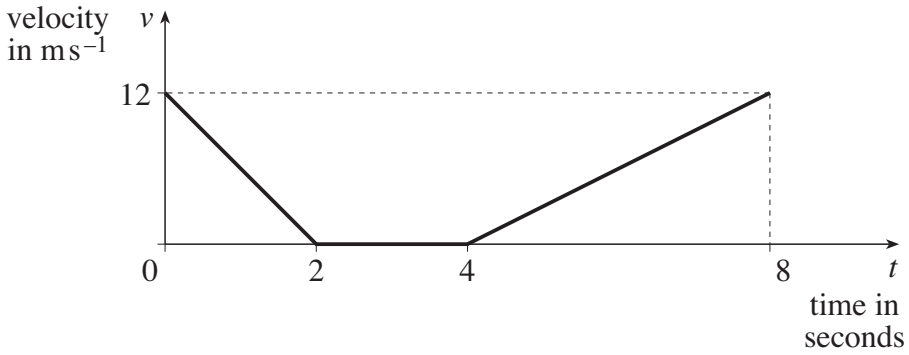


Fig. 6

(i) Calculate the distance travelled by the car from $t = 0$ to $t = 8$. [2]

(ii) How much less time would the car have taken to travel this distance if it had maintained its initial speed throughout? [1]

(iii) What is the acceleration of the car when $t = 1$? [2]

From $t = 8$ to $t = 14$, the car travels 58.5 m with a new constant acceleration, a m s^{-2}.

(iv) Find a. [2]

A second model for the velocity, v m s^{-1}, of the toy car is

$$v = 12 - 10t + \tfrac{9}{4}t^2 - \tfrac{1}{8}t^3, \text{ for } 0 \leqslant t \leqslant 8.$$

This model agrees with the values for v given in Fig. 6 for $t = 0, 2, 4$ and 6. [Note that you are not required to verify this.] Use this second model to answer the following questions.

(v) Calculate the acceleration of the car when $t = 1$. [3]

(vi) Initially the car is at A. Find an expression in terms of t for the displacement of the car from A after the first t seconds of its motion.

Hence find the displacement of the car from A when $t = 8$. [5]

(vii) Explain with a reason what this model predicts for the motion of the car between $t = 2$ and $t = 4$. [3]

[Turn over

7 A box of weight 147 N is held by light strings AB and BC. As shown in Fig. 7.1, AB is inclined at α to the horizontal and is fixed at A; BC is held at C. The box is in equilibrium with BC horizontal and α such that $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$.

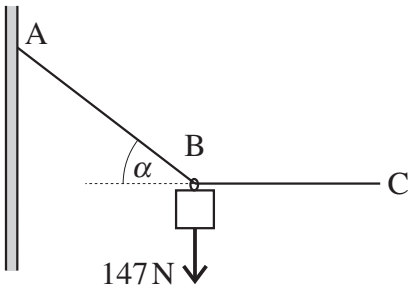


Fig. 7.1

(i) Calculate the tension in string AB. [3]

(ii) Show that the tension in string BC is 196 N. [2]

As shown in Fig. 7.2, a box of weight 90 N is now attached at C and another light string CD is held at D so that the system is in equilibrium with BC still horizontal. CD is inclined at β to the horizontal.

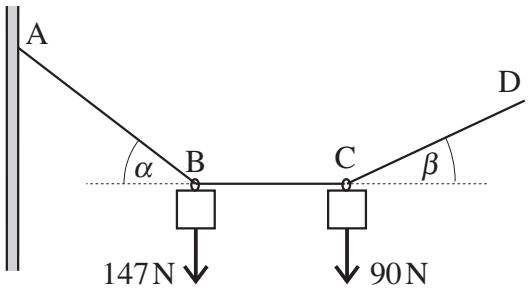


Fig. 7.2

(iii) Explain why the tension in the string BC is still 196 N. [2]

(iv) Draw a diagram showing the forces acting on the box at C.

Find the angle β and show that the tension in CD is 216 N, correct to three significant figures. [7]

The string section CD is now taken over a smooth pulley and attached to a block of mass M kg on a rough slope inclined at 40° to the horizontal. As shown in Fig. 7.3, the part of the string attached to the box is still at β to the horizontal and the part attached to the block is parallel to the slope. The system is in equilibrium with a frictional force of 20 N acting on the block **up** the slope.

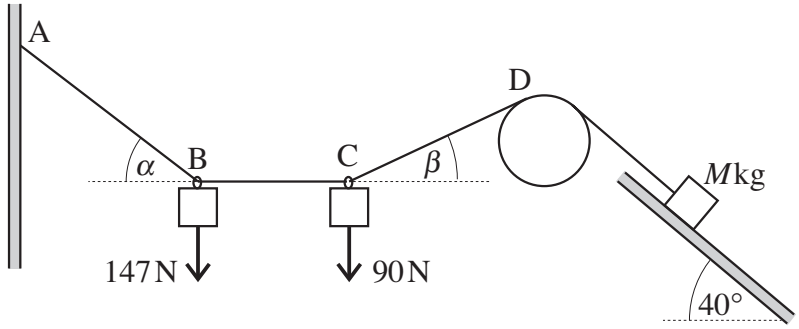


Fig. 7.3

(v) Calculate the value of M. [4]

Q 1

	mark		Sub
$0 = u - 9.8\times 3$ $u = 29.4$ so 29.4 m s^{-1}	M1 A1	*uvast* leading to u with $t = 3$ or $t = 6$ Signs consistent	
$s = 0.5\times 9.8\times 9 = 44.1$ so 44.1 m	M1 F1	*uvast* leading to s with $t = 3$ or $t = 6$ or **their** u FT **their** u if used with $t = 3$. Signs consistent. Award for 44.1, 132.3 or 176.4 seen. [Award maximum of 3 if one answer wrong]	4
			4

Q 2

		mark		Sub
(i)	$\sqrt{(-6)^2 + 13^2} = 14.31782...$ so 14.3 N (3 s. f.)	M1 A1	Accept $\sqrt{-6^2 + 13^2}$	2
(ii)	Resultant is $\begin{pmatrix}-6\\13\end{pmatrix} - \begin{pmatrix}-3\\5\end{pmatrix} = \begin{pmatrix}-3\\8\end{pmatrix}$	B1	May not be explicit. If diagram used it must have correct orientation. Give if final angle correct.	
	Require 270 + $\arctan\frac{8}{3}$	M1	Use of $\arctan\left(\pm\frac{8}{3}\right)$ or $\arctan\left(\pm\frac{3}{8}\right)$ ($\pm 20.6°$ or $\pm 69.4°$) or equivalent on **their** resultant	
	so 339.4439…° so 339°	A1	cao. Do not accept -21°.	3
(iii)	$\begin{pmatrix}-3\\5\end{pmatrix} = 5\mathbf{a}$	M1	Use of N2L with accn *used* in vector form	
	so ($-0.6\,\mathbf{i} + \mathbf{j}$) m s^{-2}	A1	Any form. Units not required. isw.	
	change in velocity is ($-6\,\mathbf{i} + 10\,\mathbf{j}$) m s^{-1}	F1	$10\mathbf{a}$ seen. Units not required. Must be a vector. [SC1 for $a = \sqrt{3^2 + 5^2}/5 = 1.17$]	3
				8

Q 3		mark		Sub
(i)	$F = 14000 \times 0.25$	M1	Use of N2L . Allow $F = mga$ and wrong mass. No extra forces.	
	so 3500 N	A1		2
(ii)	$4000 - R = 3500$ so 500 N	B1	FT F from (i). Condone negative answer.	1
(iii)	$1150 - R_T = 4000 \times 0.25$	M1	N2L applied to truck (or engine) using all forces required. No extras. Correct mass. Do not allow use of $F = mga$. Allow sign errors.	
	so 150 N	A1	cao	2
(iv)	**either**			
	Component of weight down slope is	M1	Attempt to find cpt of *weight* (allow wrong mass). Accept $\sin \leftrightarrow \cos$. Accept use of $m \sin\theta$.	
	Extra driving force is cpt of mg down slope	M1	May be implied. Correct mass. No extra forces. Must have resolved weight component. Allow $\sin \leftrightarrow \cos$	
	$14000g \sin 3°$ $= 14000 \times 9.8 \times 0.0523359... = 7180.49...$ so 7180 N (3 s. f.)	A1		
	or			
		M1	Attempt to find cpt of *weight* (allow wrong mass). Accept $\sin \leftrightarrow \cos$. Accept use of $m \sin\theta$.	
	$D - 500 - 14000g \sin 3 = 14000 \times 0.25$	M1	N2L with all terms present with correct signs and mass. No extras. FT 500 N. Accept **their** 500 + 150 for resistance. Must have resolved weight component. Allow $\sin \leftrightarrow \cos$.	
	D = 11180.49… so extra is 7180 N (3 s. f.)	A1	Must be the extra force.	3
				8

Q 4 | **mark** | **Sub**

(i) **either**

Need **j** cpt 0 so $18t^2 - 1 = 0$ — M1 — Need not solve

$\Rightarrow t^2 = \frac{1}{18}$. Only one root as $t > 0$ — E1 — Must establish only one of the two roots is valid

or

Establish sign change in **j** cpt — B1

Establish only one root — B1

2

(ii) $\mathbf{v} = 3\,\mathbf{i} + 36t\,\mathbf{j}$ — M1 — Differentiate. Allow **i** or **j** omitted

A1

Need **i** cpt 0 and this never happens — E1 — Clear explanation. Accept '**i** cpt always there' or equiv

3

(iii) $x = 3t$ and $y = 18t^2 - 1$ — B1 — Award for these two expressions seen.

Eliminate t to give

$y = 18\left(\frac{x}{3}\right)^2 - 1$ — M1 — t properly eliminated. Accept any form and brackets missing

so $y = 2x^2 - 1$ — A1 — cao

3

8

Q 5 | **mark** | **Sub**

(i) $0^2 = V^2 - 2 \times 9.8 \times 22.5$ — M1 — Use of appropriate *uvast*. Give for correct expression

$V = 21$ so 21 m s^{-1} — E1 — Clearly shown. Do not allow $v^2 = 0 + 2gs$ without explanation. Accept using $V = 21$ to show $s = 22.5$.

2

(ii) $28\sin\theta = 21$ — M1 — Attempt to find angle of projection. Allow $\sin \leftrightarrow \cos$.

so $\theta = 48.59037...$ — A1

2

(iii) Time to highest point is $\frac{21}{9.8} = \frac{15}{7}$ — B1 — Or equivalent (time of whole flight)

Distance is $2 \times \frac{15}{7} \times 28 \times \cos(\mathbf{their}\,\theta)$.. — M1 — Valid method for horizontal distance. Accept ½ time.

Do not accept 28 used for horizontal speed or vertical speed when calculating time.

B1 — Horizontal speed correct

79.3725… so 79.4 m (3 s. f.) — A1 — cao. Accept answers rounding to 79 or 80.

[If angle with vertical found in (ii) allow up to full marks in (iii). If $\sin \leftrightarrow \cos$ allow up to B1 B1 M0 A1]

[If $u^2 \sin 2\theta / g$ used then

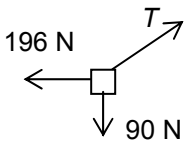
M1* Correct formula used. FT their angle.

M1 Dep on *. Correct subst. FT their angle. A2 cao]

4

8

Q 6		mark		Sub
(i)	$0.5\times2\times12+0.5\times4\times12$ so 36 m	M1 A1	Attempt at sum of areas or equivalent. No extra areas.	2
(ii)	$8-\frac{36}{12}=5$ seconds	B1	cao	1
(iii)	-6 m s^{-2}	M1 B1	Attempt at accn for $0\le t\le2$ must be - ve or equivalent	2
(iv)	$58.5=12\times6+0.5\times a\times36$ so $a=-0.75$	M1 A1	Use of *uvast* with 12 and 58.5	2
(v)	$a=-10+\frac{9}{2}t-\frac{3}{8}t^2$ $a(1)=-10+\frac{9}{2}-\frac{3}{8}=-5.875$	M1 A1 A1	Differentiation cao	3
(vi)	$s=\int\left(12-10t+\frac{9}{4}t^2-\frac{1}{8}t^3\right)\mathrm{d}t$ $=12t-5t^2+\frac{3}{4}t^3-\frac{1}{32}t^4+C$ $s=0$ when $t=0$ so $C=0$ $s(8)=32$	M1 A1 A1 A1*	Attempt to integrate At least one term correct All correct. Accept + C omitted Clearly shown cao (award even if A1* is not given)	5
(vii)	**either** $s(2)=9.5$ and $s(4)=8$ Displacement is negative Car going backwards **or** Evaluate $v(t)$ where $2<t<4$ or appeal to shape of the graph Velocity is negative Car going backwards	B1 E1 E1 B1 E1 E1	Both calculated correctly from **their** s. No further marks if **their** $s(2)\le s(4)$ Do *not* need car going backwards *throughout* the interval. e.g. $v(3)=-1.125$ No further marks if **their** $v\ge0$ Do *not* need car going backwards *throughout* the interval [Award WW2 for 'car going backwards'; WW1 for velocity or displacement negative]	3 18

Q 7		mark		Sub
(i)	$T_{AB}\sin\alpha = 147$	M1	Attempt at resolving. Accept $\sin \leftrightarrow \cos$. Must have T resolved and equated to 147.	
	so $T_{AB} = \dfrac{147}{0.6}$	B1	Use of 0.6. Accept correct subst for angle in wrong expression.	
	= 245 so 245 N	A1	Only accept answers agreeing to 3 s. f.	
			[Lami: M1 pair of ratios attempted; B1 correct sub;A1]	3
(ii)	$T_{BC} = 245\cos\alpha$	M1	Attempt to resolve 245 and equate to T, or equiv Accept $\sin \leftrightarrow \cos$	
	$= 245\times 0.8 = 196$	E1	Substitution of 0.8 clearly shown	
			[SC1 $245\times 0.8 = 196$]	2
			[Lami: M1 pair of ratios attempted; E1]	
(iii)	Geometry of A, B and C and weight of B the	E1	Mention of two of: same weight: same direction AB: same direction BC	
	same and these determine the tension	E1	Specific mention of same geometry & weight or recognition of same force diagram	2
(iv)			No extra forces.	
		B1	Correct orientation and arrows	
		B1	'T' 196 and 90 labelled. Accept 'tension' written out.	
	either			
	Realise that 196 N and 90 N are horiz and vert forces where resultant has magnitude and line of action of the tension	M1	Allow for only β or T attempted	
	$\tan\beta = 90/196$	B1	Use of arctan (196/90) or arctan (90/196) or equiv	
	$\beta = 24.6638...$ so 24.7 (3 s. f.)	A1		
	$T = \sqrt{196^2 + 90^2}$	M1	Use of Pythagoras	
	$T = 215.675\ldots$ so 216 N (3 s. f.)	E1		
	or			
	$\uparrow \quad T\sin\beta - 90 = 0$	B1	Allow if $T = 216$ assumed	
	$\rightarrow \quad T\cos\beta - 196 = 0$	B1	Allow if $T = 216$ assumed	
	Solving $\tan\beta = \dfrac{90}{196} = 0.45918...$	M1	Eliminating T, or…	
	$\beta = 24.6638...$ so 24.7 (3 s. f.)	A1	[If $T = 216$ assumed, B1 for β ; B1 for check in 2nd	
	$T = 215.675\ldots$ so 216 N (3 s. f.)	E1	equation; E0]	7
(v)	Tension on block is 215.675.. N (pulley is smooth and string is light)	B1	May be implied. Reasons not required.	
	$M\times 9.8\times\sin 40 = 215.675... + 20$	M1	*Equating* their tension on the block unresolved $\pm$ 20 to weight component. If equation in any other direction, normal reaction must be present.	
		A1	Correct	
	$M = 37.4128\ldots$ so 37.4 (3 s. f.)	A1	Accept answers rounding to 37 and 38	4
				18