

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4761

Mechanics 1

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:  
8 page answer booklet  
Graph paper  
MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .
- The total number of marks for this paper is 72.

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**This question paper consists of 5 printed pages and 3 blank pages.**

## Section A (36 marks)

- 1 A particle travels in a straight line during the time interval  $0 \leq t \leq 12$ , where  $t$  is the time in seconds. Fig. 1 is the velocity-time graph for the motion.

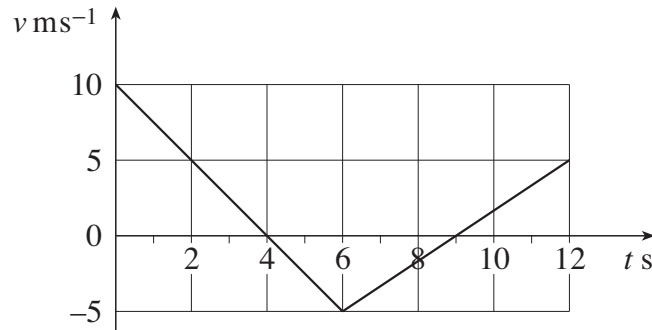


Fig. 1

- (i) Calculate the acceleration of the particle in the interval  $0 < t < 6$ . [2]
- (ii) Calculate the distance travelled by the particle from  $t = 0$  to  $t = 4$ . [2]
- (iii) When  $t = 0$  the particle is at A. Calculate how close the particle gets to A during the interval  $4 \leq t \leq 12$ . [2]
- 2 Fig. 2 shows a light string with an object of mass 4 kg attached at end A. The string passes over a smooth pulley and its other end B is attached to two light strings BC and BD of the same length. The strings BC and BD are attached to horizontal ground and are each inclined at  $20^\circ$  to the vertical.

The system is in equilibrium.

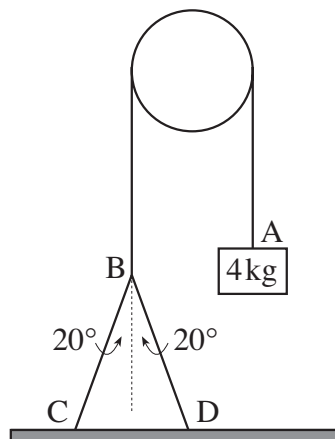


Fig. 2

- (i) What information in the question tells you that the tension is the same throughout the string AB? [1]
- (ii) What is the tension in the string AB? [1]
- (iii) Calculate the tension in the strings BC and BD. [3]

- 3 A force  $\mathbf{F}$  is given by  $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$  N, where  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors east and north respectively.
- (i) Calculate the magnitude of  $\mathbf{F}$  and also its direction as a bearing. [3]
- (ii)  $\mathbf{G}$  is the force  $(7\mathbf{i} + 24\mathbf{j})$  N. Show that  $\mathbf{G}$  and  $\mathbf{F}$  are in the same direction and compare their magnitudes. [2]
- (iii) Force  $\mathbf{F}_1$  is  $(9\mathbf{i} - 18\mathbf{j})$  N and force  $\mathbf{F}_2$  is  $(12\mathbf{i} + q\mathbf{j})$  N. Find  $q$  so that the sum  $\mathbf{F}_1 + \mathbf{F}_2$  is in the direction of  $\mathbf{F}$ . [2]
- 4 A car and its trailer travel along a straight, horizontal road. The coupling between them is light and horizontal. The car has mass 900 kg and resistance to motion 100 N, the trailer has mass 700 kg and resistance to motion 300 N, as shown in Fig. 4. The car and trailer have an acceleration of  $1.5 \text{ m s}^{-2}$ .

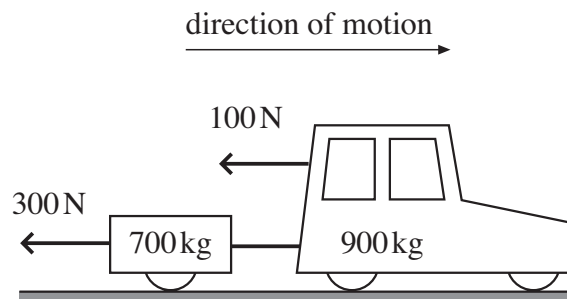


Fig. 4

- (i) Calculate the driving force of the car. [3]
- (ii) Calculate the force in the coupling. [2]
- 5 The acceleration of a particle of mass 4 kg is given by  $\mathbf{a} = (9\mathbf{i} - 4t\mathbf{j}) \text{ m s}^{-2}$ , where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors and  $t$  is the time in seconds.
- (i) Find the acceleration of the particle when  $t = 0$  and also when  $t = 3$ . [1]
- (ii) Calculate the force acting on the particle when  $t = 3$ . [1]
- The particle has velocity  $(4\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$  when  $t = 1$ .
- (iii) Find an expression for the velocity of the particle at time  $t$ . [4]

- 6 A car is driven with constant acceleration,  $a \text{ m s}^{-2}$ , along a straight road. Its speed when it passes a road sign is  $u \text{ m s}^{-1}$ . The car travels 14 m in the 2 seconds after passing the sign; 5 seconds after passing the sign it has a speed of  $19 \text{ m s}^{-1}$ .

(i) Write down two equations connecting  $a$  and  $u$ . Hence find the values of  $a$  and  $u$ . [5]

(ii) What distance does the car travel in the 5 seconds after passing the road sign? [2]

### Section B (36 marks)

- 7 Clive and Ken are trying to move a box of mass 50 kg on a rough, horizontal floor. As shown in Fig. 7, Clive always pushes horizontally and Ken always pulls at an angle of  $30^\circ$  to the horizontal. Each of them applies forces to the box in the same vertical plane as described below.

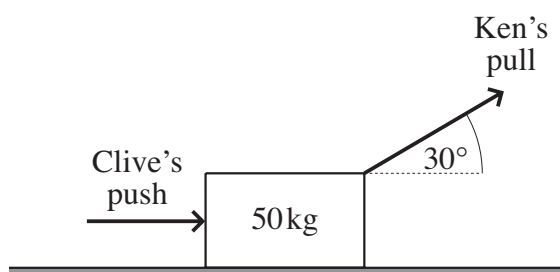


Fig. 7

Initially, the box is in equilibrium with Clive pushing with a force of 60 N and Ken not pulling at all.

(i) What is the resistance to motion of the box? [1]

Ken now adds a pull of 70 N to Clive's push of 60 N. The box remains in equilibrium.

(ii) What now is the resistance to motion of the box? [2]

(iii) Calculate the normal reaction of the floor on the box. [3]

The frictional resistance to sliding of the box is 125 N.

Clive now pushes with a force of 160 N but Ken does not pull at all.

(iv) Calculate the acceleration of the box. [2]

Clive stops pushing when the box has a speed of  $1.5 \text{ m s}^{-1}$ .

(v) How far does the box then slide before coming to rest? [4]

Ken and Clive now try again. Ken pulls with a force of  $Q \text{ N}$  and Clive pushes with a force of 160 N. The frictional resistance to sliding of the box is now 115 N and the acceleration of the box is  $3 \text{ m s}^{-2}$ .

(vi) Calculate the value of  $Q$ . [4]

- 8 A girl throws a small stone with initial speed  $14 \text{ m s}^{-1}$  at an angle of  $60^\circ$  to the horizontal from a point 1 m above the ground. She throws the stone directly towards a vertical wall of height 6 m standing on horizontal ground. The point O is on the ground directly below the point of projection, as shown in Fig. 8. Air resistance is negligible.

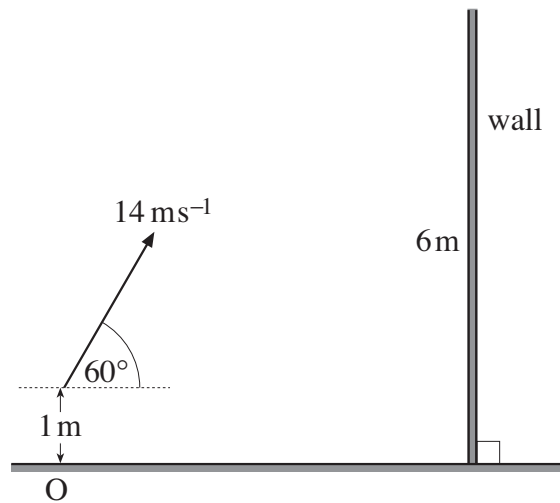


Fig. 8

- (i) Write down an expression in terms of  $t$  for the horizontal displacement of the stone from O,  $t$  seconds after projection. Find also an expression for the height of the stone above O at this time. [5]

The stone is at the top of its trajectory when it passes over the wall.

- (ii) (A) Find the time it takes for the stone to reach its highest point. [2]  
 (B) Calculate the distance of O from the base of the wall. [2]  
 (C) Show that the stone passes over the wall with 2.5 m clearance. [4]
- (iii) Find the cartesian equation of the trajectory of the stone referred to the horizontal and vertical axes, Ox and Oy. There is no need to simplify your answer. [2]

The girl now moves away a further distance  $d$  m from the wall. She throws a stone as before and it just passes over the wall.

- (iv) Calculate  $d$ . [5]

# Mark Scheme

## Section A

Q 1		mark		Sub
(i)	$\frac{-15}{6} = -2.5$ so $-2.5 \text{ m s}^{-2}$	M1 A1	Use of $\Delta v / \Delta t$ . Condone use of $v/t$ . Must have - ve sign. Accept no units.	2
(ii)	$\frac{1}{2} \times 10 \times 4 = 20 \text{ m}$	M1 A1	Attempt at area or equivalent	2
(iii)	Area under graph is $\frac{1}{2} \times 5 \times 5 = 12.5$ (and -ve)  closest is $20 - 12.5 = 7.5 \text{ m}$	M1 A1	May be implied. Area from 4 to 9 attempted. Condone missing -ve sign. Do not award if area beyond 9 is used (as well). cao	2
				6

Q 2		mark		Sub
(i)	Pulley is smooth (and the string is light)	E1	Only require pulley is smooth. Do not accept only 'string is light'.	1
(ii)	$4g = 39.2 \text{ N}$	B1	Accept either	1
(iii)	Let tension in each string be $T$  $39.2 = 2T \cos 20$  $T = 20.85788\dots$ so $20.9 \text{ N}$ (3 s.f.)	M1  B1 F1	Equating 39.2 to attempt at tensions in <b>both</b> BC and BD. Tensions need not be equal. No extra forces. Must attempt resolution. Condone $\sin \leftrightarrow \cos$ . For one occurrence of $T \cos 20$ in any equation. Accept reference to only one string. FT <b>their</b> $4g$  If Lami's Theorem used: M1 correct format B1 equation correct. FT <b>their</b> $4g$ F1 FT <b>their</b> $4g$  If Triangle of Forces used: M1 triangle with <b>their</b> $4g$ labelled <b>and</b> an	

			attempt to use this triangle. Ignore arrows. B1 for correct equation. FT <b>their</b> 4g. F1 FT <b>their</b> 4g.	3
				5

Q 3		mark		Sub
(i)	$ \mathbf{F}  = 12.5$ so 12.5 N bearing is $90 - \arctan \frac{12}{3.5}$ = (0)16.260... so (0)16.3° (3 s. f.)	B1 M1 A1	Use of arctan with 3.5 and 12 or equiv May be obtained directly as $\arctan \frac{3.5}{12}$	3
(ii)	$24/7 = 12/3.5$ or .... $\mathbf{G} = 2\mathbf{F}$ so $ \mathbf{G}  = 2 \mathbf{F} $	E1 B1	Accept <b>statement</b> following $\mathbf{G} = 2\mathbf{F}$ shown. Accept equivalent in words.	2
(iii)	$\frac{9+12}{3.5} = \frac{-18+q}{12}$  so $q = 6 \times 12 + 18 = 90$	M1 A1	Or equivalent or in scalar equations. Accept $\frac{21}{q-18}$ or $\frac{q-18}{21} = \tan(i)$ or $\tan(90 - (i))$ Accept 90j	2
				7
Q 4		mark		Sub
(i)	N2L in direction of motion $D - (100 + 300) = (900 + 700) \times 1.5$  $D = 2800$ so 2800 N	M1 A1 A1	Apply N2L. Allow 1 resistance omitted and sign error but total mass must be used. Condone use of $F = mga$ . No extra forces. All correct cao	3
(ii)	N2L on trailer $T - 300 = 700 \times 1.5$  $T = 1350$ so 1350 N	M1 A1	Use either car or trailer. All forces present. No extras. Correct mass and $a$ Allow sign error. Must use $F = ma$ . cao	2
				5

Q 5		mark		Sub
(i)	$9\mathbf{i} \text{ m s}^{-2}; (9\mathbf{i} - 12\mathbf{j}) \text{ m s}^{-2}$	B1	Award for either. Accept no units. (isw e.g. finding magnitudes)	1
(ii)	N2L $\mathbf{F} = 4(9\mathbf{i} - 12\mathbf{j}) = (36\mathbf{i} - 48\mathbf{j}) \text{ N}$	B1	Accept factored form. isw. FT $\mathbf{a}(3)$ . Accept 60 N or <b>their</b> $4 \mathbf{a} $	1
(iii)	$\mathbf{v} = \int \begin{pmatrix} 9 \\ -4t \end{pmatrix} dt = \begin{pmatrix} 9t + C \\ -2t^2 + D \end{pmatrix}$ <p>Using <math>\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}</math> when <math>t = 1</math></p> $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 + C \\ -2 + D \end{pmatrix}$ <p><math>\Rightarrow C = -5, D = 4</math> so <math>\mathbf{v} = (9t - 5)\mathbf{i} + (4 - 2t^2)\mathbf{j}</math></p>	M1 A1 M1  A1	Integration. At least one term correct. Neglect arbitrary constant(s) Sub at $t = 1$ to find arb const(s)  Any form	4
				6

Q 6		mark		Sub
(i)	$14 = 2u + 0.5a \times 4$ $19 = u + 5a$  Solving gives $u = 4$ and $a = 3$	M1 A1 A1  M1 F1	Use of appropriate <i>uvast</i> for either equn Any form Any form  Attempt at solution of 2 eqns in 2 unknowns. At least one value found . Must have complete correct solution to <b>their</b> eqns. .	5
(ii)	$19^2 = 4^2 + 2 \times 3 \times s$ or $s = 4 \times 5 + 0.5 \times 3 \times 25$ $s = 57.5$ so 57.5 m	M1  A1	Use of appropriate <i>uvast</i> and <b>their</b> $u, a$ & $t = 5$ . cao [Accept 50 if $t = 7$ instead of $t = 5$ in (i) for 2/2]	2
				7



## Section B

Q 7		mark		Sub
(i)	60 N	B1		1
(ii)	$60 + 70 \cos 30 = 120.62\dots$ so 121 N (3 s. f.)	M1 A1	70 cos30 or 70 sin 30 used only with 60N. Accept sign errors. cao. Any reasonable accuracy	2
(iii)	resolve $\uparrow$ $R + 70 \sin 30 - 50g = 0$  $R = 455$ so 455 N	M1 A1 A1	Resolve $\uparrow$ All forces present. No extras. Allow sign errors and $\sin \leftrightarrow \cos$ . All correct. cao	3
(iv)	N2L $\rightarrow$ $160 - 125 = 50a$ $a = 0.7$ so $0.7 \text{ m s}^{-2}$	M1 A1	N2L. No extra forces. Accept 125 N omitted but not use of $F = mga$	2
(v)	N2L $\rightarrow$ $-125 = 50a$ $a = -2.5$ $0 = 1.5^2 + 2 \times -2.5 \times s$  $s = 0.45$ so 0.45 m	M1 A1 M1 A1	N2L to find new accn. Accept +125 but not $F = mga$ . May be implied. Accept +2.5 Appropriate (sequence of) <i>uvast</i> using a new value for acceln. Allow use of $\pm$ <b>their new</b> $a$ cao. Signs must be justified.	4
(vi)	N2L $\rightarrow$ $160 + Q \cos 30 - 115 = 50 \times 3$  $Q = 121.24\dots$ so 121 (3 s. f.)	M1  B1 A1 A1	Use of N2L with cpt of $Q$ attempted. Accept 115 omitted or taken to be 125 and $a$ wrong. Do not allow $F = mga$ . $Q \cos 30$ seen in any equn. All correct cao	4
				16

Q 8		mark		Sub
(i)	$x = 14 \cos 60t$  so $x = 7t$  $y = 14 \sin 60t - 4.9t^2 + 1$   $y = 7\sqrt{3}t - 4.9t^2 + 1$ $(y = 12.124...t - 4.9t^2 + 1)$	M1  A1  M1  A1  A1	Consider motion in $x$ direction. Need not resolve. Allow $\sin \leftrightarrow \cos$ . Condone +1 seen. Need not be simplified. Suitable $uvast$ used for $y$ with $g = \pm 9.8, \pm 10, \pm 9.81$ soi Need not resolve. Allow $\sin \leftrightarrow \cos$ . Allow +1 omitted. Any form and 2 s. f. Need not be simplified All correct. +1 need not be justified. Accept any form and 2 s. f. Need not be simplified.	5
(ii) (A)	time taken to reach highest point $0 = 7\sqrt{3} - 9.8T$   so $\frac{5\sqrt{3}}{7}$ s (1.23717.... = 1.24 s (3 s. f.))	M1    A1	Appropriate $uvast$ . Accept $u = 14$ and $\sin \leftrightarrow \cos$ and $u \leftrightarrow v$ . Require $v = 0$ or equivalent. $g = \pm 9.8, \pm 10, \pm 9.81$ soi.  cao  [If time of flight attempted, do not award M1 if twice interval obtained]	2
(B)	distance from base is $7 \times \frac{5\sqrt{3}}{7} = 5\sqrt{3}$ m (= 8.66025... so 8.66 m (3 s. f.))	M1  B1	Use of <b>their</b> $x = 7t$ with <b>their</b> $T$  FT <b>their</b> $T$ only in $x = 7t$ . Accept values rounding to 8.6 and 8.7.	2
(C)	<b>either</b> Height at this time is $H = 7\sqrt{3} \times \frac{5\sqrt{3}}{7} - 4.9 \times \left(\frac{5\sqrt{3}}{7}\right)^2 + 1$   = 8.5	M1  A1  A1	Subst in <b>their quadratic</b> $y$ with <b>their</b> $T$ .  Correct subst of <b>their</b> $T$ in <b>their</b> $y$ which has attempts at all 3 terms. Do not accept $u = 14$ .	

	<p>clearance is <math>8.5 - 6 = 2.5</math> m</p> <p><b>or</b> for height above pt of projection</p> $0 = (7\sqrt{3})^2 + 2 \times -9.8 \times s$ <p><math>s = 7.5</math> so clearance is <math>7.5 - 5 = 2.5</math> m</p>	<p>E1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>E1</p>	<p>Clearly shown.</p> <p>Appropriate <i>uvast</i> . Accept <math>u = 14</math>. <math>g = \pm 9.8, \pm 10, \pm 9.81</math> so i</p> <p>Attempt at vert cpt accept <math>\sin \leftrightarrow \cos</math> .Accept sign errors but not <math>u = 14</math>.</p> <p>Clearly shown.</p>	4
(iii)	See over			

Q 8	continued	mark		su b
(iii)	<p>Elim <math>t</math> between <math>y = 7\sqrt{3}t - 4.9t^2 + 1</math> and <math>x = 7t</math></p> <p>so <math>y = 7\sqrt{3}\frac{x}{7} - 4.9\left(\frac{x}{7}\right)^2 + 1</math></p> <p>so <math>y = \sqrt{3}x - 0.1x^2 + 1</math></p>	<p>M1</p> <p>F1</p>	<p>Must see <b>their</b> <math>t = x/7</math> fully substituted in <b>their</b> quadratic <math>y</math> (accept bracket errors) Accept any form correctly written. FT <b>their</b> <math>x</math> and 3 term quadratic <math>y</math> (neither using <math>u = 14</math>)</p>	2
(iv)	<p><b>either</b></p> <p>need <math>6 = 7\sqrt{3}t - 4.9t^2 + 1</math></p> <p>so <math>4.9t^2 - 7\sqrt{3}t + 5 = 0</math></p> <p><math>t = \frac{5(\sqrt{3} \pm 1)}{7}</math> (0.52289.... or 1.95146...)</p> <p>moves by <math>\left(\frac{5(\sqrt{3}+1)}{7} - \frac{5\sqrt{3}}{7}\right) \times 7</math></p> <p><math>[(1.95146.. - 1.23717...) \times 7]</math></p> <p><math>= 5</math> m</p> <p><b>or</b></p> <p>using equation of trajectory with <math>y = 6</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p><b>their</b> quadratic <math>y</math> from (i) = 6, or equivalent. Dep. Attempt to solve this 3 term quadratic. (Allow <math>u = 14</math>).</p> <p>for either root</p> <p>Moves by <b>their</b> root - <b>their</b> (ii)(A) <math>\times 7</math> or equivalent. Award this for recognition of correct dist (no calc)</p> <p>cao [If new distance to wall found must have larger of 2 +ve roots for 3<sup>rd</sup> M and award max 4/5 for 13.66]</p>	

	$6 = \sqrt{3}x - 0.1x^2 + 1$ Solving $x^2 - 10\sqrt{3}x + 50 = 0$  $x = 5(\sqrt{3} \pm 1)$ ( 13.660... or 3.6602....) distance is $5(\sqrt{3} + 1) - 5\sqrt{3}$  = 5 m	M1 M1  A1 M1  A1	Equating <b>their</b> quadratic trajectory equn to 6 Dep. Attempt to solve this 3 term quadratic. (Allow $u = 14$ ).  for either root distance is <b>their</b> root – <b>their</b> (ii)(B) Award this for recognition of correct dist (no calc) Cao [If new distance to wall found must have larger of 2 + ve roots for 3 <sup>rd</sup> M and award max 4/5 for 13.66]	5 20
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