OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g $\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Registered Charity 1066969
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Section A (36 marks)

1 A particle travels along a straight line. Its *acceleration* during the time interval $0 \leqslant t \leqslant 8$ is given by the acceleration–time graph in Fig. 1.

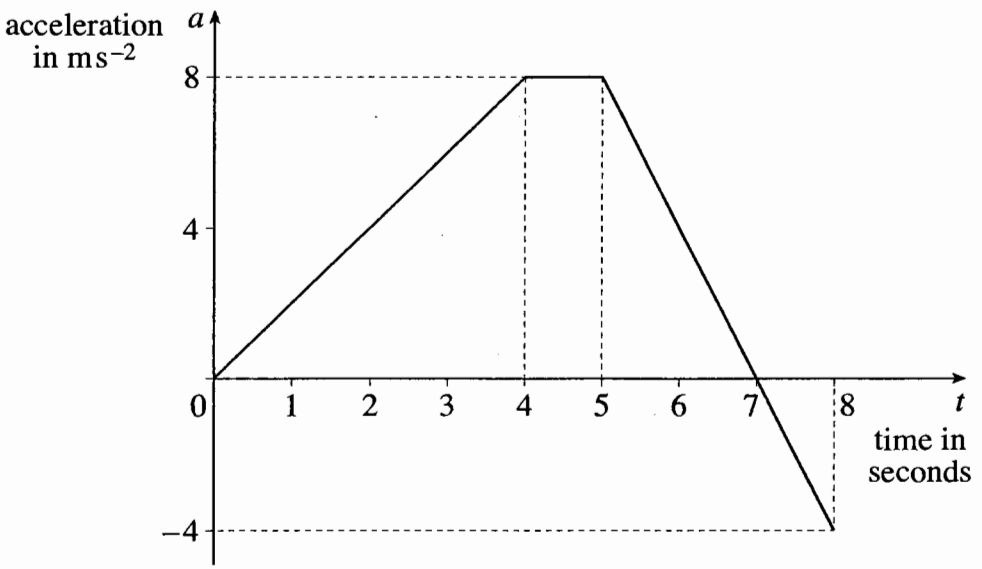


Fig. 1

(i) Write down the acceleration of the particle when $t = 4$. Given that the particle starts from rest, find its speed when $t = 4$. [2]

(ii) Write down an expression in terms of t for the acceleration, a ms^{-2}, of the particle in the time interval $0 \leqslant t \leqslant 4$. [1]

(iii) Without calculation, state the time at which the *speed* of the particle is greatest. Give a reason for your answer. [2]

(iv) Calculate the change in speed of the particle from $t = 5$ to $t = 8$, indicating whether this is an increase or a decrease. [3]

2 A particle moves along the x-axis with velocity, v m s^{-1}, at time t given by

$$v = 24t - 6t^2.$$

The positive direction is in the sense of x increasing.

(i) Find an expression for the acceleration of the particle at time t. [2]

(ii) Find the times, t_1 and t_2, at which the particle has zero speed. [2]

(iii) Find the distance travelled between the times t_1 and t_2. [4]

3 A particle rests on a smooth, horizontal plane. Horizontal unit vectors **i** and **j** lie in this plane. The particle is in equilibrium under the action of the three forces $(-3\mathbf{i} + 4\mathbf{j})$N and $(21\mathbf{i} - 7\mathbf{j})$N and **R**N.

(i) Write down an expression for **R** in terms of **i** and **j**. [2]

(ii) Find the magnitude of **R** and the angle between **R** and the **i** direction. [4]

4 A block of mass 4 kg is in equilibrium on a rough plane inclined at 60° to the horizontal, as shown in Fig. 4. A frictional force of 10 N acts up the plane and a vertical string AB attached to the block is in tension.

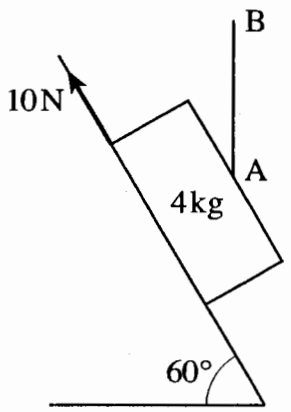


Fig. 4

(i) Draw a diagram showing the four forces acting on the block. [1]

(ii) By considering the components of the forces parallel to the slope, calculate the tension in the string. [3]

(iii) Calculate the normal reaction of the plane on the block. [3]

5 The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where **i** and **j** are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

(iii) Find the coordinates of the point where the particle is moving at 45° to both Ox and Oy. [3]

[Turn over

Section B (36 marks)

6 A car of mass 1000 kg is travelling along a straight, level road.

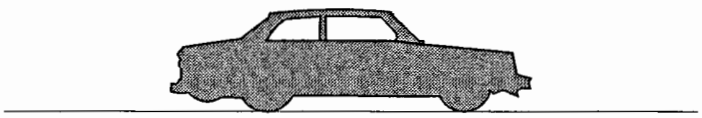


Fig. 6.1

(i) Calculate the acceleration of the car when a resultant force of 2000 N acts on it in the direction of its motion.

How long does it take the car to increase its speed from $5\,m\,s^{-1}$ to $12.5\,m\,s^{-1}$? [3]

The car has an acceleration of $1.4\,m\,s^{-2}$ when there is a driving force of 2000 N.

(ii) Show that the resistance to motion of the car is 600 N. [2]

A trailer is now atached to the car, as shown in Fig. 6.2. The car still has a driving force of 2000 N and resistance to motion of 600 N. The trailer has a mass of 800 kg. The tow-bar connecting the car and the trailer is light and horizontal. The car and trailer are accelerating at $0.7\,m\,s^{-2}$.

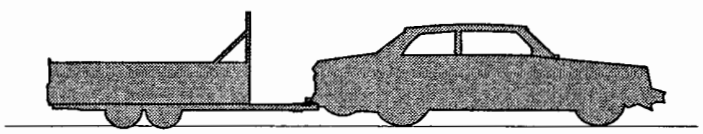


Fig. 6.2

(iii) Show that the resistance to the motion of the trailer is 140 N. [3]

(iv) Calculate the force in the tow-bar. [3]

The driving force is now removed and a braking force of 610 N is applied to the car. All the resistances to motion remain as before. The trailer has no brakes.

(v) Calculate the new acceleration. Calculate also the force in the tow-bar, stating whether it is a tension or a thrust (compression). [6]

7 In this question take the value of g to be $10\,\text{m s}^{-2}$.

A particle A is projected over horizontal ground from a point P which is 9 m above a point O on the ground. The initial velocity has horizontal and vertical components of $10\,\text{m s}^{-1}$ and $12\,\text{m s}^{-1}$ respectively, as shown in Fig. 7. The trajectory of the particle meets the ground at X. Air resistance may be neglected.

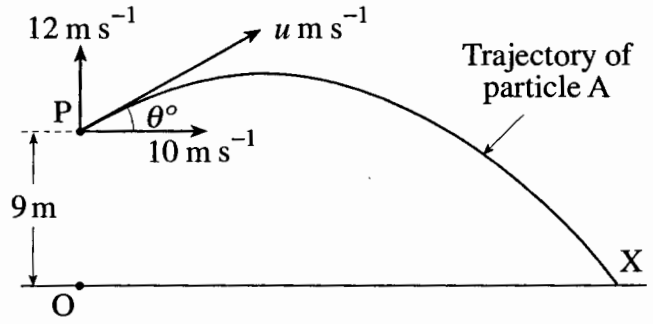


Fig. 7

(i) Calculate the speed of projection $u\,\text{m s}^{-1}$ and the angle of projection $\theta°$. [3]

(ii) Show that, t seconds after projection, the height of particle A above the ground is $9 + 12t - 5t^2$. Write down an expression in terms of t for the horizontal distance of the particle from O at this time. [4]

(iii) Calculate the maximum height of particle A above the point of projection. [2]

(iv) Calculate the distance OX. [4]

A second particle, B, is projected from O with speed $20\,\text{m s}^{-1}$ at 60° to the horizontal. The trajectories of A and B are in the same vertical plane. Particles A and B are projected at the same time.

(v) Show that the horizontal displacements of A and B are always equal. [2]

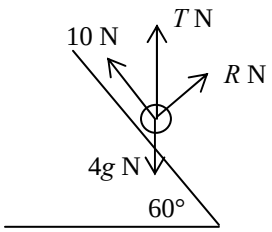
(vi) Show that, t seconds after projection, the height of particle B above the ground is $10\sqrt{3}t - 5t^2$. [1]

(vii) Show that the particles collide 1.7 seconds after projection (correct to two significant figures). [3]

Q 1		**mark**		**Sub**
(i)	Acceleration is 8 m s^{-2} speed is $0 + 0.5 \times 4 \times 8 = 16$ m s^{-1}	B1 B1		2
(ii)	$a = 2t$	B1		1
(iii)	$t = 7$ $a > 0$ for $t < 7$ and $a < 0$ for $t > 7$	B1 E1	Full reason required	2
(iv)	Area under graph $0.5 \times 2 \times 8 - 0.5 \times 1 \times 4 = 6$ so 6 m s^{-1} Increase	M1 B1 E1	Both areas under graph attempted. Accept both positive areas. If 2×3 seen accept ONLY IF reference to average accn has been made. Award for $v = -2t^2 + 28t + c$ seen or 24 and 30 seen Award if 6 seen. Accept '24 to 30'. This must be clear. Mark dept. on award of M1	3
	total	8		

Q 2		**mark**		**Sub**
(i)	$a = 24 - 12t$	M1 A1	Differentiate cao	2
(ii)	Need $24t - 6t^2 = 0$ $t = 0, 4$	M1 A1	Equate $v = 0$ and attempt to factorise (or solve). Award for one root found. Both. cao.	2
(iii)	$s = \int_0^4 \left(24t - 6t^2\right) \mathrm{d}t$ $= \left[12t^2 - 2t^3\right]_0^4$ $(12 \times 16 - 2 \times 64) - 0$ = 64 m	M1 A1 M1 A1	Attempt to integrate. No limits required. Either term correct. No limits required Sub $t = 4$ in integral. Accept no bottom limit substituted or arb const assumed 0. Accept reversed limits. FT **their** limits. cao. Award if seen. [If trapezium rule used. M1 At least 4 strips: M1 enough strips for 3 s. f. A1 (dep on 2nd M1) One strip area correct: A1 cao]	4
	total	8		

Q 3		**mark**		**Sub**
(i)	$\mathbf{R} + \begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}21\\-7\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$	M1	Sum to zero	
	$\mathbf{R} = \begin{pmatrix}-18\\3\end{pmatrix}$	A1	Award if seen here or in (ii) or used in (ii). [SC1for $\begin{pmatrix}18\\-3\end{pmatrix}$]	2
(ii)	$\lvert\ \mathbf{R}\ \rvert = \sqrt{18^2+3^2}$	M1	Use of Pythagoras	
	= 18.248… so 18.2 N (3 s. f.)	A1	Any reasonable accuracy. FT **R** (with 2 non-zero cpts)	
	angle is $180 - \arctan\left(\frac{3}{18}\right) = 170.53…°$	M1	Allow $\arctan\left(\frac{\pm 3}{\pm 18}\right)$ or $\arctan\left(\frac{\pm 18}{\pm 3}\right)$	
	so 171° (3 s. f.)	A1	Any reasonable accuracy. FT **R** provided their angle is obtuse but not 180°	4
	total	6		

Q 4		**mark**		**Sub**
(i)		B1	All forces present. No extras. Accept *mg*, *w* etc. All labelled with arrows. Accept resolved parts only if clearly additional. Accept no angles	1
(ii)	Resolve parallel to the plane $10 + T\cos 30 = 4g\cos 30$	M1	All terms present. Must be resolution in at least 1 term. Accept $\sin \leftrightarrow \cos$. If resolution in another direction there must be an equation only in T with no forces omitted. No extra forces.	
		A1	All correct	
	T = 27.65299… so 27.7 N (3 s. f.)	A1	Any reasonable accuracy	3
(iii)	Resolve perpendicular to the plane $R + 0.5\,T = 2g$	M1	At least one resolution correct . Accept resolution horiz or vert if at least 1 resolution correct. All forces present. No extra forces.	
		A1	Correct. FT T if evaluated.	
	R = 5.7735… so 5.77 N (3 s. f.)	A1	Any reasonable accuracy. cao.	3
	total	7		

Q 5		mark		Sub
(i)	$x = 2 \Rightarrow t = 4$ $t = 4 \Rightarrow y = 16 - 1 = 15$	B1 F1	cao FT **their** t and y. Accept 15 **j**	2
(ii)	$x = \frac{1}{2}t$ and $y = t^2 - 1$ Eliminating t gives $y = \left((2x)^2 - 1\right) = 4x^2 - 1$	M1 E1	Attempt at elimination of expressions for x and y in terms of t Accept seeing $(2x)^2 - 1 = 4x^2 - 1$	2
(iii)	**either** We require $\frac{dy}{dx} = 1$ so $8x = 1$ $x = \frac{1}{8}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$ **or** Differentiate to find **v** equate **i** and **j** cpts so $t = \frac{1}{4}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	 M1 B1 A1 M1 M1 A1	 This may be implied Differentiating correctly to obtain $8x$ Equating the **i** and **j** cpts of **their v**	3
	total	7		

Q 6		mark		Sub
(i)	$2000 = 1000a$ so $a = 2$ so 2 m s^{-2}	B1		
		M1	Use of appropriate *uvast* for t	
	$12.5 = 5 + 2t$ so $t = 3.75$ so 3.75 s	A1	cao	3
(ii)	$2000 - R = 1000 \times 1.4$	M1	N2L. Accept $F = mga$. Accept sign errors. Both forces present. Must use $a = 1.4$	
	$R = 600$ so 600 N (AG)	E1		2
(iii)	$2000 - 600 - S = 1800 \times 0.7$	M1	N2L overall or 2 paired equations. $F = ma$ and use 0.7. Mass must be correct. Allow sign errors and 600 omitted.	
		A1	All correct	
	$S = 140$ so 140 N (AG)	E1	Clearly shown	3
(iv)	$T - 140 = 800 \times 0.7$	M1	N2L on trailer (or car). $F = 800a$ (or $1000a$). Condone missing resistance otherwise all forces present. Condone sign errors.	
		B1	Use of 140 (or 2000 – 600) and 0.7	
	$T = 700$ so 700 N	A1		3
(v)	N2L in direction of motion car and trailer			
	$-600 - 140 - 610 = 1800a$	M1	Use of $F = 1800a$ to find new accn. Condone 2000 included but not T. Allow missing forces.	
		A1	All forces present; no extra ones Allow sign errors.	
	$a = -0.75$	A1	Accept $\pm$. cao.	
	For trailer $T - 140 = -0.75 \times 800$	M1	N2Lwith their a ($\neq 0.7$) on trailer or car. Must have correct mass and forces. Accept sign errors	
	so $T = -460$ so 460	A1	cao. Accept ± 460	
	thrust	F1	Dep on M1. Take tension as +ve unless clear other convention	6
	total	17		

Q 7		mark		Sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$ $\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3s.f.)	B1 M1 A1	Accept any accuracy 2 s. f. or better Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** $15.62\cos\theta = 10$ or **their** $15.62\sin\theta = 12$) [FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u\cos\theta = 10$ and $u\sin\theta = 12$ seen]	3
(ii)	vert $12t - 0.5\times10t^2 + 9$ $= 12t - 5t^2 + 9$ (AG) horiz $10t$	M1 A1 E1 B1	Use of $s = ut + 0.5at^2$, $a = \pm9.8$ or ±10 and u = 12 or 15.62.. Condone $-9 = 12t - 0.5\times10t^2$, condone $y = 9 + 12t - 0.5\times10t^2$. Condone g. All correct with origin of u = 12 clear; accept 9 omitted Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	4
(iii)	$0 = 12^2 - 20s$ s = 7.2 so 7.2 m	M1 A1	Use of $v^2 = u^2 + 2as$ or equiv with u = 12, v = 0. Condone $u \leftrightarrow v$ From CWO. Accept 16.2.	2
(iv)	We require $0 = 12t - 5t^2 + 9$ Solve for t the + ve root is 3 range is 30 m	M1 M1 A1 F1	Use of y equated to 0 Attempt to solve a 3 term quadratic Accept no reference to other root. cao. FT root and **their** x. [If range split up M1 all parts considered; M1 valid method for each part; A1 final phase correct; A1]	4
(v)	Horiz displacement of B: $20\cos 60t = 10t$ Comparison with Horiz displacement of A	B1 E1	Condone unsimplified expression. Award for $20\cos60 = 10$ Comparison clear, must show $10t$ for each or explain.	2
(vi)	vertical height is $20\sin 60t - 0.5\times10t^2 = 10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and $20\sin60 = 10\sqrt{3}$ not explained.	1
(vii)	Need $10\sqrt{3}t - 5t^2 = 12t - 5t^2 + 9$ $\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$ t = 1.6915… so 1.7 s (2 s. f.) (AG)	M1 A1 E1	Equating the **given** expressions Expression for t obtained in any form Clearly shown. Accept 3 s. f. or better as evidence. Award M1 A1 E0 for 1.7 sub in each ht	3
	total	19		